SSNV219 – Method of the solutions manufactured in contact 3D and great deformations

Summary:

The objective of this test is to check the modeling of the contact 3D in great deformations thanks to the method of the manufactured solutions [bib1].

1 Problem of reference

1.1 Geometry

One considers a cube of with dimensions $1\,m$.

1.2 Properties of material

$E=1\mathrm{MPa}$	Young modulus
$\nu=0.15$	Poisson's ratio

1.3 Boundary conditions and loadings

On the edge HAUT , one forces a displacement (see paragraph 2).
On the edges BORDX , BORDMX , BORDY , BORDMY , BORDZ and ESCLAVE , one forces a pressure (see paragraph 2).
In all the field, one forces a force of volume (see paragraph 2).
Surface MAITRE of natural paraboloid is described by the equation:

$$Z=-0.2\times(1.+X^2+Y^2)-0.3 \qquad (1)$$

1.4 Initial conditions

Nothing

2 Reference solution

2.1 Method of calculating

The analytical reference solution is given by:

$$\begin{array}{lll} Ux & = & 0.2\times Z^2\times X\times Y \\ Uy & = & 0.2\times Z^2\times X\times Y \\ Uz & = & -0.2\times(1.+X^2+Y^2)\times(1+0.01\times Z)-0.01*Z-0.3 \end{array} \qquad (2)$$

The conditions of Dirichlet, Neumann and the source term are obtained by the method of the manufactured solutions [bib1].
One starts by determining the gradient of the transformation $\underline{\underline{F}}$:

$$\underline{\underline{F}}=\nabla\underline{U}+\underline{\underline{Id}} \qquad (3)$$

Knowing the normal $\underline{N}=[0,0,-1]^T$ on surface $\mathrm{ESCLAVE}$ in the configuration not-deformation, one obtains his expression in the configuration deformed by the formula of Nanson:

$$\underline{n}=\frac{\underline{\underline{F}}^{-T}\underline{N}}{\|\underline{\underline{F}}^{-T}\underline{N}\|} \quad (4)$$

Knowing the tensor of Hooke $\underline{\underline{\underline{\underline{A}}}}$ and the tensor of Green-Lagrange $\underline{\underline{E}}$, one calculates the second tensor of Piola-Kirchhoff $\underline{\underline{S}}$:

$$\underline{\underline{E}}=\frac{1}{2}(\underline{\underline{F}}^T\cdot\underline{\underline{F}}-\underline{\underline{Id}}) \quad (5)$$

$$\underline{\underline{S}}=\underline{\underline{\underline{\underline{A}}}}:\underline{\underline{E}} \quad (6)$$

It is pointed out that the second tensor of Piola-Kirchhoff $\underline{\underline{S}}$ allows to obtain efforts in configuration not deformed per not deformed unit of area:

$$\frac{d\,\underline{f}_0}{dA}=\underline{\underline{S}}\cdot\underline{N} \quad (7)$$

As we seek to determine efforts in deformed configuration, we will determine the first tensor of Piola-Kirchhoff $\underline{\underline{\Pi}}$

$$\underline{\underline{\Pi}}=\underline{\underline{F}}\cdot\underline{\underline{S}} \quad (8)$$

One can thus determine the forces of volume $\underline{f}_{vol}$:

$$\underline{f}_{vol}=\text{-}div\,\underline{\underline{\Pi}} \quad (9)$$

Knowing the normal in initial configuration on the various faces and the first tensor of Piola-Kirchhoff $\underline{\underline{\Pi}}$, one can calculate the efforts of surface in deformed configuration:

$$\underline{f}_{surf}=\underline{\underline{\Pi}}\cdot\underline{N} \quad (10)$$

On surface BAS who is in contact, one needs a particular treatment. Indeed, the normal efforts are taken there into account by the contact:

$$\begin{aligned}\underline{f}_{surf}^{BAS} &= \underline{f}_{surf_n}^{BAS} + \underline{f}_{surf_t}^{BAS}\\ &= \underline{f}_{contact} + \underline{f}_{surf_t}^{BAS}\\ &= p*\underline{n} + \underline{f}_{surf_t}^{BAS}\end{aligned} \quad (11)$$

Where p indicate the contact pressure. It can be given by the expression:

$$p=(\underline{\underline{\Pi}}\cdot\underline{N})\cdot\underline{n} \quad (12)$$

One thus should apply only the tangential stresses to it. One calculates them by the expression:

$$\begin{aligned}\underline{f}_{surf_t}^{BAS} &= \underline{f}_{surf}^{BAS}-\underline{f}_{surf_n}^{BAS}\\ &= \underline{f}_{surf}^{BAS}-(\underline{f}_{surf_n}^{BAS}\cdot\underline{n})\,\underline{n}\end{aligned} \quad (13)$$

Concerning the efforts of contact, *it is absolutely essential to build the manufactured solution so that they check the equations of the contact* [bib2], namely:

$$\begin{aligned} gap(\underline{U}) &\geqslant 0 \\ p &\leqslant 0 \\ p \cdot gap(\underline{U}) &= 0 \end{aligned} \quad (14)$$

This checking is done after having calculated in an analytical way the pressure and the jump of displacement associated with the manufactured solution, in general with a formal computational tool (in fact, it acts of the module Python *sympy*). One must then visualize them, in order to check *retrospectively* that the solution which one has contruite checks well (14). In the case of this test, we represented pressure and analytical jump of displacement in figures 2.1-1 and 2.1-2. It is noticed that they check $p<0$ and $\mathrm{gap}(\underline{U})=0$, which is characteristic of a surface entirely contacting, and conforms to (14).

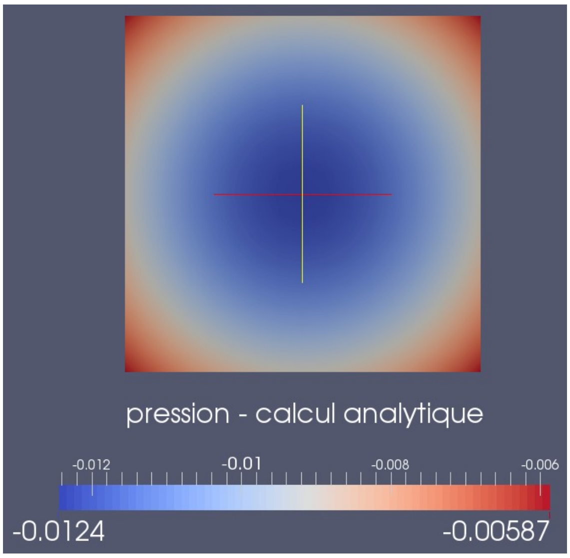

Figure 2.1-1: Validity of the manufactured solution: pressure p

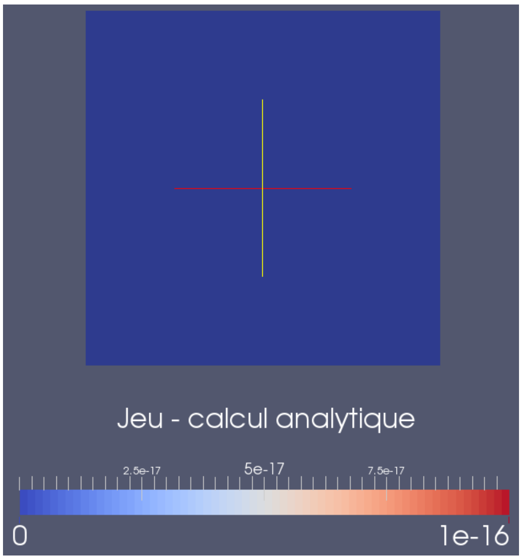

Figure 2.1-2: Validity of the manufactured solution: game Gap (U)

2.2 Sizes and results of reference

The value of the difference between solutions analytical and calculated on the grid: $\sum_{\text{noeuds } n} |\underline{U}_n^{\text{calc}} - \underline{U}_n^{\text{ref}}|$ and $\sum_{\text{noeuds } n} |p_n^{\text{calc}} - p_n^{\text{ref}}|$.

In the case of modelings which carry out an analysis of convergence with the smoothness of the grid, the speed of convergence with the smoothness of the grid of the solution calculated towards the analytical solution in standard L_2 :

– greatest reality $\alpha_U > 0$ such as $\|\underline{U}^{\text{calc}} - \underline{U}^{\text{ref}}\|_{0,\Omega} < C_U \times h^{\alpha_U}$ where C_U is independent of h for displacement;

– greatest reality $\alpha_p > 0$ such as $\|p^{\text{calc}} - p^{\text{ref}}\|_{0,\Gamma_c} < C_p \times h^{\alpha_p}$ where C_p is independent of h for the contact pressure.

2.3 Uncertainties on the solution

None

2.4 Bibliographical references

[1] U2.08.08 document, Use of the Method of the Solutions Manufactured for the software validation, Documentation U2 de Code_Aster

[2] R5.03.50 document, discrete Formulation of contact-friction, Documentation R of Code_Aster

3 Modeling A

3.1 Characteristics of modeling

A modeling is used `3D` and method `CONTINUOUS` of treatment of the contact.

3.2 Characteristics of the grid

The grid contains 1 element of the type `QUAD8`, 768 elements of the type `TRIA6` and 3072 elements of the type `TETRA10`.

3.3 Sizes tested and results

One tests the sum of the absolute values of the difference between the calculated solution and the analytical solution.

Identification	Type of reference	Value of reference
$\sum^{\text{noeuds } n} \lvert \underline{U}_{\text{n}}^{\text{calc}} - \underline{U}_{\text{n}}^{\text{ref}} \rvert$	'NON_REGRESSION'	0.0410211809958

4 Modeling B

4.1 Characteristics of modeling

A modeling is used `3D` and method `CONTINUOUS` of treatment of the contact.

4.2 Characteristics of the grid

One carries out a study of convergence with the smoothness of the grid of the solution calculated towards the analytical solution. A succession of grids obtained by uniform refinement using the order `MACR_ADAP_MAIL` is used:

– e-mailold 0: 1 element of the type `QUAD8`, 12 elements of the type `TRIA6` and 6 elements of the type `TETRA10`
– grid 1: 1 element of the type `QUAD8`, 48 elements of the type `TRIA6` and 48 elements of the type `TETRA10`
– grid 2: 1 element of the type `QUAD8`, 192 elements of the type `TRIA6` and 384 elements of the type `TETRA10`
– grid 3: 1 element of the type `QUAD8`, 768 elements of the type `TRIA6` and 3072 elements of the type `TETRA10`

4.3 Sizes tested and results

One tests the speed of convergence with the smoothness of the grid of the solution calculated towards the analytical solution in standard L_2 :

– greatest reality $\alpha_U > 0$ such as $\lVert \underline{U}^{\text{calc}} - \underline{U}^{\text{ref}} \rVert_{0,\Omega} < C_U \times h^{\alpha_U}$ where C_U is independent of h for displacement;
– greatest reality $\alpha_p > 0$ such as $\lVert p^{\text{calc}} - p^{\text{ref}} \rVert_{0,\Gamma_c} < C_p \times h^{\alpha_p}$ where C_p is independent of h for the contact pressure.

One tests also the sum of the absolute values of the difference between the calculated solution and the analytical solution for displacement.

Identification	Type of reference	Value of reference
$\sum^{\text{noeuds } n} \lvert \underline{U}_n^{\text{calc}} - \underline{U}_n^{\text{ref}} \rvert$	'NON_REGRESSION'	4.13935026178E-05
α_U	'ANALYTICAL'	3.0
α_p	'NON_REGRESSION'	2.534025066720
α_p	'ANALYTICAL'	2.5

5 Modeling C

5.1 Characteristics of modeling

A modeling is used 3D and method DISCRETE gradient combined project (GCP) of treatment of the contact.

5.2 Characteristics of the grid

One carries out a study of convergence with the smoothness of the grid of the solution calculated towards the analytical solution. A succession of grids obtained by uniform refinement using the order MACR_ADAP_MAIL is used:

- e-mailold 0: 14 elements of the type TRIA6 and 6 elements of the type TETRA10
- grid 1: 50 elements of the type TRIA6 and 48 elements of the type TETRA10
- grid 2: 194 elements of the type TRIA6 and 384 elements of the type TETRA10
- grid 3: 770 elements of the type TRIA6 and 3072 elements of the type TETRA10

It is noted that, compared to modelings A and B, the base is with a grid with 2 TRIA6 instead of one QUAD8. Indeed, the discrete methods are not adapted to the use of these elements (see [R5.03.50]).

5.3 Sizes tested and results

One tests the speed of convergence with the smoothness of the grid of the solution calculated towards the analytical solution in standard L_2 :

- greatest reality $\alpha_U > 0$ such as $\lVert \underline{U}^{\text{calc}} - \underline{U}^{\text{ref}} \rVert_{0,\Omega} < C_U \times h^{\alpha_U}$ where C_U is independent of h for displacement in the field Ω ;
- greatest reality $\alpha_s > 0$ such as $\lVert U^{\text{calc}} - U^{\text{ref}} \rVert_{0,\Gamma_C} < C_s \times h^{\alpha_s}$ where C_s is independent of h for displacement on surface Γ_C .

One tests also the sum of the absolute values of the difference between the calculated solution and the analytical solution for displacement.

Identification	Type of reference	Value of reference
$\sum^{\text{noeuds } n} \lvert \underline{U}_n^{\text{calc}} - \underline{U}_n^{\text{ref}} \rvert$	'NON_REGRESSION'	4.25881911029E-05
α_U	'ANALYTICAL'	3.0
α_p	'ANALYTICAL'	3.5

6 Modeling D

6.1 Characteristics of modeling

A modeling is used 3D and method DISCRETE gradient combined project (GCP) of treatment of the contact.

6.2 Characteristics of the grid

One carries out a study of convergence with the smoothness of the grid of the solution calculated towards the analytical solution. A succession of grids obtained by uniform refinement using the order MACR_ADAP_MAIL is used:

- e-mailold 0: 2 elements of the type TRIA6, 6 elements of the type QUAD9 and 1 element of the type HEXA27
- grid 1: 2 elements of the type TRIA6, 24 elements of the type QUAD9 and 8 element of the type HEXA27
- grid 2: 2 elements of the type TRIA6, 96 elements of the type QUAD9 and 64 element of the type HEXA27
- grid 3: 2 elements of the type TRIA6, 384 elements of the type QUAD9 and 512 element of the type HEXA27

It is noted that, compared to modelings A and B, the base is with a grid with 2 TRIA6 instead of one QUAD8. Indeed, the discrete methods are not adapted to the use of these elements (see [R5.03.50]).

6.3 Sizes tested and results

One tests the sum of the absolute values of the variation enters the calculated solution and the analytical solution for displacement. One does not test the rate of convergence because the element HEXA27 provides the exact solution with a single element.

Identification	Type of reference	Value of reference
$\sum^{\text{noeuds } n} \lvert \underline{U}_{\text{n}}^{\text{calc}} - \underline{U}_{\text{n}}^{\text{ref}} \rvert$	'NON_REGRESSION'	0

7 Summary of the results

The results are in very good agreement with the theory.