

## SSNV245 - Imposition of conditions of Dirichlet on elements XFEM Heaviside using a function of space

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### Summary:

It is a question of validating the imposition of conditions of Dirichlet using a function of space on elements the Heaviside XFEM, as well quadratic as linear. In particular, one will use a discontinuous function through the interface.

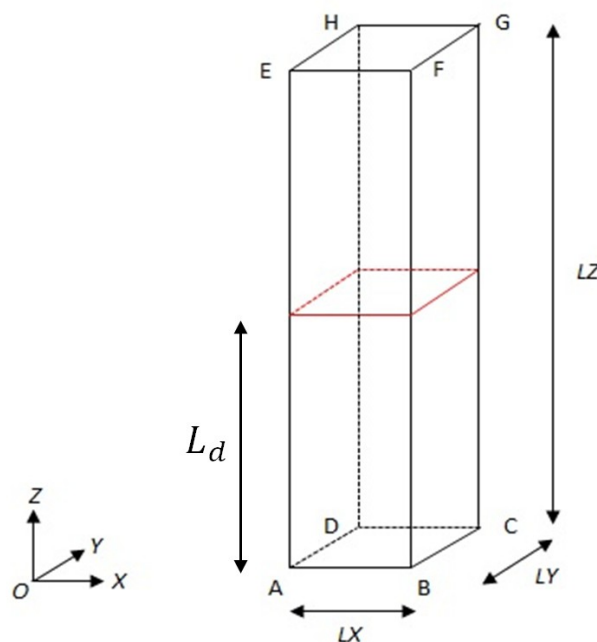
## 1 Problem of reference

### 1.1 Geometry of the problem

It is about a column height  $LZ=5m$ , length  $LX=1m$  and of width  $LY=1m$ . This column presents in  $Z=L_d$  a discontinuity of the type interfaces (interface nonwith a grid which is introduced into the model via the level-sets thanks to the operator `DEFI_FISS_XFEM`). The column is thus entirely crossed by discontinuity (on the level of the approximation of the field of displacements, one takes into account only Heaviside enrichment). The height of the interface  $Y=L_d$  will be different in modelings C and D.

One represents on the Figure 1.1-a geometry of the column.

Figure 1.1-a: Geometry of the problem 3D



### 1.2 Properties materials

Young modulus:  $E=5800\text{ MPa}$

Poisson's ratio:  $\nu=0$

Thermal dilation coefficient:  $\alpha=0\text{ K}^{-1}$

### 1.3 Boundary conditions

#### Case 2D

Boundary conditions which one bracket are of Neuman type. One imposes:

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- In all the field  $u_y(x, y) = f_y(x, y)$  with  $f_y(x, y) = \begin{cases} -0.01 * y & \text{si } Y < L_d \\ 0.01 * (LY - y) & \text{si } Y > L_d \end{cases}$
  - On side [AD]  $u_x(x, y) = f_{null}$  with  $f_{null}$  the constant function of zero value
  - On side [BC]  $u_x(x, y) = f_x(x, y)$
- In modeling A (linear)  $f_x(x, y) = \begin{cases} 0.01 * y & \text{si } Y < L_d \\ -0.01 * (LY - y) & \text{si } Y > L_d \end{cases}$
- In modeling B (quadratic)  $f_x(x, y) = \begin{cases} 0.01 * y^2 & \text{si } Y < L_d \\ -0.01 * (LY - y)^2 & \text{si } Y > L_d \end{cases}$

## Case 3D

Boundary conditions which one bracket are of Neuman type. One imposes:

- In all the field  $u_z(x, y, z) = f_z(x, y, z)$  with  $f_z(x, y, z) = \begin{cases} -0.01 * z & \text{si } Z < L_d \\ 0.01 * (LZ - z) & \text{si } Z > L_d \end{cases}$
  - On face [ADEH]  $u_x(x, y, z) = f_{null}$  with  $f_{null}$  the constant function of zero value
  - On face [BCGF]  $u_x(x, y, z) = f_x(x, y, z)$
- In modeling C (linear)  $f_x(x, y, z) = \begin{cases} 0.01 * z & \text{si } Z < L_d \\ -0.01 * (LZ - z) & \text{si } Z > L_d \end{cases}$
- In modeling D (quadratic)  $f_x(x, y, z) = \begin{cases} 0.01 * z^2 & \text{si } Z < L_d \\ -0.01 * (LZ - z)^2 & \text{si } Z > L_d \end{cases}$
- In all the field  $u_y(x, y, z) = f_{null}$

## 2 Reference solution

### 2.1 Method of calculating

#### Case 2D

The Poisson's ratio  $\nu$  being null, the solution is written independently according to the direction  $x$  and direction  $y$ .

By neglecting gravity, the equation is written (in total constraints):

$$\text{Div}(\boldsymbol{\sigma}) = \mathbf{0}$$

Being in the elastic case, one has  $\boldsymbol{\sigma} = E \boldsymbol{\epsilon}$ , that is to say finally  $\text{Div}(\boldsymbol{\epsilon}) = \mathbf{0}$ .

According to  $x$ ,  $\frac{\partial \epsilon_{xx}}{\partial x} = 0$  from where:

- displacements with the top of the interface are written  $u_x(x, y) = \frac{-0.01 * x}{LX} * (LY - y)$
- displacements below the interface are written  $u_x(x, y) = f_x(x, y) * \frac{x}{LX}$

According to  $y$ , displacements are imposed everywhere thus:

- displacements with the top of the interface are written  $u_y(x, y) = f_y(x, y)$
- displacements with the lower part of the interface are written  $u_y(x, y) = f_y(x, y)$

#### Case 3D

The Poisson's ratio  $\nu$  being null, the solution is written independently according to the direction  $x$ , direction  $y$  and direction  $z$ .

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According to  $x$ ,  $\frac{\partial \epsilon_{xx}}{\partial x} = 0$  from where:

- displacements with the top of the interface are written  $u_x(x, y, z) = f_x(x, y, z) * \frac{x}{LX}$
- displacements below the interface are written  $u_x(x, y, z) = f_x(x, y, z) * \frac{x}{LX}$

According to  $z$ , displacements are imposed everywhere thus:

- displacements with the top of the interface are written  $u_z(x, y, z) = f_z(x, y, z)$
- displacements with the lower part of the interface are written  $u_z(x, y, z) = f_z(x, y, z)$

According to  $y$ , displacements are worthless everywhere.

## 2.2 Sizes and results of reference

One tests displacements with the top and below the interface.

### 2.2.1 In 2D

$$\text{In modeling A (linear) } f_x(x, y) = \begin{cases} 0.01 * y & \text{si } Y < L_d \\ -0.01 * (LY - y) & \text{si } Y > L_d \end{cases}$$

$$\text{In modeling B (quadratic) } f_x(x, y) = \begin{cases} 0.01 * y^2 & \text{si } Y < L_d \\ -0.01 * (LY - y)^2 & \text{si } Y > L_d \end{cases}$$

$$\text{with } LX = 1\text{m}, LY = 5\text{m} \text{ and } f_y(x, y) = \begin{cases} -0.01 * y & \text{si } Y < L_d \\ 0.01 * (LY - y) & \text{si } Y > L_d \end{cases}$$

According to  $x$  :

- displacements with the top of the interface are written  $u_x(x, y) = f_x(x, y) * \frac{x}{LX}$
- displacements below the interface are written  $u_x(x, y) = f_x(x, y) * \frac{x}{LX}$

According to  $y$ , displacements are imposed everywhere thus:

- displacements with the top of the interface are written  $u_y(x, y) = f_y(x, y)$
- displacements with the lower part of the interface are written  $u_y(x, y) = f_y(x, y)$

$$\text{For modeling A, } Y = L_d = \frac{13 * LY}{25}.$$

One tests displacement according to  $y$  of the two nodes of the interface respectively on the lower lip and higher of the crack.

Sizes tested	Type of reference	Value of reference
DY (in lower part)	'ANALYTICAL'	-2.6E-02
DY (with the top)	'ANALYTICAL'	2.4E-02

One also tests displacement according to  $x$  of the two nodes of the interface respectively on the lower lip and higher of the crack in  $x = 1\text{m}$ .

Sizes tested	Type of reference	Value of reference
DX (in lower part)	'ANALYTICAL'	2.6E-02
DX (with above)	'ANALYTICAL'	-2.4E-02

$$\text{For modeling B, } Y = L_d = \frac{13 * LY}{25}.$$

One tests displacement according to  $y$  of the two nodes of the interface respectively on the lower lip and higher of the crack.

Sizes tested	Type of reference	Value of reference
DY (in lower part)	'ANALYTICAL'	-2.6E-02
DY (with the top)	'ANALYTICAL'	2.4E-02

One also tests displacement according to  $x$  of the two nodes of the interface respectively on the lower lip and higher of the crack in  $x=1m$ .

Sizes tested	Type of reference	Value of reference
DX (in lower part)	'ANALYTICAL'	6.76E-02
DX (with above)	'ANALYTICAL'	-5.76E-02

## 2.2.2 In 3D

$$\text{In modeling C (linear) } f_x(x, y, z) = \begin{cases} 0.01 * z & \text{si } Z < L_d \\ -0.01 * (LZ - z) & \text{si } Z > L_d \end{cases}$$

$$\text{In modeling D (quadratic) } f_x(x, y, z) = \begin{cases} 0.01 * z^2 & \text{si } Z < L_d \\ -0.01 * (LZ - z)^2 & \text{si } Z > L_d \end{cases}$$

$$\text{with } LX = 1m, LY = 1m, LZ = 5m \text{ and } f_z(x, y, z) = \begin{cases} -0.01 * z & \text{si } Z < L_d \\ 0.01 * (LZ - z) & \text{si } Z > L_d \end{cases}.$$

According to  $x$  :

- displacements with the top of the interface are written  $u_x(x, y, z) = f_x(x, y, z) * \frac{x}{LX}$
- displacements below the interface are written  $u_x(x, y, z) = f_x(x, y, z) * \frac{x}{LX}$

According to  $z$ , displacements are imposed everywhere thus:

- displacements with the top of the interface are written  $u_z(x, y, z) = f_z(x, y, z)$
- displacements with the lower part of the interface are written  $u_z(x, y, z) = f_z(x, y, z)$

According to  $y$ , displacements are worthless everywhere.

$$\text{For modeling C, } Z = L_d = \frac{2 * LZ}{5}.$$

One tests displacement according to  $z$  of the two nodes of the interface respectively on the lower lip and higher of the crack.

Sizes tested	Type of reference	Value of reference
DZ (in lower part)	'ANALYTICAL'	-2.0E-02
DZ (with the top)	'ANALYTICAL'	3.0E-02

One also tests displacement according to  $x$  of the two nodes of the interface respectively on the lower lip and higher of the crack in  $x=1m$ .

Sizes tested	Type of reference	Value of reference
DX (in lower part)	'ANALYTICAL'	2.0E-02
DX (with above)	'ANALYTICAL'	-3.0E-02

For modeling D,  $Z = L_d = \frac{LZ}{2}$ .

One tests displacement according to  $z$  of the two nodes of the interface respectively on the lower lip and higher of the crack.

Sizes tested	Type of reference	Value of reference
DZ (in lower part)	'ANALYTICAL'	-2.5E-02
DZ (with the top)	'ANALYTICAL'	2.5E-02

One also tests displacement according to  $x$  of the two nodes of the interface respectively on the lower lip and higher of the crack in  $x=1m$ .

Sizes tested	Type of reference	Value of reference
DX (in lower part)	'ANALYTICAL'	6.25E-02
DX (with above)	'ANALYTICAL'	-6.25E-02

## 2.3 Uncertainties on the solution

No, the values tested are analytical.

### 3 Modeling A

#### 3.1 Characteristics of modeling

It is about a modeling  $D\_PLAN$  using linear elements XFEM.

The interface is nonwith a grid and cuts the central element in  $Y=L_d = \frac{13*LY}{25}$ . Thus there are 3 classical elements XFEM and 2 elements. As indicated on the Figure 3.1-a, 3 elements XFEM undergo under cutting under triangles (to carry out the integration of Gauss-Legendre on both sides of lips of the interface, but these triangular subelements are not elements of the grid).

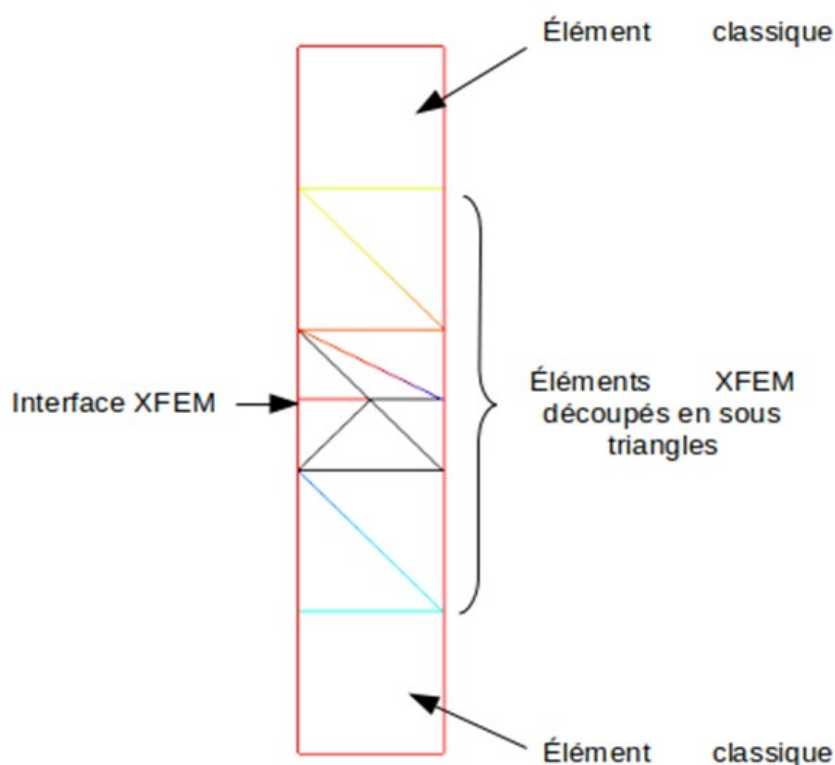


Figure 3.1-a: Characteristics of modeling

#### 3.2 Characteristic of the grid

Grid made up of 5  $QUAD4$  is represented on the Figure 3.2-a.



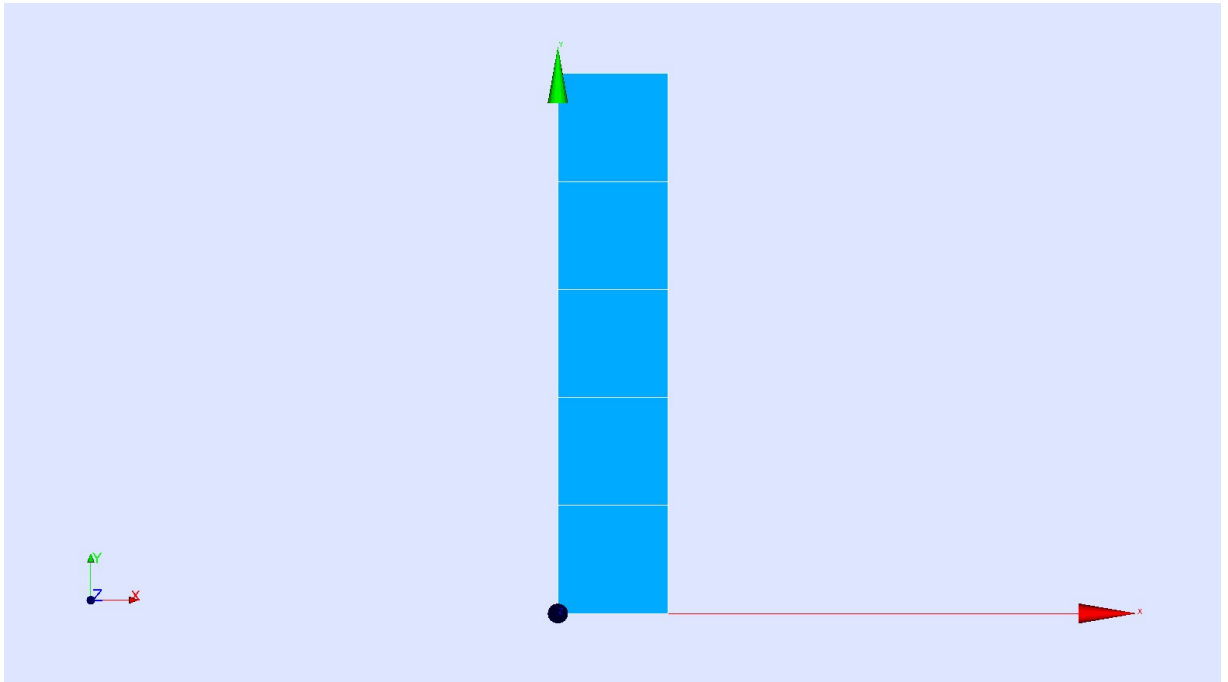


Figure 3.2-a: Grid 2D

### 3.3 Sizes tested and results

The results (resolution with `STAT_NON_LINE`) are synthesized in the table below for the direction  $y$ . Following displacement is tested  $y$  nodes of the interface.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DY (in lower part) MIN	'ANALYTICAL'	-2.6E-02	0,001
DY (with the top) MAX	'ANALYTICAL'	2.4E-02	0,001

One also tests displacement according to  $x$  of the two nodes of the interface located on side [BC] and respectively on the lower lip and higher of the crack.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DX (in lower part side [BC])	'ANALYTICAL'	2.6E-02	0,001
DX (with above side [BC])	'ANALYTICAL'	-2.4E-02	0,001

One presents the fields of displacements according to the direction  $y$  (Figure 3.3-a) and  $x$  (Figure 3.3-b).

Figure 3.3-a: Field of displacements according to direction (OY)

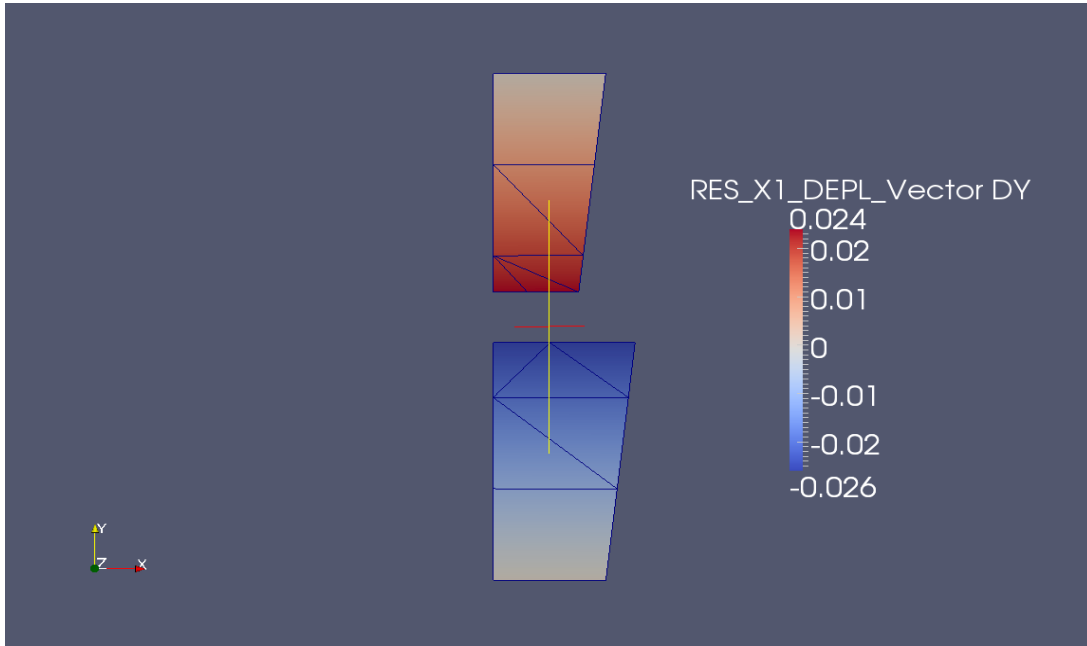
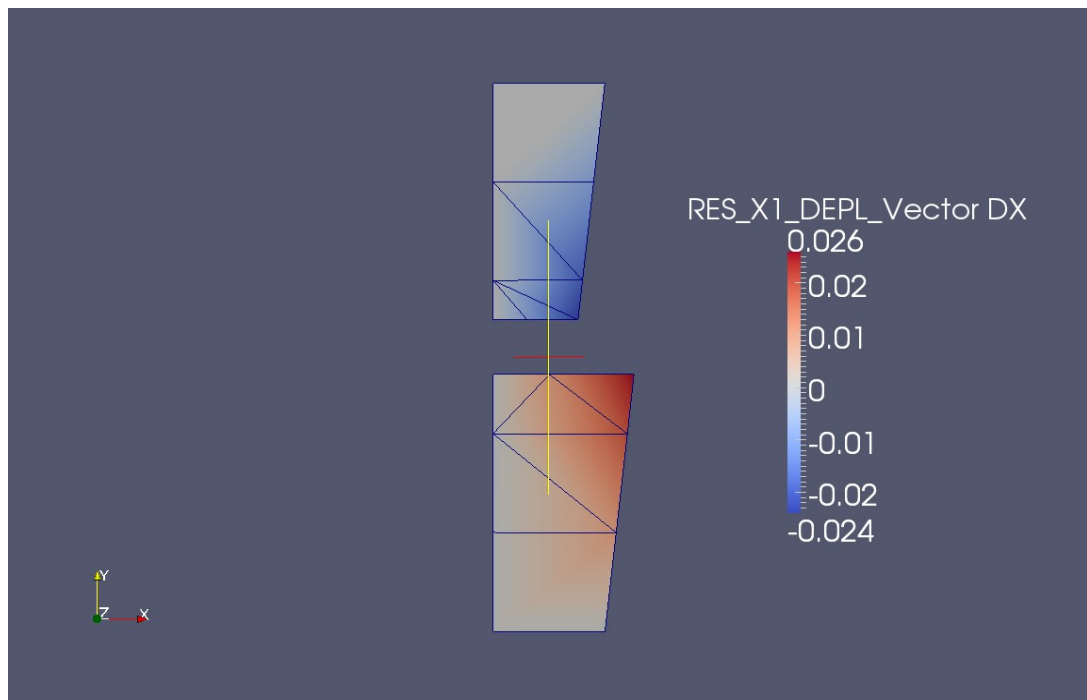


Figure 3.3-b: Field of displacements according to direction (OX)



## 4 Modeling B

### 4.1 Characteristics of modeling

It is about a modeling `D_PLAN` using quadratic elements XFEM. The bar on which one carries out modeling is divided into 5 `QUAD8`. The interface is nonwith a grid and cuts the central element in

$Y = L_d = \frac{13 * LY}{25}$ . There are 3 classical elements XFEM and 2 elements.

### 4.2 Characteristic of the grid

The grid is identical to that of the modeling A, except which it is quadratic. It is composed of 5 `QUAD8`.

### 4.3 Sizes tested and results

The results (resolution with `STAT_NON_LINE`) are synthesized in the table below for the direction `y`. Following displacement is tested `y` nodes of the interface.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DY (in lower part) MIN	'ANALYTICAL'	-2.6E-02	0,001
DY (with the top) MAX	'ANALYTICAL'	2.4E-02	0,001

One also tests displacement according to `x` of the two nodes of the interface located on side [BC] and respectively on the lower lip and higher of the crack.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DX (in lower part side [BC])	'ANALYTICAL'	6.76E-02	0,001
DX (with the top side [BC])	'ANALYTICAL'	-5.76E-02	0,001

One presents the field of displacements according to the direction `y` (Figure 4.3-a) and `x` (Figure 4.3-b).

Figure 4.3-a: Field of displacements according to direction (OY)

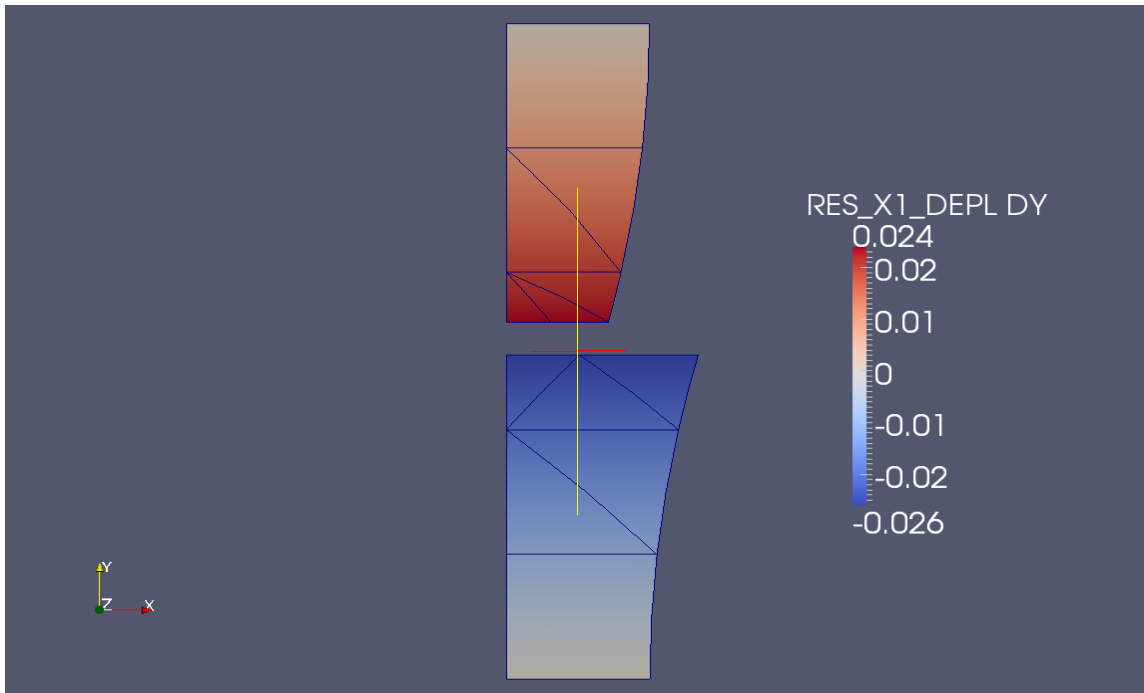
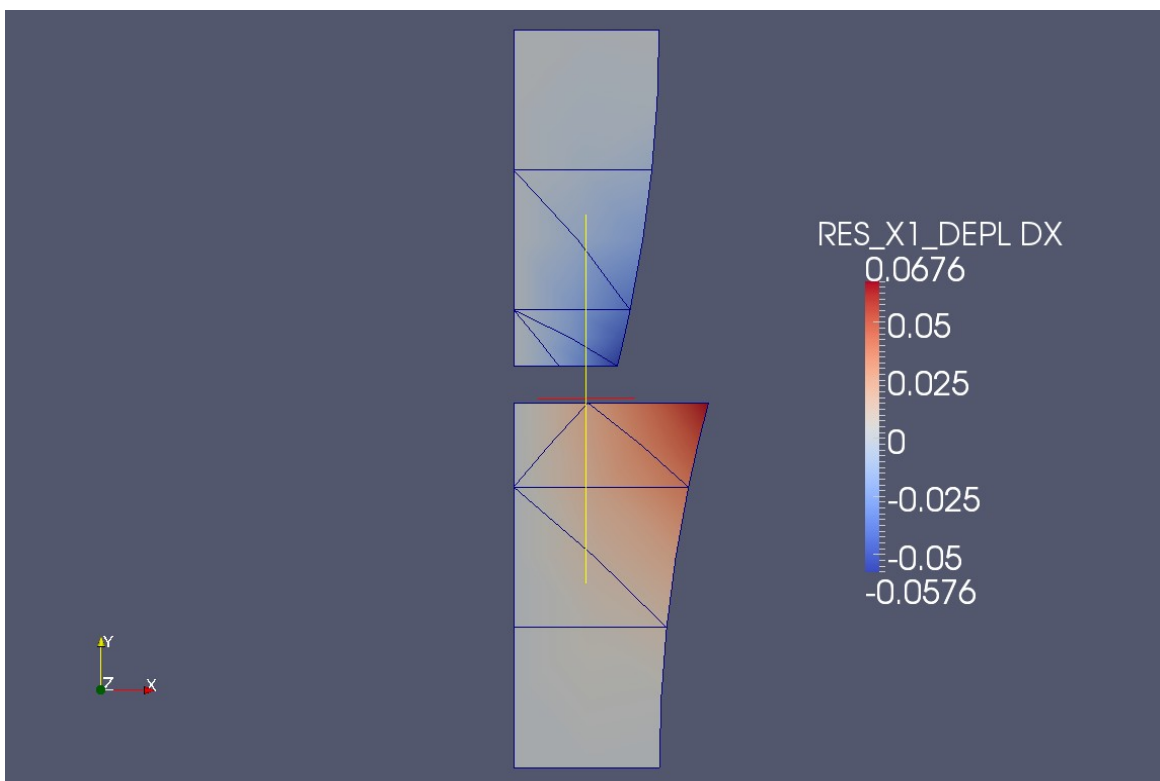


Figure 4.3-b: Field of displacements according to direction (OX)



## 5 Modeling C

### 5.1 Characteristics of modeling

It is about a modeling 3D using linear elements XFEM. The column on which one carries out modeling is divided into 5 `HEXA8`. The interface is nonwith a grid and cuts the column in  $Z = L_d = \frac{2 * LZ}{5}$ . The interface is thus in conformity with the grid. There are 2 classical elements XFEM and 3 elements.

### 5.2 Characteristics of the grid

Grid made up of 5 `HEXA8` is represented on Figure 5.2-a.

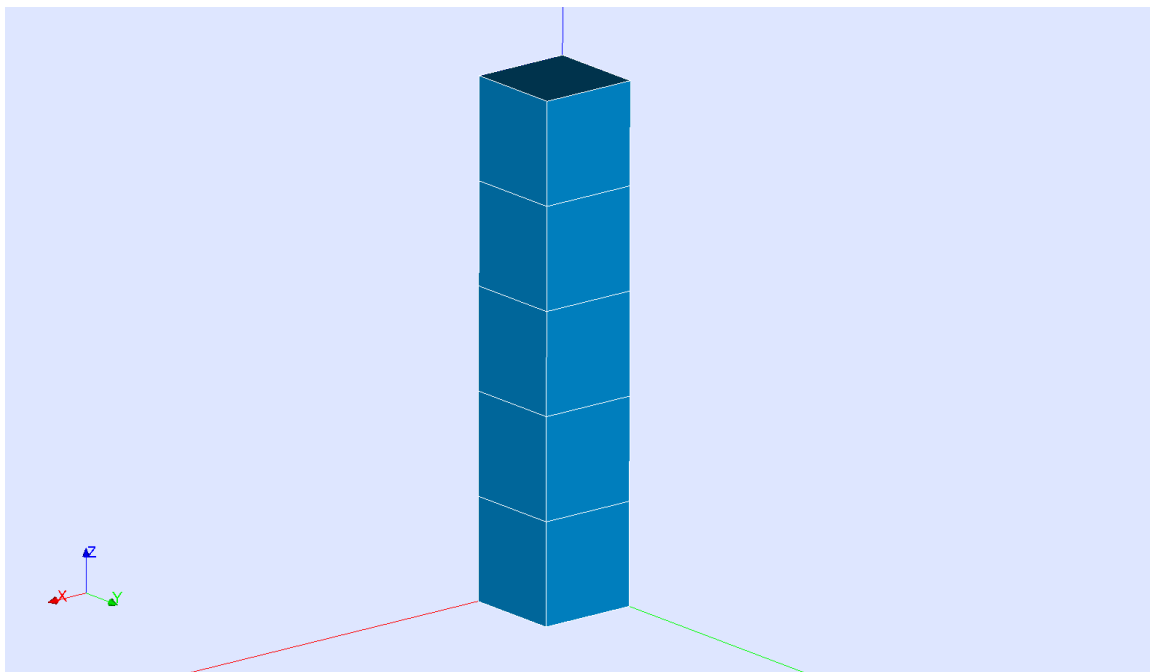


Figure 5.2-a: Grid 2D

### 5.3 Sizes tested and results

The results (resolution with `STAT_NON_LINE`) are synthesized in the table below for the direction  $z$ . Following displacement is tested  $z$  nodes of the interface.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DZ (in lower part) MIN	'ANALYTICAL'	-2.0E-02	0,001
DZ (with the top) MAX	'ANALYTICAL'	3.0E-02	0,001

One also tests displacement according to  $x$  of two nodes of the interface located on face [BCFG] and respectively on the lower lip and higher of the crack.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DX (in lower part face [BCFG])	'ANALYTICAL'	2.0E-02	0,001
DX (with above face [BCFG])	'ANALYTICAL'	-3.0E-02	0,001

One presents the field of displacements according to the direction  $z$  (Figure 5.3-a) and  $x$  (Figure 5.3-b).

**Figure 5.3-a: Field of displacements according to direction (OZ)**

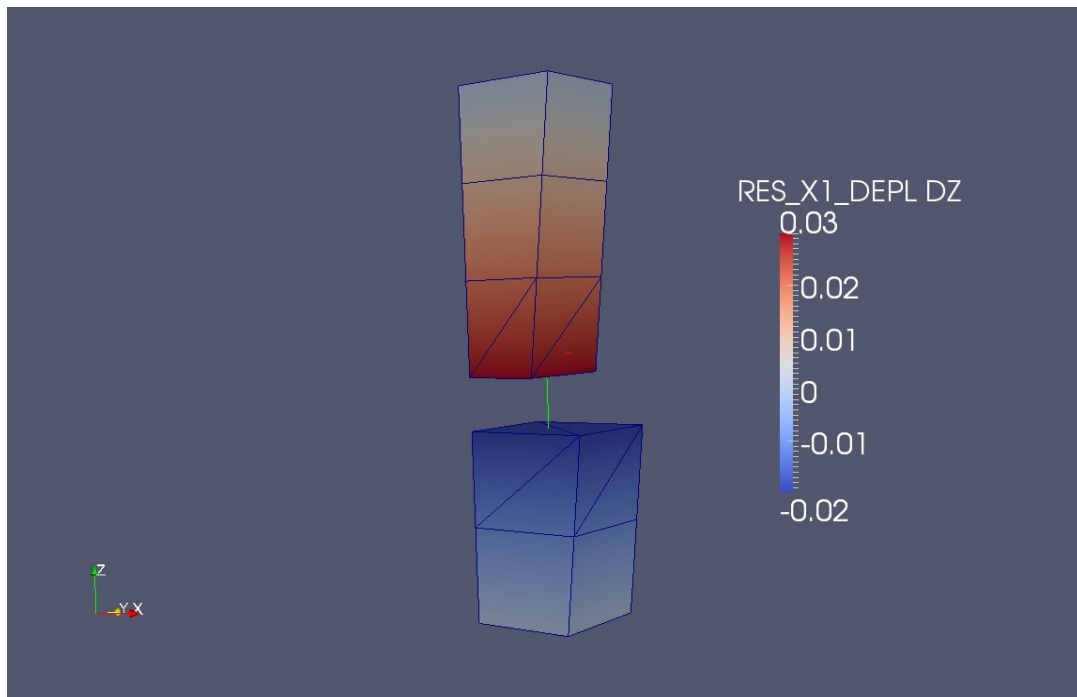
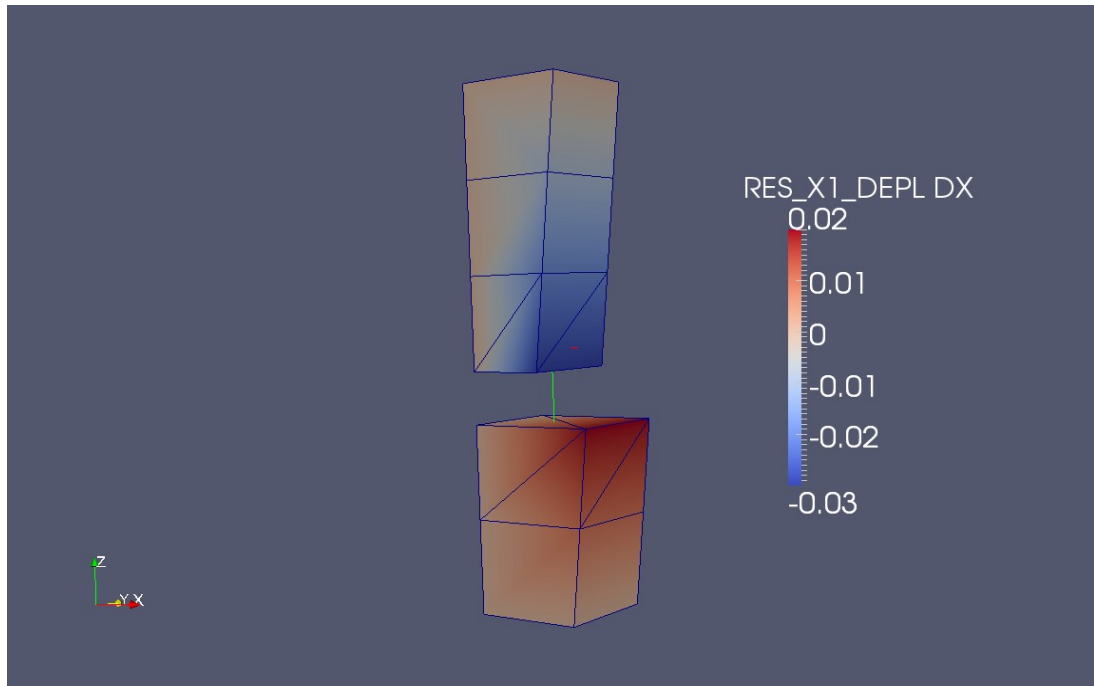


Figure 5.3-b: Field of displacements according to direction (OX)



## 6 Modeling D

### 6.1 Characteristics of modeling

It is about a modeling 3D using quadratic elements XFEM. The column on which one carries out modeling is divided into 5 HEXA20. The interface is nonwith a grid and cuts the column in

$Z = L_d = \frac{LZ}{2}$ . The interface is thus in conformity with the nodes mediums of the central element.

There are 3 classical elements XFEM and 2 elements.

### 6.2 Characteristics of the grid

The grid is identical to that of the modeling C except which it is quadratic. It is compound 5 HEXA20.

### 6.3 Sizes tested and results

The results (resolution with STAT\_NON\_LINE) are synthesized in the table below for the direction  $z$ . Following displacement is tested  $z$  nodes of the interface.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DZ (in lower part) MIN	'ANALYTICAL'	-2.5E-02	0,001
DZ (with the top) MAX	'ANALYTICAL'	2.5E-02	0,001

One also tests displacement according to  $x$  of two nodes of the interface located on face [BCFG] and respectively on the lower lip and higher of the crack.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DX (in lower part face [BCFG])	'ANALYTICAL'	6.25E-02	0,001
DX (with above face [BCFG])	'ANALYTICAL'	-6.25E-02	0,001

One presents the field of displacements according to the direction  $z$  (Figure 6.3-a) and  $x$  (Figure 6.3-b).



Figure 6.3-a: Field of displacements according to direction (OZ)

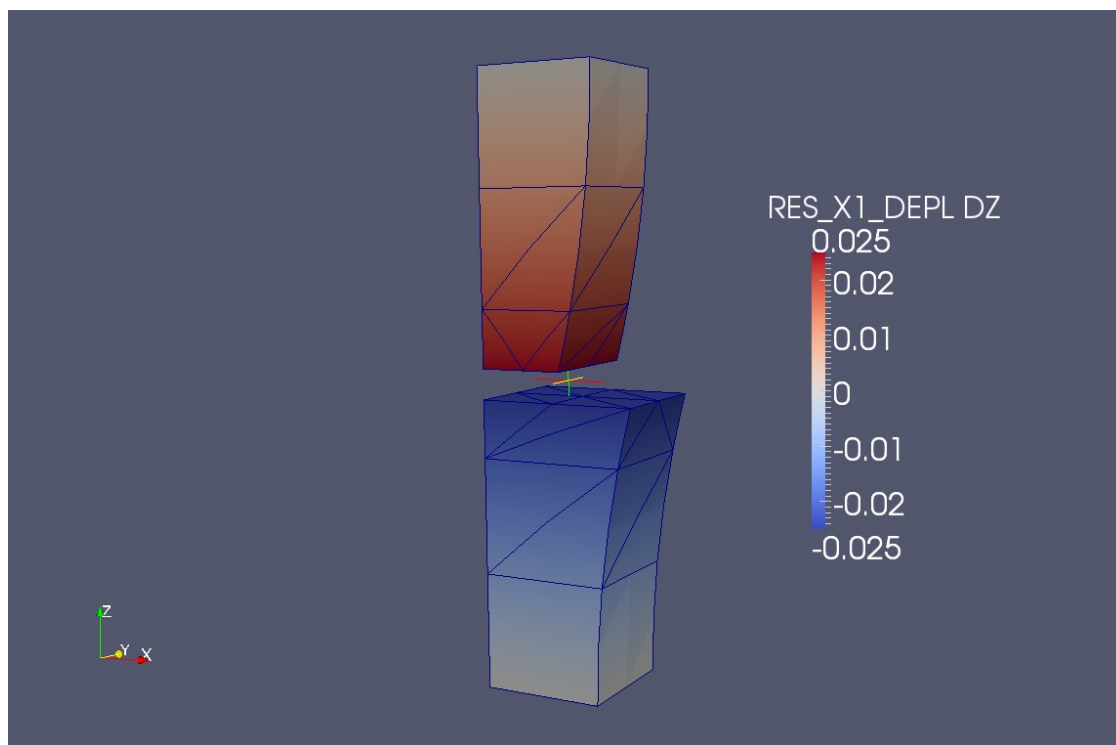
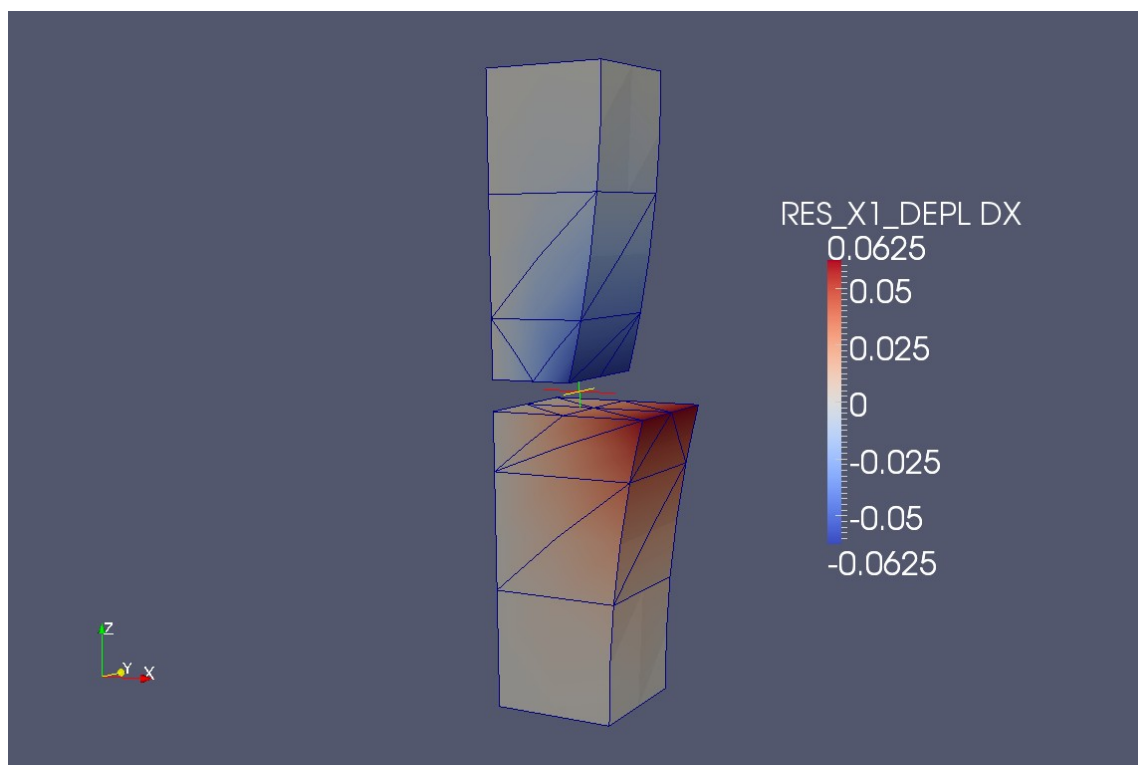


Figure 6.3-b: Field of displacements according to direction (OX)



## 7 Summary of the results

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The imposition of conditions of Dirichlet on elements XFEM Heaviside using a function of space is validated, as well on the elements 2D as 3D, for linear and quadratic modelings.

One also took care to carry out a modeling in which the interface is in conformity with the grid (modeling c) and one modeling in which the interface passes by a node medium (modeling d). This functionality makes it possible in particular to impose conditions discontinuous kinematics through the interface.