

## FORMA04 - Practical works of the formation “advanced Use”: contact of Hertz

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### Summary:

This milked TP of the contact of Hertz, one of the rare noncommonplace cases of contact for which one has an analytical solution.

The model consists of two half-spheres which one comes to put in contact one against the other.

This simple test will make it possible to illustrate the setting in data of the contact, the various manners of solving the problem and finally the tools at disposal post-to treat a calculation of contact.

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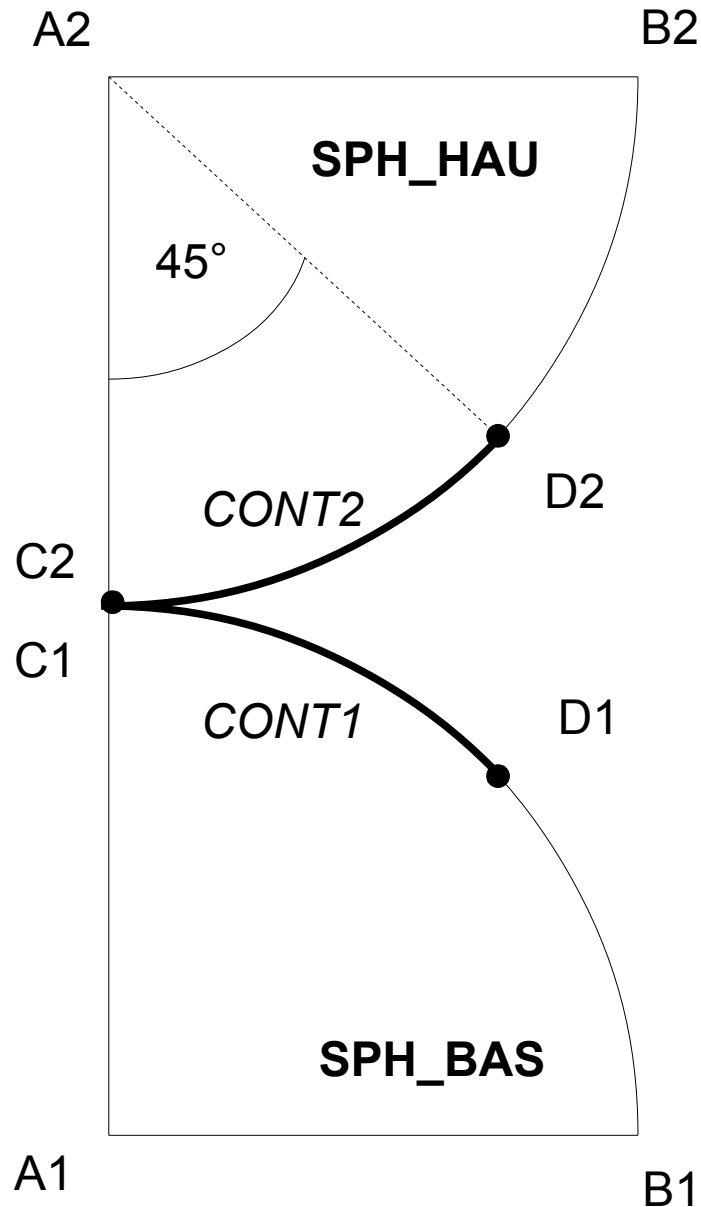
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## 1 Problème de référence

### 1.1 Geometry

The model consists of two half-spheres. One chooses to deal with the problem in **axisymmetric**, only quarters of disc will thus be modelled. The geometrical data are the following ones:

- Ray of the spheres:  $R=50\text{ mm}$



### 1.2 Properties of materials

The material is elastic linear:

- Young modulus:  $E=20000\text{ MPa}$
- Poisson's ratio:  $\nu=0.3$

## 1.3 Boundary conditions and loadings

- Following displacement  $DX$  blocked on the axis  $A1A2$  (this condition is implicit into axisymmetric, one imposes it nevertheless for the points which would not be perfectly on the axis)
- Following displacement  $DY$  imposed:
  - +2 mm on  $A1B1$
  - 2 mm on  $A2B2$

## 2 Reference solution

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### 2.1 Method of calculating

In 1881, Hertz established under certain assumptions a solution with the problem which bears its name[1]. Thus by supposing a contact without friction and for which the half-width of contact  $a$  is very small in front of the ray of the spheres  $R$  ( $a \ll R$ ), the contact pressure at the points  $C1$  and  $C2$  is worth [2] :

$$P_0 = - \frac{E}{\pi(1-\nu^2)} \sqrt{\frac{2h}{R}} \quad \text{éq 2.1-1}$$

where  $h$  corresponds to the imposed crushing, which is worth here  $4 \text{ mm}$  .

The half-width of contact  $a$  express yourself according to imposed crushing and of the ray of the spheres:

$$a = \sqrt{\frac{Rh}{2}} \quad \text{éq 2.1-2}$$

The surface of contact is a disc of ray  $a$ , distribution of pressure according to the ray  $r$  in this zone is the following one:

$$\text{If } r \leq a \text{ then } P(r) = P_0 \sqrt{1 - \left(\frac{r}{a}\right)^2} \quad \text{éq 2.1-3}$$

### 2.2 Sizes and results of reference

One will compare the pressure obtained with the center of the zone of contact and the half-width of contact to the analytical solution above.

### 2.3 Uncertainties on the solution

No (under the assumptions given above).

### 2.4 References

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- 1: HERTZ H, Über die Berührung fester elastischer Körper, 1881
- 2: DUMONT G, La méthode des contraintes actives appliquée au contact unilatéral, 1993

## 3 Modeling A

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### 3.1 Implementation of the TP

#### 3.1.1 Geometry

##### ***To read well the statement in entirety before starting***

The geometry is carried out with Salomé, module GEOM. The tool “ *Sketcher* ” (Finely *New Entity BASIC 2D Sketch*) allows to build **the contour of the higher disc** (for the arc of a circle one can enter an angle and a negative ray). One can then build **a face on this contour** (Finely *New Entity Build Face*). The point  $D2$  will be created during the construction of contour (the arc of a circle is made in 2 times).

By symmetry, one can generate the second sphere Finely ( *Operations Transformation Mirror image*). **It is necessary to build a horizontal line or a plan to carry out symmetry.**

It remains **to assemble the two spheres** to constitute one object GEOM Finely ( *New Entity Build Compound*). Finally it is necessary **to create groups** on this geometry (with the names suggested in the figure above).

**One draws attention to the fact that the groups are created in dernier .**

To create the points  $C1$  and  $C2$  , one uses the functionality allowing to select under element of an entity GEOM (In the dialog box *Create Group*, notch “ *Only Sub-shapes of the Second Shape* ” then for  $C2$  for example, to select like “ *Second Shape* ” the higher half-sphere in the tree of the objects).

To create the groups for the application of the boundary conditions: long DE the axis of axisymetry (  $A1A2$  ) and on the upper parts and lower of the half-spheres (  $A1B1$  and  $A2B2$  ).

For the modeling of the contact, one will need for groups of meshes representing potential surfaces of contact: to create the groups  $CONT1$  and  $CONT2$  with *Create Group*.

#### 3.1.2 Grid

Module SMESH is used. The grid is carried out in triangles. One will use the assumptions of automatic grid “ *Assign has set of assumptions 2D: Automatic triangulation* ” by choosing a maximum length of 2. One then obtains a grid containing approximately 3000 triangles and 1500 nodes.

One will import then the groups starting from the geometry (“ *Create groups from geometry* ”).

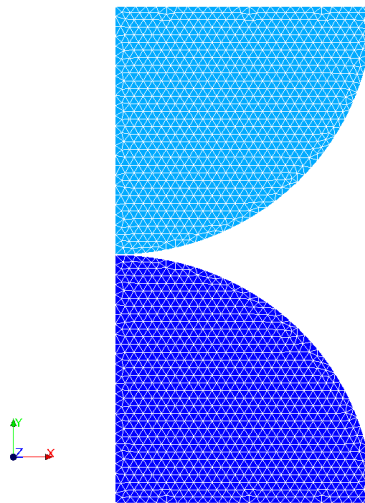


Illustration 3.1: Grid obtained for modeling A

### 3.1.3 Command file

- By using the non-linear operator `STAT_NON_LINE`, to build the command file relating to the CAS-test by not making the contact counts some initially.  
**What does one note on convergence? What would be necessary to make to converge?**

*Index: how the by default convergence criteria is builds?*

- To add the definition of the contact: **to direct surfaces**, to define the load of contact with the operator `DEFI_CONTACT` and to adapt the options of the nonlinear solver.

*An alarm appears during calculation with contact. Once understood, it can be removed by informing the keyword `SYME` of `STAT_NON_LINE` (under `SOLVEUR`). One will preserve the method of contact by default initially (`FORMULATION=' DISCRETE '` and `'FORCED ALGORITHME_CONT=' '`).*

- No-claims bonus question: how many y-a it movements of rigid body in axisymmetric modeling?

*Recall: a rigid movement of body is a movement with worthless deformation.*

### 3.1.4 Pressure of Hertz

- Digital application: to calculate with the hand with the facts of the case, the value of the pressure of Hertz and half-width of contact. To compare with the values obtained by calculation.  
**What does one note? Why this variation?**

*Note: the component at first approximation will be taken `SIYY` of `SIEF_NOEU` like contact pressure*

- To trace the contact pressure according to the ray in the zone of contact, to compare it with the analytical solution.

*The orders will be used `DEFI_GROUP` (`CREA_GROUP_NO / OPTION=' NOEUD_ORDO '`) in order to create a group of nodes with a directed curvilinear X-coordinate, `POST_RELEVE_T` to extract in a table the component `SIYY` along this group (a column of this table called `ABSC_CURV` defines the curvilinear X-coordinate) and finally `RECU_FONCTION` (`TABLE / PARA_X / PARA_Y`) to create*

the function. One will print it with the format `XMGRACE` with `IMPR_FONCTION` (`COURBE/FONCTION`) while selecting a "pilot" `INTERACTIVE`.

## 3.1.5 Constraint of Von Mises

- To display the constraint of Von Mises on the configuration deformed with Salomé. **How does this constraint in the sphere vary? Where is the maximum?**

The component will be displayed `VMIS_SG` field of the equivalent constraints `SIEQ_NOEU` calculated with the operator `CALC_CHAMP`.

## 3.2 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ not <i>CI</i>	'ANALYTICAL'	-2798,3 <i>Mpa</i>	1.0%



## 4 Modeling B

One will set out again of modeling A. the grid is the same one as that of modeling A **but it passed into quadratic.**

### 4.1 Implementation of the TP

#### 4.1.1 Continuous formulation

- To finely convert the grid of modeling A into quadratic grid in module SMESH ( *Modification Convert to/from quadratic*)

*In order to benefit fully from the quadratic elements and to obtain a smooth profile of pressure, it is important to place the nodes mediums on the geometry during conversion (to notch for that the box "Medium nodes one geometry" during conversion).*

- To change method of contact and to use the formulation 'CONTINUES'. **To observe the contact pressure obtained using the degree of freedom LAGS\_C field DEPL.** To compare with the method of contact by default. What does one note? Which of the methods provides the best approximation?

*To obtain a correct profile of pressure, it is necessary to use a diagram of integration of the terms of contact at the points of Gauss (INTEGRATION=' GAUSS' in DEFI\_CONTACT).*  
*In order to remove the alarm which appears with the adjustments by default, it is necessary to impose REAC\_ITER=1 under the keyword factor NEWTON (in continuous formulation, the tangent matrix is inevitably rebuilt with each iteration).*

### 4.2 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
<i>DEPL LAGS<sub>C</sub> not CI</i>	'ANALYTICAL'	<i>-2798,3 Mpa</i>	3.0%

## 5 Modeling C

In this modeling, one carries out a grid in quadrangles, with a refinement near the zone of contact.

### 5.1 Implementation of the TP

#### 5.1.1 Realization of the geometry

In this modeling, one must partitionner the geometry so as to be able to net it in quadrangles. An illustration of the possible partition is given in the figure 5.1.

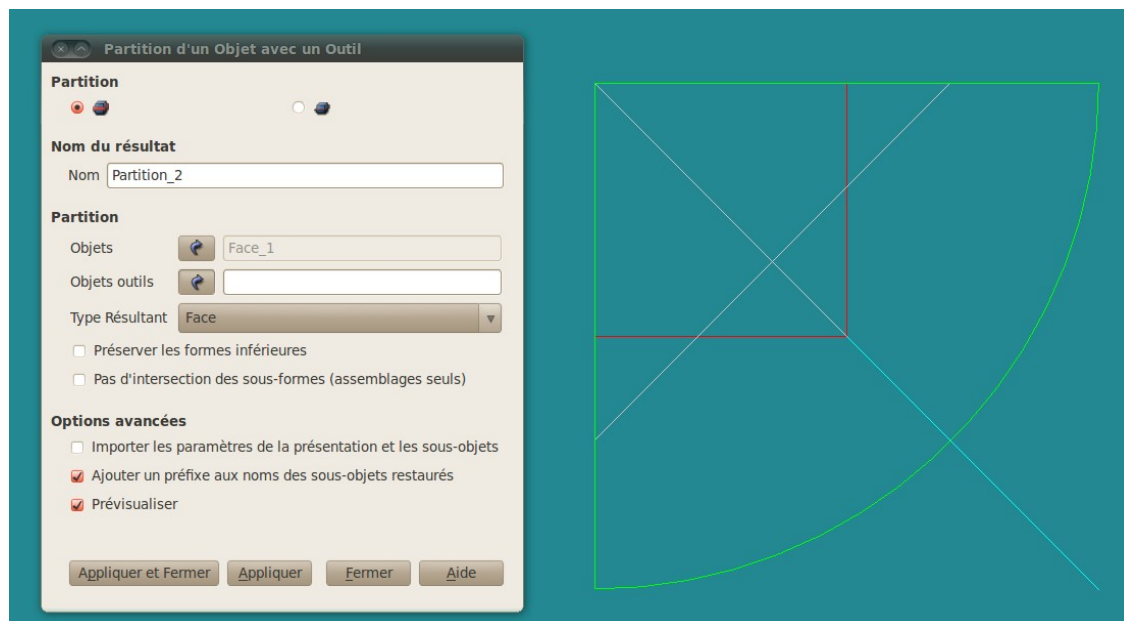


Illustration 5.1: Partition of the higher disc

#### 5.1.2 Realization of the grid

For the grid, one chooses to refine around the zone of contact by using a geometric progression on the circumference and the ray. It can be necessary to reorientate certain segments in the module *SMESH*.

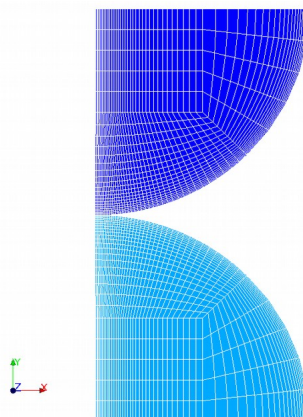


Illustration 5.2: Grid in quadrangles obtained for modeling C

The grid comprises 2200 elements for 2332 nodes.

## 5.2 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\sigma_{yy}$ not <i>CI</i>	'ANALYTICAL'	-2798,3 Mpa	5.0%