SSNV223 - Elementary validation of the law ENDO_SCALAIRE and of piloting PRED_ELAS for modeling GRAD_VARI

Summary:

The purpose of this test is to validate the algorithm of integration of the law of behavior ENDO_SCALAIRE with gradient of internal variables as well as piloting PRED_ELAS available for this law. The studied problem corresponds to a request with imposed homogeneous deformation for which one can obtain an analytical solution.

Various treated modelings are following:

- **Modeling A** ($2D$): Modeling is employed D_PLAN_GRAD_VARI.
- **Modeling B** ($3D$): Modeling is employed 3D_GRAD_VARI.

1 Problem of reference

1.1 Geometry

According to modeling $2D$ or $3D$, one respectively considers a square or a cube on side 2 mm.

1.2 Properties of material

The material obeys the law of elastic behavior fragile ENDO_SCALAIRE with gradient of damage (D_PLAN_GRAD_VARI and 3D_GRAD_VARI). The macroscopic data correspond to:

$E=30\,000\,\mathrm{MPa}$	Young modulus
$\nu=0.2$	Poisson's ratio
$G_f=0.1\,\mathrm{N/mm}$	Energy of cracking
$p=5$	Parameter of form
$f_t=3\,\mathrm{MPa}$	Limit in traction
$f_c=15\,\mathrm{MPa}$	Limit in compression
$\tau=4\,\mathrm{MPa}$	Limit in shearing
$D=50\,\mathrm{mm}$	Half-width of the band of damage

internal parameters of the model, as described in the booklet [U4.43.01], are obtained by the formulas which are presented there. They lead to the following results:

Key word: ELAS
- E = 3.E4
- NAKED = 0.2

ENDO_SCALAIRE
- K = 31.5E-3
- M = 10
- P = 5
- C_VOLU = 3.68
- C_COMP = 1.847520861

NON_LOCAL
- C_GRAD_VARI = 1,875

1.3 Boundary conditions and loadings

Displacements are imposed in all the nodes of the structure, of kind to correspond to the desired homogeneous deformation. More precisely, displacement in a node of coordinates X is worth: $u(x)=\varepsilon\cdot x$

1.4 Initial conditions

None.

2 Reference solution

2.1 Method of calculating used for the reference solution

This problem admits an analytical solution. One determines the relation which with the imposed deformation $\boldsymbol{\varepsilon}$ associate the homogeneous level of damage a (reciprocal problem where more generally not-room does not admit any more a simple analytical solution). The problem being homogeneous, the damage (in load) and the deformation are bound by the relation of coherence (function threshold):

$A'(a)\,\Gamma(\boldsymbol{\varepsilon})+\omega'(a)=0$

with

$$\omega(a)=ka\,,\quad A(a)=\frac{(1-a)^2}{(1-a)^2+ma(1+pa)}\quad \text{et}\quad \Gamma(\boldsymbol{\varepsilon})=\left[c_T\, tr\,\boldsymbol{\varepsilon}+\sqrt{c_H\, tr^2\,\boldsymbol{\varepsilon}+c_S\,\varepsilon_{eq}^2}\right]$$

where parameters c_T, c_H et c_S result from the facts of the case by:

$$c_S=\frac{E}{2}\left[(1-2\nu)\,c_{comp}+(1+\nu)\sqrt{\frac{1-2\nu}{2(1+\nu)}c_{volu}+1}\right]\quad ;c_T=c_{comp}\sqrt{c_S}\quad ;c_H=\frac{1+\nu}{2(1-2\nu)}c_{volu}\,c_S$$

One adopts a uniaxial deformation of the form:

$\boldsymbol{\varepsilon}=\varepsilon\,\boldsymbol{n}\otimes\boldsymbol{n}\quad \text{où}\quad \|\boldsymbol{n}\|=1\quad \text{et}\quad \varepsilon>0$

In this case the function threshold in deformation is written simply: $\Gamma(\boldsymbol{\varepsilon})=\left[c_T+\sqrt{c_H+c_S}\right]\varepsilon$

To reach the damage a given, by requesting the deformation in the direction $\boldsymbol{n}$, it is thus necessary to impose an intensity of deformation:

$$\varepsilon=\frac{-k}{A'(a)\left[c_T+\sqrt{c_H+c_S}\right]}$$

For the reference solution we thus adopt a following strategy: one sets the level of damage is one checks by calculations EF, that for an estimated theoretical deformation one reaches this same level of damage.

2.2 Results of reference

In plane deformations, one adopts a direction of request $\boldsymbol{n}=(1/\sqrt{5},\ 2/\sqrt{5})$. In 3D, it is worth $\boldsymbol{n}=(1/\sqrt{14},\ 2/\sqrt{14},\ 3/\sqrt{14})$. One sets like target a damage $a=0.6$; that corresponds to an intensity of request $\varepsilon=9.574237\times 10^{-4}$ according to the reference solution above.

The loading is applied with the help of the technique of piloting `PRED_ELAS` in which one fixes the maximum terminal of kind to reach the level of deformation ε above. It will be checked that the damage corresponding reached well 0.6 .

2.3 Uncertainties on the solution

Nothing

2.4 Bibliographical references

Without object

3 Modeling A

3.1 Characteristics of modeling

A modeling D_PLAN_GRAD_VARI with a single mesh, element QUAD8 .
Loading in the direction $\boldsymbol{n}=(1/\sqrt{5},\ 2/\sqrt{5})$.

3.2 Characteristics of the grid

Many nodes: 8
Number and types of meshs: 1 QUAD8, 4 SEG3

3.3 Sizes tested and results of modeling A

One tests the damage in three nodes of the mesh, the value with the nodes being obtained by extrapolation (CHAM_NO 'VARI_NOEU', component V1).

Identification	Reference	Type	Tolerance
V1 ($X=2$, $Y=0$)	0.6	ANALYTICAL	RELATIVE – 0.1%
V1 ($X=2$, $Y=1$)	0.6	ANALYTICAL	RELATIVE – 0.1%
V1 ($X=2$, $Y=2$)	0.6	ANALYTICAL	RELATIVE – 0.1%

4 Modeling B

4.1 Characteristics of modeling

A modeling 3D_GRAD_VARI with a single mesh, element HEXA20 .
Loading in the direction $\boldsymbol{n}=(1/\sqrt{14},\ 2/\sqrt{14},\ 3/\sqrt{14})$.

4.2 Characteristics of the grid

Many nodes: 20
Number and types of meshs: 1 HEXA20 , 6 QUAD8, 8 SEG3

4.3 Sizes tested and results of modeling B

One tests the damage in three nodes of the mesh, the value with the nodes being obtained by extrapolation (CHAM_NO 'VARI_NOEU', component V1).

Identification	Reference	Type	Tolerance
V1 ($X=2$, $Y=0$)	0.6	ANALYTICAL	RELATIVE – 0.1%
V1 ($X=2$, $Y=1$)	0.6	ANALYTICAL	RELATIVE – 0.1%
V1 ($X=2$, $Y=2$)	0.6	ANALYTICAL	RELATIVE – 0.1%

5 Summary of the results

This CAS-tests is realized on only one nets, consequently it is the homogeneous answer of damage which is found numerically. The reference solution is obtained while being put on the threshold of damage. One notes very a good agreement between the modeling and the reference solution. Not-local part of the law nevertheless is not tested.