SSNV221 – Hydrostatic test with a behavior DRUCK_PRAGER linear and parabolic

Summary:

The case test proposes a purely hydrostatic loading for the associated law Drücker-Prager [R7.01.16]. The formulation of this plastic law, often used for the grounds, is made at the same time on the deviatoric and hydrostatic part; nevertheless, surface criterion presents a singularity for a purely hydrostatic state of stress. This analytical CAS-test is used to check correct work hardening in this singularity.

The test is carried out on a material point with the order SIMU_POINT_MAT. One works with imposed deformations.

One makes a test with linear work hardening (modeling A) and another with parabolic work hardening (modeling B).

1 Problem of reference

1.1 Properties of material

Rubber bands :

$E=3000\ MPa$ Young modulus
$\nu=0{,}25$ Poisson's ratio

`DRUCK_PRAGER` linear (modeling A):

$\alpha=0{,}20$ Coefficient of dependence in pressure
$p_{ultm}=0{,}04$ Ultimate cumulated plastic deformation
$\sigma_Y=6\ MPa$ Plastic constraint
$h=100\ MPa$ Module of work hardening

`DRUCK_PRAGER` parabolic (modeling B):

$\alpha=0{,}20$ Coefficient of dependence in pressure
$p_{ultm}=0{,}04$ Ultimate cumulated plastic deformation
$\sigma_Y=6\ MPa$ Plastic constraint
$\sigma_Y^{ult}=10\ MPa$ Ultimate plastic constraint

1.2 Loadings and boundary conditions

A voluminal deformation is imposed $\varepsilon_v=\mathrm{tr}(\boldsymbol{\varepsilon})$. The loading is not monotonous: one charges initially until the voluminal deformation ε_{v1}, by exceeding the threshold of plasticization, then one discharges on a null level of deformation; then one still charges with the deformation ε_{v2} by thus exceeding the ultimate cumulated plastic deformation p_{ultm}, beyond which one finds a perfect plasticity; one still discharges with worthless constraint (deformation equal to the plastic deformation ε_{v2}^p) and one reloads while plasticizing later on until the deformation ε_{v3}. The time of loading (see Table 1.2-1) is fictitious because the plastic laws are independent of time.

t	ε_v
0	0
10	$\varepsilon_{v1}=0{,}018$
14	0
26	$\varepsilon_{v2}=0{,}045$
30	$\varepsilon_{v2}^p=0{,}03667$
40	$\varepsilon_{v3}=0{,}06$

Table 1.2-1: imposed voluminal deformation.

1.3 Initial conditions

All the components of the constraints and deformations are worthless at the beginning of the loading.

2 Reference solution

Modeling checks the behavior of the law with linear work hardening.

2.1 Method of calculating

The equations which interest us for analytical calculation are ($I_1 = \mathrm{tr}(\boldsymbol{\sigma})$: trace of the tensor of the constraints, ε_v^p : voluminal plastic deformation):

- plastic constitutive law on the voluminal part:

$$I_1 = 3\mathrm{K}(\varepsilon_v - \varepsilon_v^p) \quad \textbf{(éq 2.1-1)}$$

- surface criterion, by posing worthless the constraint of Von Mises ($\sigma_{eq} = 0$) :

$$F(\boldsymbol{\sigma}, p) = \alpha I_1 - R(p) \quad \textbf{(éq 2.1-2)}$$

- relation between the voluminal plastic deformation and the cumulated plastic deformation (variable intern of the plastic law):

$$\dot{\varepsilon}_v^p = 3\alpha \dot{p} \quad \text{thus while integrating: } \varepsilon_v^p = 3\alpha p \quad \textbf{(éq 2.1-3)}$$

- expression of work hardening
  - linear:

$$\begin{aligned} R(p) &= \sigma_Y + h\,p & si\ p \leqslant p_{ult} \\ R(p) &= \sigma_Y + h\,p_{ult} = \sigma_Y^{ult} & si\ p > p_{ult} \end{aligned} \quad \textbf{(éq 2.1-4)}$$

  - parabolic:

$$\begin{aligned} R(p) &= \sigma_Y \left(1 - \left(1 - \sqrt{\frac{\sigma_Y^{ult}}{\sigma_Y}}\right) \frac{p}{p_{ult}}\right)^2 & si\ p \leqslant p_{ult} \\ R(p) &= \sigma_Y^{ult} & si\ p > p_{ult} \end{aligned} \quad \textbf{(éq 2.1-5)}$$

It is observed that, as in the linear case, $R(p) = \sigma_Y$ if $p = 0$ and there is perfect plasticity if $p > p_{ult}$.

2.1.1 Deformation in extreme cases elastic initial

This deformation is obtained for $\varepsilon_v^p = p = 0$.

If one poses $F(\boldsymbol{\sigma}, p) = 0$ (plastic evolution) one a:

$$I_1^{el} = \frac{R(p)}{\alpha} = \frac{\sigma_Y}{\alpha}$$

$$\varepsilon_v^{el} = \frac{I_1^{el}}{3K}$$

2.1.2 Ultimate deformation

Ultimate deformation is called ε_v^{ult} that obtained for $p = p_{ult}$.

The trace of constraints easily is found I_1^{ult} and plastic deformation ε_v^{pult} corresponding:

$$I_1^{ult} = \frac{R(p)}{\alpha} = \frac{\sigma_Y^{ult}}{\alpha}$$

$$\varepsilon_v^{pult}=3\,\alpha\,p_{ult}$$
$$\varepsilon_v^{ult}=\frac{I_1^{ult}}{3K}+\varepsilon_v^{pult}$$

2.1.3 Deformation enters the yield stress and the ultimate deformation

One calculates initially the cumulated plastic deformation.

- By combining the equations (2.1-1), (2.1-2), (2.1-3) and (2.1-4) with $F(\boldsymbol{\sigma},p)=0$ for **linear work hardening** one a:

$$p=\frac{3\,K\,A\,\varepsilon_{vI}-\sigma_Y}{9\,K\,\alpha^2+h} \quad \textbf{(éq 2.1-6)}$$

- By combining the equations (2.1-1), (2.1-2), (2.1-3) and (2.1-5) with $F(\boldsymbol{\sigma},p)=0$ for L'**parabolic work hardening** one arrives at the equation of dismantled 2:

$$A_1\,\bar{p}^2+B_1\,\bar{p}+C_1=0$$

$$\begin{aligned} A_1&=\sigma_Y(1-\gamma)^2\\ B_1&=9\,K\,\alpha^2\,p_{ult}-2\,\sigma_Y(1-\gamma)\\ C_1&=\sigma_Y-3\,K\,\alpha\,\varepsilon_{v1}\\ \gamma&=\sqrt{\frac{\sigma_Y^{ult}}{\sigma_Y}}\\ \bar{p}&=\frac{p}{p_{ult}} \end{aligned} \quad \textbf{(eq 2.1-7)}$$

One uses the equations then (2.1-3) (2.1-1) to find the plastic deformation ε_v^p and traces it constraints I_1.

If one makes discharge elastic material of way until worthless constraint, one finds a residual deformation equal to the plastic deformation; it is on the other hand necessary to charge material in compression to obtain a worthless total deflection. This second branch is also elastic, because the material of Drücker-Prager cannot plasticize in a hydrostatic state of compression. In this last case, the trace of the constraints, negative, is:

$$I_1^c=-3\,K\,\varepsilon_v^p \quad \textbf{(éq 2.1-8)}$$

2.1.4 Deformation higher than the ultimate deformation

All the quantities of interest are easily found, because the trace of constraints is known a priori and equal to I_1^{ult}.

$$\varepsilon_v^p=\varepsilon_v-\frac{I_1^{ult}}{3\,K}$$

$$p=\frac{\varepsilon_v^p}{3\alpha}$$

2.2 Sizes and results of reference

The module of compressibility K is:

$$K=\frac{E}{3(1-2\nu)}=2000\ MPa$$

2.2.1 Deformation in extreme cases elastic

For two modelings one finds easily:

$$I_1^{el}=30\ MPa$$
$$\varepsilon_v^{el}=0{,}005$$

2.2.2 Ultimate deformation

For two modelings one finds:

$$I_1^{ult}=50\ MPa$$
$$\varepsilon_v^{pult}=0{,}024$$
$$\varepsilon_v^{ult}\approx 0{,}03233$$

2.2.3 Deformation equalizes to 0.018 and discharges with worthless deformation

This value of deformation $\varepsilon_{v1}=0{,}018$ is higher than the yield stress ε_v^{el} and lower than ε_v^{ult}. One calculates initially the plastic deformation cumulated with the equations (2.1-7) and (2.1-8), then plastic deformation and the trace of the constraints:

- linear work hardening:

$$p_1=\frac{3K\,A\,\varepsilon_{v1}-\sigma_Y}{9K\alpha^2+h}\approx 0{,}019$$
$$\varepsilon_{v1}^p=3\,\alpha\,p_1=0{,}0114$$
$$I_1^1=3\,K(\varepsilon_{v1}-\varepsilon_{v1}^p)\approx 39{,}51\ MPa$$

- parabolic work hardening:

$$p_1\approx 0.0192$$
$$\varepsilon_{v1}^p=3\,\alpha\,p_1\approx 0.0115$$
$$I_1^1=3\,K(\varepsilon_{v1}-\varepsilon_{v1}^p)\approx 38.956\ MPa$$

The trace of the constraints with worthless deformation is:

- linear work hardening:

$$I_1^{1c}=-3\,K\,\varepsilon_{v1}^p\approx -68{,}49\ MPa$$

- parabolic work hardening:

$$I_1^{1c}=-3\,K\,\varepsilon_{v1}^p\approx -69{,}044\ MPa$$

Indeed, the difference between the parabolic and linear case is very weak.

2.2.4 Loading until deformation EGA to 0.045 and 0.06

One reloads material up to the values of deformation $\varepsilon_{v2}=0{,}045$ and $\varepsilon_{v3}=0{,}06$, higher than ε_v^{ult}.

The results are the same ones for two modelings.

For $\varepsilon_{v2}=0{,}045$:

$$\varepsilon_{v2}^{p}=\varepsilon_{v2}-\frac{I_1^{ult}}{3K}\approx 0{,}03667$$

$$p_2=\frac{\varepsilon_{v2}^{p}}{3\alpha}\approx 0{,}0611$$

Following the elastic discharge (until worthless constraint), one finds $\varepsilon_v=\varepsilon_{v2}^{p}$, $p=p_2$.

For $\varepsilon_{v3}=0{,}06$:

$$\varepsilon_{v3}^{p}=\varepsilon_{v3}-\frac{I_1^{ult}}{3K}\approx 0{,}051667$$

$$p_3=\frac{\varepsilon_{v3}^{p}}{3\alpha}\approx 0{,}0861$$

2.2.5 Stress-strain curves

In the Figures (2.2.5-a) and (2.2.5-b) one represents the curve (ε_v, I_1) for linear and parabolic work hardening. In red are the points tested by the CAS-test.

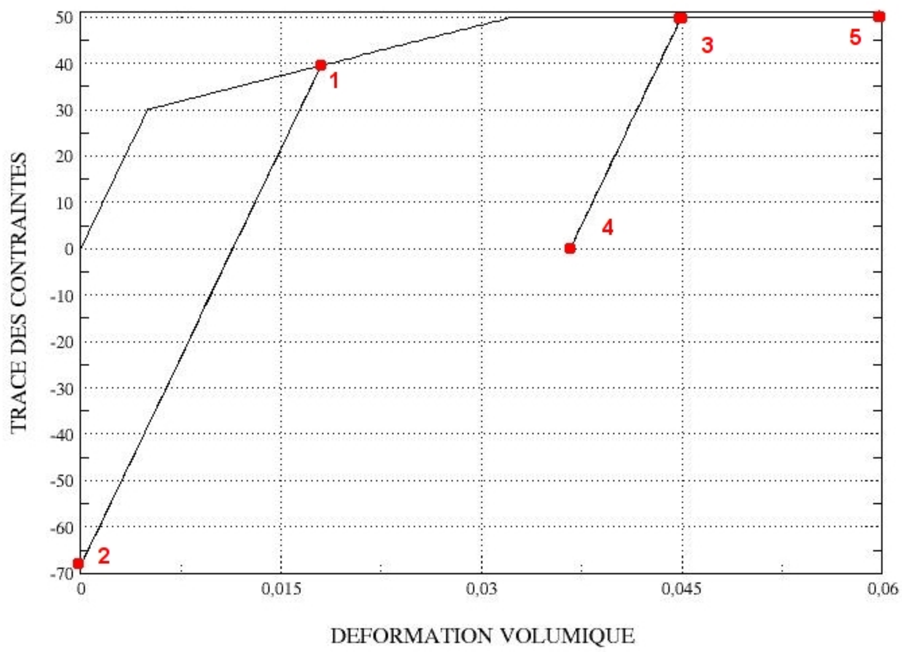

Figure 2.2.5-a: stress-strain curves for linear work hardening.

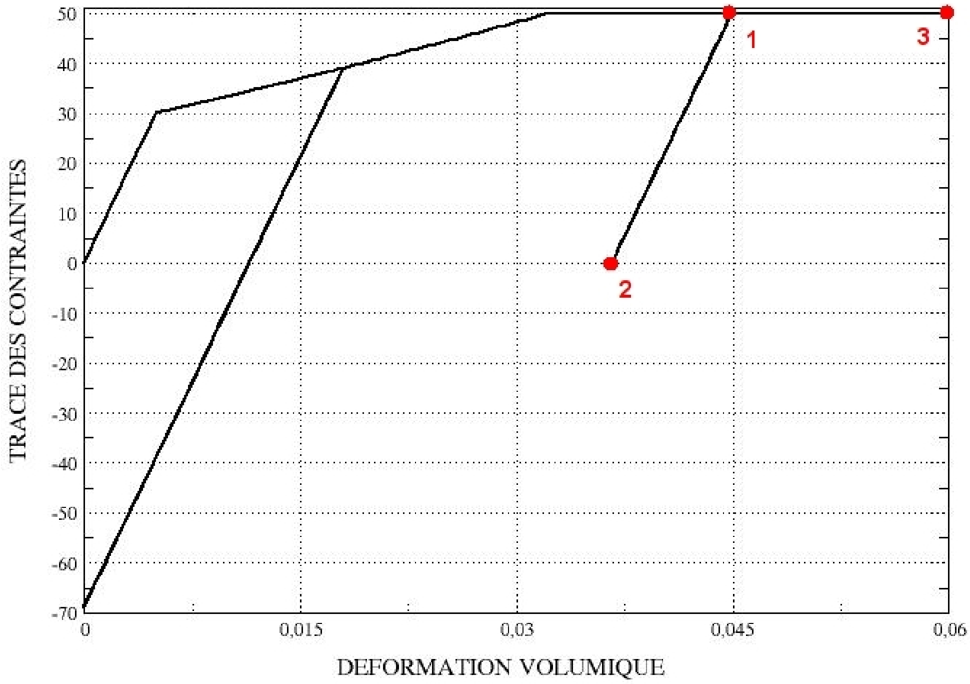

Figure 2.2.5-b: stress-strain curves for parabolic work hardening.

2.3 Uncertainties on the solution

The solution is analytical.

2.4 Bibliographical references

[1] Document [R3.01.16], Integration of the elastoplastic mechanical behavior of Drücker-Prager DRUCK_PRAGER and postprocessings. Handbook of Code_Aster reference.

3 Modeling A

3.1 Characteristics of modeling

The test is carried out on a material point with the order `SIMU_POINT_MAT` . One works with imposed deformations.
Work hardening is linear.

3.2 Sizes and results of reference

Not on Figure 2.2.5-a	Checked quantity	Value of reference	Type of reference	Tolerance (relative)
1	Trace of the constraints	$I_1^1=39,51\ MPa$	ANALYTICAL	10^{-6} %
2	Trace of the constraints	$I_1^{1c}=-68,49\ MPa$	ANALYTICAL	10^{-6} %
3 or 4	Spherical part of the plastic deformation	$\varepsilon_{v2}^p=0,03667$	ANALYTICAL	10^{-6} %
3 or 5	Trace of constraints	$I_1^{ult}=50\ MPa$	ANALYTICAL	10^{-6} %
5	Spherical part of the plastic deformation	$\varepsilon_{v3}^p=0,051667$	ANALYTICAL	10^{-6} %

4 Modeling B

4.1 Characteristics of modeling

The test is carried out on a material point with the order `SIMU_POINT_MAT` . One works with imposed deformations.
Work hardening is parabolic.

4.2 Sizes and results of reference

Not on Figure 2.2.5-b	Checked quantity	Value of reference	Type of reference	Tolerance (relative)
1 or 2	Spherical part of the plastic deformation	$\varepsilon_{v2}^{p}=0{,}03667$	ANALYTICAL	10^{-6} %
1 or 3	Trace of constraints	$I_1^{ult}=50\ MPa$	ANALYTICAL	10^{-6} %
3	Spherical part of the plastic deformation	$\varepsilon_{v3}^{p}=0{,}051667$	ANALYTICAL	10^{-6} %

5 Summary of the results

The results of the CAS-test are satisfactory, *Code_Aster* reproduced the analytical results with a high precision.