SSNV507 – Rotation of a rigid inclusion with X-FEM

Summary:

This test is used to validate the approach X-FEM on the assumption of the taking into account of great rotations (`DISPLACEMENT = 'GROT_GDEP'` in the operator `STAT_NON_LINE`).

Whatever the modeling, one considers a square structure having a discontinuity defining a first disc inside a square. One defines as rigid inclusion, a second disc inside the first. One then imposes a rotation finished on this inclusion.

Modelings A, B, C, D, E and F consist in operating great rotation without taking into account of the contact. For that, one blocks rigid displacements of body by applying boundary conditions of blocking to the edges of the square. All the points belonging to the first disc are pulled by the rotation of inclusion and undergo same rotation. Outside the disc, the points do not undergo any displacement. The validation is done on the values of displacements of the points which are inside the disc and which must correspond to the rotation movement finished imposed.

Modelings G, H and I consist in operating great rotation by activating the contact. One applies a displacement to the edges of the square. This displacement is selected so as to obtain a normal constraint uniform on the level of discontinuity. The validation is done on the values of Lagrange of contact.

1 Problem of reference

1.1 Geometry

The structure is a healthy square in which a circle of radius R an interface (discontinuity) defines. A circle of radius R_1 delimit inclusion and material. Dimensions of the structure are:

$L=20\,m$

$R=8\,m$

$R_1=3\,m$

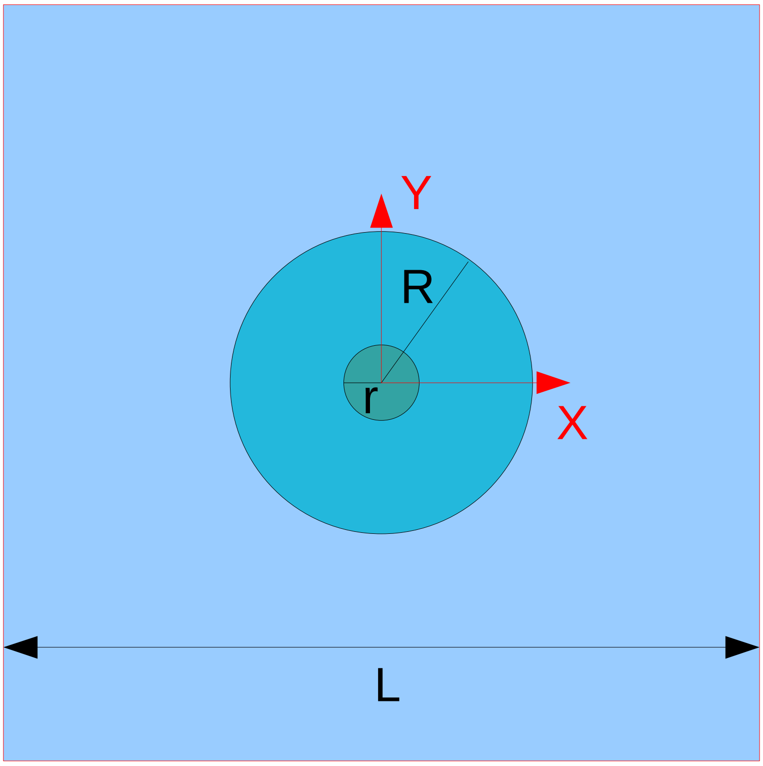

Figure 1.1-a: Geometry of the structure.

1.2 Properties of material

•Young modulus: $E=100\,MPa$
•Without contact: Poisson's ratio: $\nu=0.3$
•With contact: Poisson's ratio: $\nu=0$

1.3 Boundary conditions and loadings

1.3.1. Without contact

The blocking of horizontal displacements is imposed on the sides left and right square (Figure 1.3-a). The blocking of vertical displacements is imposed on with dimensions the inferior and superior of the square.

A time finished rotation of a full rotation is imposed on central inclusion. Its application is done according to a function crawls classical, in 4 pas de charges. The digital values of imposed rotation are thus:

$$\omega=(90°,180°,270°,360°)$$

Concretely, with the step of time i, for a node pertaining to the inclusion, which has as real coordinates of reference (X, Y), following displacements are imposed:

$$Depl_X(X,Y,i)=\sqrt{X^2+Y^2}(\cos(\arctan(\frac{Y}{X})-\omega(i)))-X$$
$$Depl_Y(X,Y,i)=\sqrt{X^2+Y^2}(\sin(\arctan(\frac{Y}{X})-\omega(i)))-Y \quad \text{éq 1.1}$$

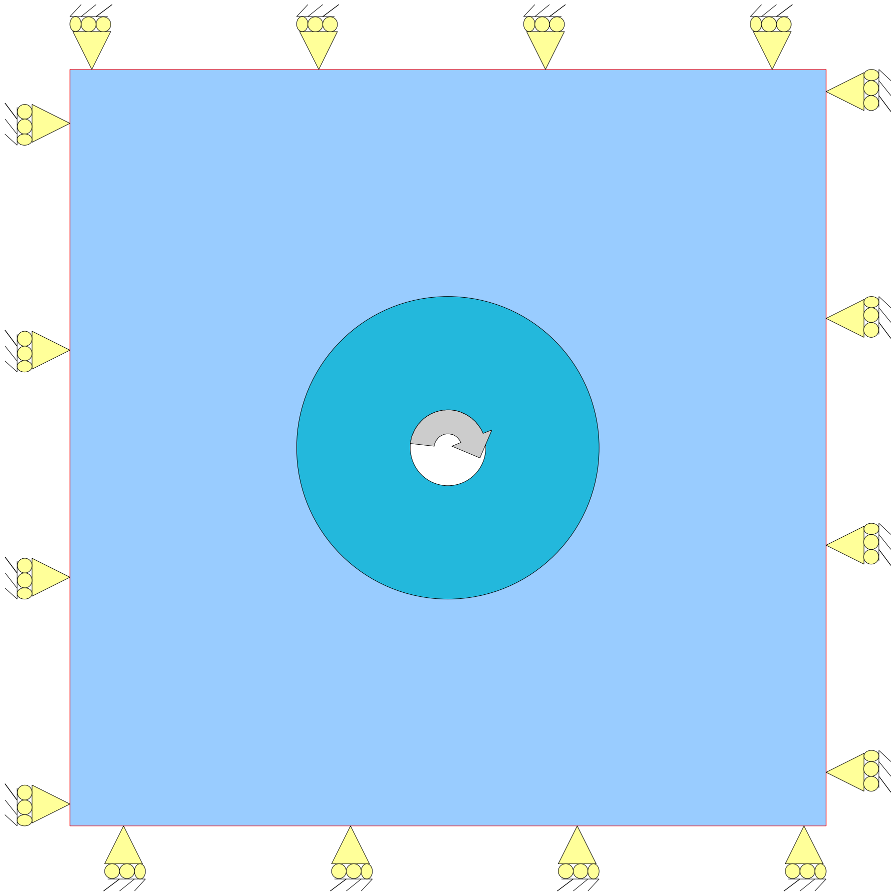

Figure 1.3-a: Illustration of the boundary conditions and the loadings.

1.3.2. With contact

A displacement u_r is imposed on all the edges of the square, it is equivalent to a radial displacement applied to a circle of radius $R_2=\frac{L}{2}$:

$$u_r=ar+\frac{b}{r} \quad \text{éq 1. 2}$$

Coefficients a and b are selected so as to satisfy $\sigma_{\theta\theta}(r=R_1)=0$ and $\sigma_{rr}(r=R_2)=-p$ where $p=1.10^6 Pa$:

$$a=\frac{-p}{E}\frac{R_2^2}{R_1^2+R_2^2}$$
$$b=\frac{p}{E}\frac{R_1^2 R_2^2}{R_1^2+R_2^2} \quad \text{éq 1.3}$$

A time finished rotation of a quarter of turn is imposed on central inclusion. Its application is done according to a function crawls classical, in 4 pas de charges. The digital values of imposed rotation are thus:

$$\omega=(\frac{\pi}{8},\frac{2\pi}{8},\frac{3\pi}{8},\frac{\pi}{2})$$

Concretely, with the step of time i , for a node pertaining to the inclusion, which has as real coordinates of reference (X,Y) , following displacements are imposed:

$$Depl_X(X,Y,i)=\sqrt{X^2+Y^2}(\cos(\arctan(\frac{Y}{X})-\omega(i)))-X$$
$$Depl_Y(X,Y,i)=\sqrt{X^2+Y^2}(\sin(\arctan(\frac{Y}{X})-\omega(i)))-Y \quad \text{éq 1.4}$$

2 Reference solution

2.1 Without contact

One tests each node inside the disc delimited by the interface. For each one of these nodes, the reference solution is provided analytically by the calculation of its displacement. This displacement must correspond to the rotation imposed on inclusion: it is thus calculated by using equation 1.1.

2.2 With contact

One tests each node pertaining to the interface. For each one of these nodes, the reference solution is provided analytically by the calculation of the contact pressure. It is calculated by using equation 1.2:

$$\lambda=aE-\frac{bE}{R^2} \quad \text{éq 2.1}$$

3 Modeling A

3.1 Characteristics of modeling

It is about a modeling X-FEM, in plane deformations (D_PLAN). Discontinuity is defined by a function of level (level set noted normal Ln) directly introduced into the command file using the operator DEFI_FISS_XFEM [U4.82.08]. This function represents the circle of radius R :

$$\mathrm{ln}=R^2-X^2-Y^2 \qquad \text{Éq 3.1}$$

3.2 Characteristics of the grid

The grid (Figure 3.2-a) comprises 361 meshs of the type QUAD4.

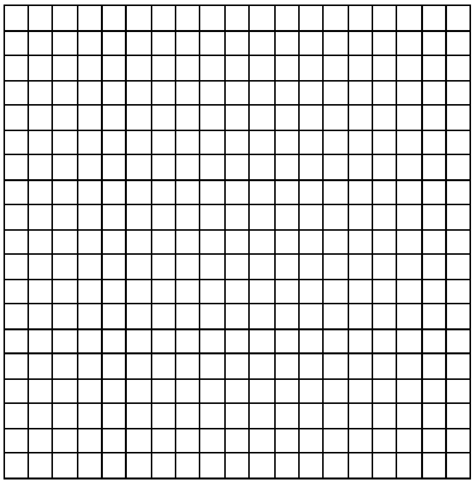

Figure 3.2-a: Grid of modeling A.

3.3 Sizes tested and results

For each step of load, one calculates for all the nodes inside the disc, the absolute difference between the displacement obtained by calculation aster and that calculated analytically. For each one of these nodes, the difference must be worthless. To go more quickly one tests the sum of all these differences which must also be worth zero.

Not	Identification	Reference	Type	Tolerance (%)
1	SOMM	0	ANALYTICAL	1,00E-008
2	SOMM	0	ANALYTICAL	1,00E-008
3	SOMM	0	ANALYTICAL	1,00E-008
4	SOMM	0	ANALYTICAL	1,00E-008

3.4 Comments

One imposed a rigid rotation on the nodes of inclusion. The test shows that inclusion involves all the other nodes of the disc (nonconcerned by the limiting condition of rotation) of the same rotation.
If one launches this case test while replacing 'GROT_GDEP' by 'SMALL' in the operand DISPLACEMENT, the disc 'inflates' and displacements of the nodes tested do not follow any more finished rotation.

4 Modeling B

4.1 Characteristics of modeling

Modeling: C_PLAN .

4.2 Characteristics of the grid

The grid (Figure 4.2-a) comprises 722 meshs of the type TRIA3.

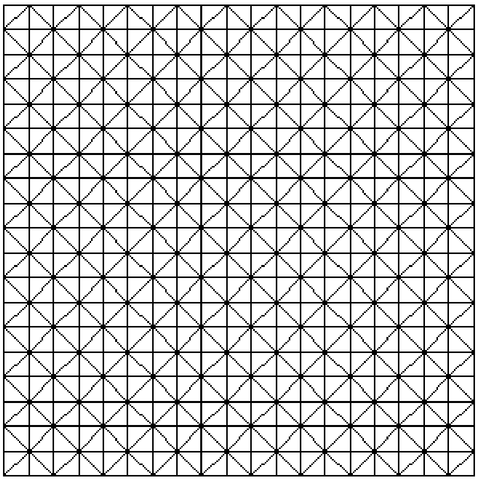

Figure 4.2-a: Grid of modeling B.

4.3 Sizes tested and results

One tests the sum of the differences of displacements as for modeling A .

Not	Identification	Reference	Type	Tolerance (%)
1	SOMM	0	ANALYTICAL	1.0e-008
2	SOMM	0	ANALYTICAL	1.0e-008
3	SOMM	0	ANALYTICAL	1.0e-008
4	SOMM	0	ANALYTICAL	1.0e-008

4.4 Comments

Idem modeling A

5 Modeling C

5.1 Characteristics of modeling

They is the same characteristics of modeling as those of modeling A but in 3D :
- one extrudes of 2 meters in the direction Z (1 element),
- one blocks the movements of rigid body in this direction,
- the level set normal (defined by equation 3.1) represents a cylinder now.

5.2 Characteristics of the grid

The grid (Figure 5.2-a) comprises 361 meshs of the type HEXA8.

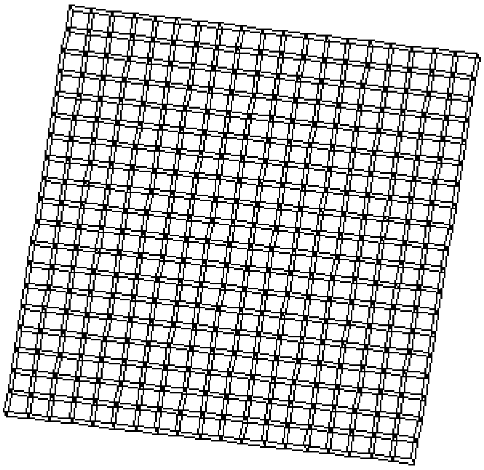

Figure 5.2-a: Grid of modeling C.

5.3 Sizes tested and results

One tests the sum of the differences of displacements as for modeling A .

Not	Identification	Reference	Type	Tolerance (%)
1	SOMM	0	ANALYTICAL	1.0e-008
2	SOMM	0	ANALYTICAL	1.0e-008
3	SOMM	0	ANALYTICAL	1.0e-008
4	SOMM	0	ANALYTICAL	1.0e-008

5.4 Comments

Idem modeling A

6 Modeling D

6.1 Characteristics of modeling

They is the same characteristics of modeling as those of modeling C.

6.2 Characteristics of the grid

The grid (Figure 6.2-a) comprises 2166 meshs of the type TETRA4.

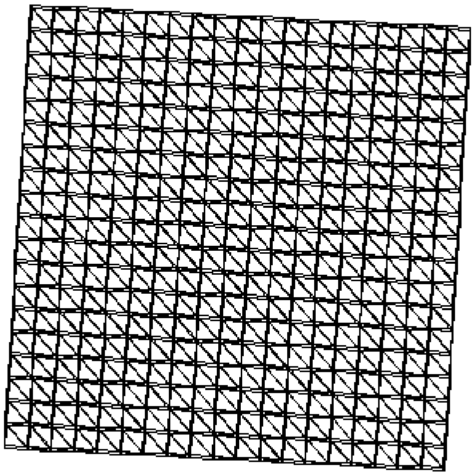

Figure 6.2-a: Grid of modeling D.

6.3 Sizes tested and results

One tests the sum of the differences of displacements as for modeling A.

Not	Identification	Reference	Type	Tolerance (%)
1	SOMM	0	ANALYTICAL	1.0e-008
2	SOMM	0	ANALYTICAL	1.0e-008
3	SOMM	0	ANALYTICAL	1.0e-008
4	SOMM	0	ANALYTICAL	1.0e-008

6.4 Comments

Idem modeling A

7 Modeling E

7.1 Characteristics of modeling

Modeling: D_PLAN .

7.2 Characteristics of the grid

The grid (Figure 7.2-a) comprises 361 meshs of the type QUAD8.

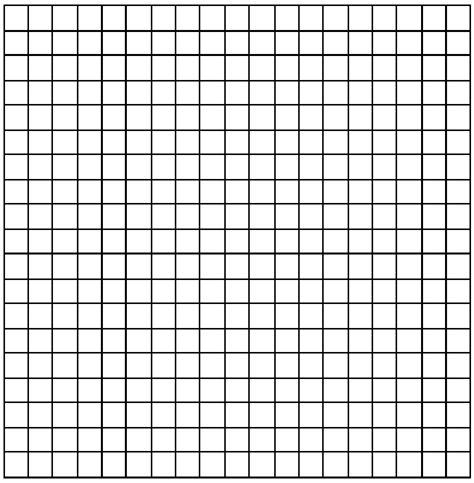

Figure 7.2-a: Grid of modeling E.

7.3 Sizes tested and results

For each step of load, one calculates for all the nodes inside the disc, the absolute difference between the displacement obtained by calculation of Code_Aster and that calculated analytically. For each one of these nodes, the difference must be worthless. To go more quickly one tests the sum of all these differences which must also be worth zero.

Not	Identification	Reference	Type	% tolerance
1	SOMM	0	ANALYTICAL	1E-8
2	SOMM	0	ANALYTICAL	1E-8
3	SOMM	0	ANALYTICAL	1E-8
4	SOMM	0	ANALYTICAL	1E-8

7.4 Comments

This test validates the calculation of the matrix of rigidity in great rotations with elements QUAD8 .

8 Modeling F

8.1 Characteristics of modeling

Modeling: C_PLAN .

8.2 Characteristics of the grid

The grid (Figure 9.2-a) comprises 722 meshs of the type TRIA6.

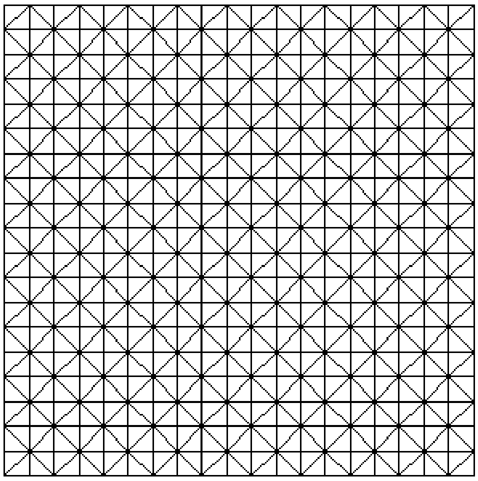

Figure 9.2-a: Grid of modeling F.

8.3 Sizes tested and results

One tests the sum of the differences of displacements as for modeling A .

Not	Identification	Reference	Type	% tolerance
1	SOMM	0	ANALYTICAL	1E-8
2	SOMM	0	ANALYTICAL	1E-8
3	SOMM	0	ANALYTICAL	1E-8
4	SOMM	0	ANALYTICAL	1E-8

8.4 Comments

This test validates the calculation of the matrix of rigidity in great rotations with elements TRIA6.

9 Modeling G

9.1 Characteristics of modeling

Modeling: D_PLAN .
The contact is treated with elements X-FEM linear P1 (displacement) P1 (pressure), i.e. carrying the degrees of freedom of displacement and Lagrange of contact on all the nodes (formulation contact with the nodes) .

9.2 Characteristics of the grid

The grid (Figure 9.2-a) comprises 10201 meshs of the type QUAD4.

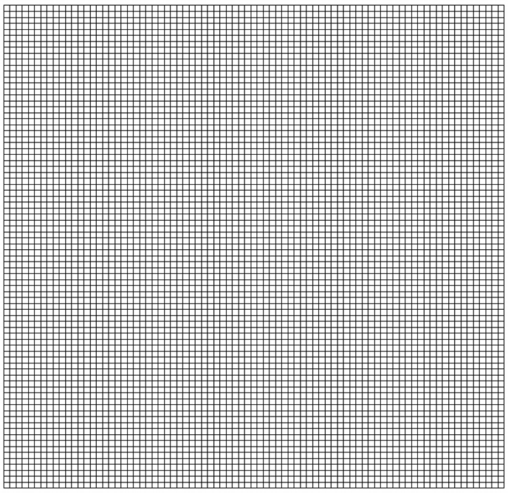

Figure 9.2-a: Grid of modeling G.

9.3 Features tested

One uses a diagram of integration reduced to 3 points of Gauss per facet of contact. Friction is not taken into account and the contact is active as of the 1 era iteration of active constraints. The algorithm aiming at restricting the space of the multipliers of Lagrange is the n°2.

9.4 Sizes tested and results

One tests the maximum and minimal value of the normal pressure of contact post-treated with each step of time.

Not	Identification	Reference	Type	% tolerance
1	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	22.0
2	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	22.0
3	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	22.0
4	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	22.0
5	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	22.0
6	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	22.0

	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	22.0
7	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	22.0
8	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	22.0
9	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	22.0
10	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	22.0

9.5 Comments

This valid test:

- réappariemment,
- the calculation of the matrix of rigidity in great rotations,
- the calculation of the matrices of contact in great slips (integration on one SE2 at the points of Gauss),
- postprocessing X-FEM elements P1P1 .

10 Modeling H

10.1 Characteristics of modeling

Modeling: D _PLAN .
The contact is treated with elements X-FEM linear P2 (displacement) P1 (pressure), i.e. carrying the degrees of freedom of displacement on all the nodes and them Lagrange of contact on the nodes tops (contact formulation with the nodes) .

10.2 Characteristics of the grid

The grid (Figure 10.2-a) comprises 2500 meshs of the type QUAD8.

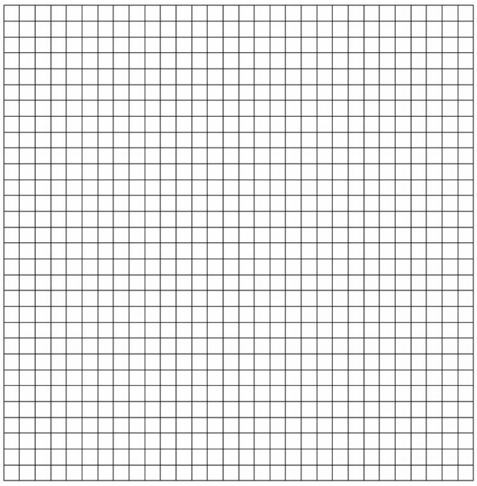

Figure 10.2-a: Grid of modeling I.

10.3 Features tested

One uses a diagram of integration reduced to 3 points of Gauss per facet of contact. Friction is not taken into account and the contact is active as of the 1 era iteration of active constraints. The algorithm aiming at restricting the space of the multipliers of Lagrange is the n°2.

10.4 Sizes tested and results

One tests the maximum and minimal value of the normal pressure of contact post-treated with each step of time.

Not	Identification	Reference	Type	% tolerance
1	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.5
2	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.5
3	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.5
4	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.5
5	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.5
6	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.5

	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.5
7	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.5
8	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.5
9	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.5
10	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.5

10.5 Comments

This valid test:

- réappariemment (curved interface and elements on right board),
- the calculation of the matrix of rigidity in great rotations,
- the calculation of the matrices of contact in great slips (integration on one SE3 at the points of Gauss),
- postprocessing X-FEM elements P2P1 .

11 Modeling I

11.1 Characteristics of modeling

Modeling: D_PLAN .
The contact is treated with elements X-FEM linear P2 (displacement) P1 (pressure), i.e. carrying the degrees of freedom of displacement on all the nodes and them Lagrange of contact on the nodes tops (contact formulation with the nodes) .

11.2 Characteristics of the grid

The grid (Figure 11.2-a) comprises 2601 meshs of the type TRIA6.

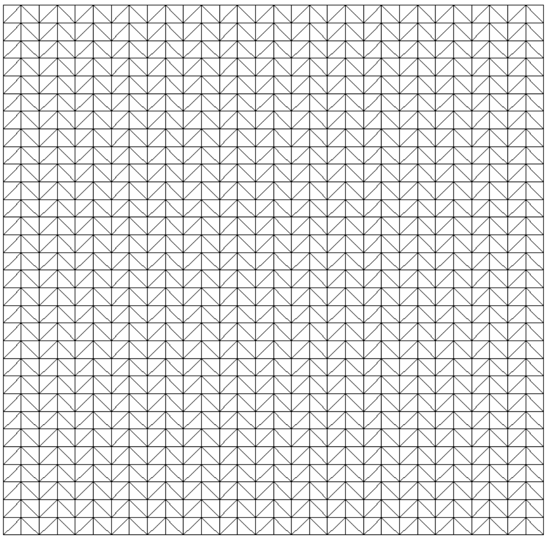

Figure 11.2-a: Grid of modeling J.

11.3 Features tested

One uses a diagram of integration reduced to 3 points of Gauss per facet of contact. Friction is not taken into account and the contact is active as of the 1 era iteration of active constraints. The algorithm aiming at restricting the space of the multipliers of Lagrange is the n°2.

11.4 Sizes tested and results

One tests the maximum and minimal value of the normal pressure of contact post-treated with each step of time.

Not	Identification	Reference	Type	% tolerance
1	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.0
2	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.0
3	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.0
4	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.0
5	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.0

6	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.0
7	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.0
8	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.0
9	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.0
10	LAGS_C (MAX)	-1.11552e6	ANALYTICAL	5.0
	LAGS_C (MIN)	-1.11552e6	ANALYTICAL	5.0

11.5 Comments

This valid test:

- réappariemment (curved interface and elements on right board),
- the calculation of the matrix of rigidity in great rotations,
- the calculation of the matrices of contact in great slips (integration on one SE3 at the points of Gauss),
- postprocessing X-FEM elements P2P1 .

12 Summary of the results

The goals of this test are achieved. They were to show the feasibility of the taking into account great rotations with X-FEM :

- in 2D (plane strains and plane stresses) for linear elements QUAD4 and TRIA3, and quadratic QUAD8 and TRIA6,
- in 3D for elements HEXA8 and TETRA4.

It was a question of showing the feasibility of the taking into account of the contact on the lips of the interface in great slips:

- in 2D P1 (displacement) P1 (pressure) on a grid quadrangle and triangle.
- in 2D P2 (displacement) P1 (pressure) on a grid quadrangle and triangle.