SSNV508 – Bloc in plane constraints with interface, traction and side compression, for quadratic elements X-FEM

Summary:

The purpose of this test is to validate the deformation of an interface introduced into a rectangular parallelepipedic very thin plate within a framework X-FEM. The structure is requested in traction and is subjected to a linear side pressure. The interface is represented by a level set plane and horizontal cutting elements or coinciding with their edges. It utilizes the elements X-FEM [R7.02.12] $P2$ (displacement) which has degrees of freedom of displacement in each node. With the problem is dealt in 2D and 3D.

1 Problem of reference

1.1 Geometry 2D

The structure is a rectangle made up of two of the same plates material, separated by an interface.

Dimensions of the plate, to which the pressures are applied, are:
$L_X=2\text{m}$, $L_Y=1{,}8\,m$
The second plate has following dimensions:
$L_X=2\text{m}$, $L_Y\,'=1{,}2\,m$

The position of the points of reference east:

	x	y
A	-1	0
B	1	0
C	1	1.8
D	-1	1.8
O	0	0

1.2 Geometry 3D

The structure is a rectangular parallelepiped made up by two of the same plates material, separated by an interface.

Dimensions of the plate of the top, to which the pressures are applied, are:
$L_X=2\text{m}$, $L_Y=1{,}8\,m$, $L_Z=0{,}01\,m$
The second plate in lower part has following dimensions:
$L_X=2\text{m}$, $L_Y\,'=1{,}2\,m$, $L_Z=0{,}01\,m$
The position of the points of reference east:

	x	y	z
A	-1	0	0
B	1	0	0
C	1	1.8	0
D	-1	1.8	0
O	0	0	0
A'	-1	0	-0.01
B'	1	0	-0.01
C'	1	1.8	-0.01
D'	-1	1.8	-0.01
O'	0	0	-0.01

1.3 Material properties

Poisson's ratio: 0.2
Young modulus: $1.10^{10} N/m^2$

1.4 Boundary conditions and loadings

The lower plate $(y<0)$ is blocked by an embedding of its lower face.
The plan $ABCD$ is blocked in the direction e_z.
In the case 3D, the plan $(x=0)$ is blocked in the direction e_x, and in the case 2D, it is the line $(x=0)$ who is blocked in the direction e_x.
The higher plate $(y>0)$ is subjected to a pressure distributed horizontal acting on the side faces $P=\pm(-5.10^4 y+1.10^5) N/m^2$ (according to the principle of compression). One applies a quadratic displacement of traction to it to the higher face $d=-2,5.10^{-6} x^2+1.10^{-5} m$.

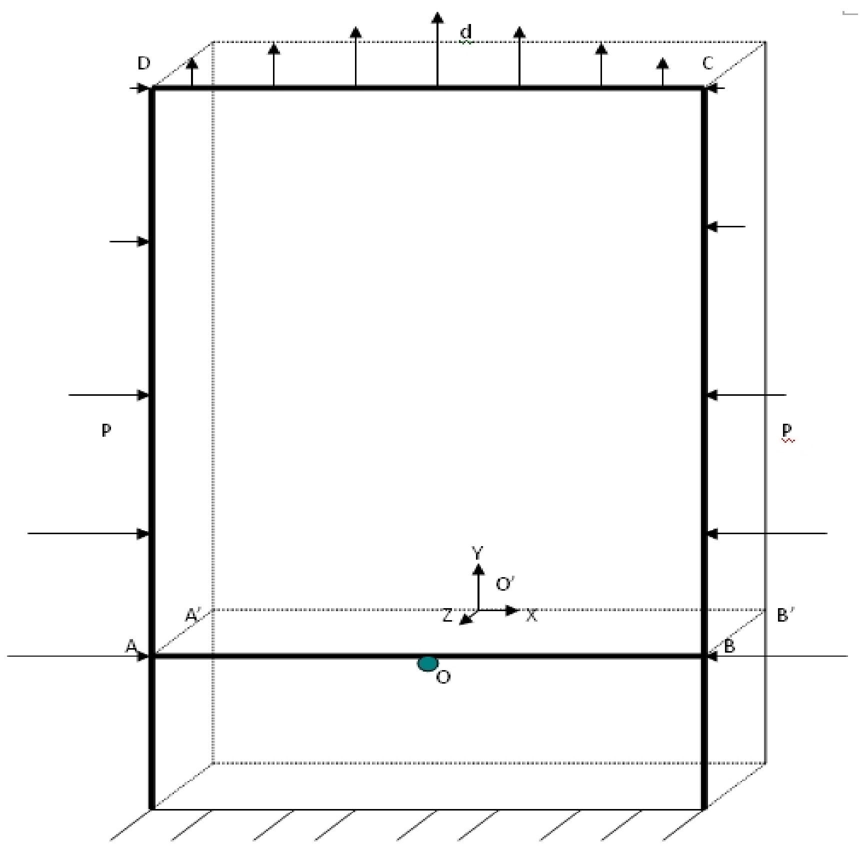

Figure 1: Geometry of the structure and positioning of the interface and loadings 3D

2 Reference solution: analytical in plane constraints

2.1 Solution 2D

While placing itself in the Cartesian reference mark (x, y), the displacement of any point $M(x, y)$ higher plate is written:

$$u(x, y)=u_x(x, y)\vec{e}_x+u_y(x, y)\vec{e}_y \quad \textbf{éq 2-1}$$

Remarks :

- *The higher plate and the lower plate are dissociated owing to the fact that the interface separates the plate into two completely. The plate lower being embedded than its base, it results from it that it is completely motionless and that one makes carry the analytical study only on the higher plate.*

One breaks up the components of displacement in the base $\{1, x, y, x^2, y^2, xy, x^2 y, xy^2\}$:

$$u_x(x, y)=I_1+I_2 x+I_3 y+I_4 x^2+I_5 y^2+I_6 xy+I_7 x^2 y+I_8 x y^2 \quad \textbf{éq 2-2}$$

$$u_y(x, y)=J_1+J_2 x+J_3 y+J_4 x^2+J_5 y^2+J_6 xy+J_7 x^2 y+J_8 x y^2 \quad \textbf{éq 2-3}$$

The problem has a geometrical symmetry and of loading compared to the y-axis. That implies:

$$I_1=I_3=I_4=I_5=I_7=J_2=J_6=J_8=0 \quad \textbf{éq 2-4}$$

The equilibrium equations local expressed in the Cartesian reference mark gives:

$$I_8=J_7=0 \quad \textbf{éq 2-5}$$

$$I_6=-\frac{2(1-\nu)}{1+\nu}J_4-\frac{4}{1+\nu}J_5 \quad \textbf{éq 2-6}$$

By applying the limiting conditions of Dirichlet of the higher face $d=d_2 x^2+d_0$, one from of deduced:

$$J_4=d_2=-2{,}5.10^{-6} \quad \textbf{éq 2-7}$$

$$J_1+J_3 L_y+J_5 L_y^2=d_0 \quad \textbf{éq 2-8}$$

By applying the limiting conditions of Neumann of the side edges $P=p_1 y+p_0$ on the constraints resulting from the law of Hooke generalized:

$$J_5=-\frac{d_2}{2+\nu}+\frac{p_1(1+\nu)^2}{2E(2+\nu)}=-0{,}5.10^{-6} \quad \textbf{éq 2-9}$$

$$I_6=-\frac{2d_2(1-\nu)}{1+\nu}p_0-\frac{4J_5}{1+\nu}=-\frac{5}{3}.10^{-6} \quad \textbf{éq 2-10}$$

$$I_2=-\frac{1-\nu^2}{E}p_0-\nu J_3 \quad \textbf{éq 2-11}$$

The interface is a free edge, i.e. the vector forced in any point of this surface in the normal direction external with the structure is null:

$$J_3=\frac{\nu\, p_0}{E}=2.10^{-6} \quad \textbf{éq 2-12}$$

$$I_2=\frac{-p_0}{E}=-1.10^{-6} \quad \textbf{éq 2-13}$$

Consequently, by combining the results and expressions obtained, one draws J_1 :

$$J_1=d_0-J_3 L_y-J_5 L_y^2=8{,}02.10^{-6} \quad \textbf{éq 2-14}$$

The solution obtained is the following one:

$$u_x(x,y)=-1.10^{-6}(10\mathrm{x}+\frac{5}{3}xy)$$ **éq 2-15**

$$u_y(x,y)=1.10^{-6}(8{,}02+2\mathrm{y}-2{,}5x^2-0{,}5y^2)$$ **éq 2-16**

Maybe on the interface the following result:

$$u_x(x,y=0)=-1.10^{-5}x$$ **éq 2-17**

$$u_y(x,y=0)=1.10^{-6}(8{,}02-2{,}5x^2)$$ **éq 2-18**

2.2 3D solution

According to the assumption of the plane constraints the stress field 3D does not vary in the direction z, which implies that the deformations of it are also independent. The problem can then be brought back to the problem in 2D (plan $ABCD$) for the resolution of the constraints and deformations.
In the case 3D, the solution on u_x and u_y thus have the following form:

$$u_x(x,y,z)=1.10^{-6}(-10\mathrm{x}-\frac{5}{3}xy+h_x(z))$$ **éq 2-19**

$$u_y(x,y,z)=1.10^{-6}(8{,}02+2\mathrm{y}-2{,}5x^2+h_y(z))$$ **éq 2-20**

Moreover, deformation on e_z is written:

$$\epsilon_{zz}(x,y,z)=-\frac{\nu}{1-\nu}(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y})=1.10^{-6}(2+\frac{2}{3}y)$$ **éq 2-21**

With:

$$\epsilon_{zz}(x,y,z)=\frac{\partial u_z(x,y,z)}{\partial z}$$ **éq 2-22**

Consequently, by combining the results and expressions obtained, one obtains:

$$u_z(x,y,z)=1.10^{-6}[(2+\frac{2}{3}y)z+g(x,y)]$$ **éq 2-23**

According to $\epsilon_{xz}=0$ and $\epsilon_{yz}=0$, one obtains:

$$h_x(z)=C_1z+C_0$$ **éq 2-24**

$$h_y(z)=-\frac{z^2}{3}+C_3z+C_5$$ **éq 2-25**

$$g(x,y)=-C_{1\mathrm{x}}-C_3y+C_4$$ **éq 2-26**

The plan $ABCD$ is blocked on the direction e_z , one obtains:

$$u_z(x,y,z=0)=0$$ **éq 2-27**

That implies: $C_1=C_3=C_4=0$.

The plan is blocked on the direction e_x , one obtains:

$$u_x(x=0,y,z)=0$$ **éq 2-28**

That implies: $C_0=0$.

Moreover, the displacement imposed on the upper surface leads to: $C_5=0$.

Finally, one obtains:

$$u_x(x,y,z)=-(10\mathrm{x}+\frac{5}{3}xy).10^{-6}$$ **éq 2-29**

$$u_y(x,y,z)=(8{,}02+2\mathrm{y}-2{,}5x^2-0{,}5y^2-\frac{1}{3}z^2).10^{-6}$$ **éq 2-30**

$$u_z(x,y,z)=(2+\frac{2}{3}y)z.10^{-6}$$ **éq 2-31**

Maybe on the interface the following result:

$$u_x(x,y=0,z)=-1.10^{-5}x$$ **éq 2-32**

$$u_y(x,y=0,z)=(8{,}02-2{,}5x^2-\frac{1}{3}z^2).10^{-6}$$ **éq 2-33**

$$u_z(x,y=0,z)=2.10^{-6}z$$ **éq 2-34**

3 Modeling A

3.1 Characteristics of modeling

Modeling: `C_PLAN` .

The structure is a healthy rectangle, into which an interface is introduced directly into the command file using the operator `DEFI_FISS_XFEM` [U4.82.08]. The interface is present at a distance $L_Y = 1,8\,m$ higher edge of the plate.

3.2 Characteristics of the grid

Many nodes: 661
Many meshs and types: 200 `QUAD8` for the plate and 50 `SEG3` for the edges.

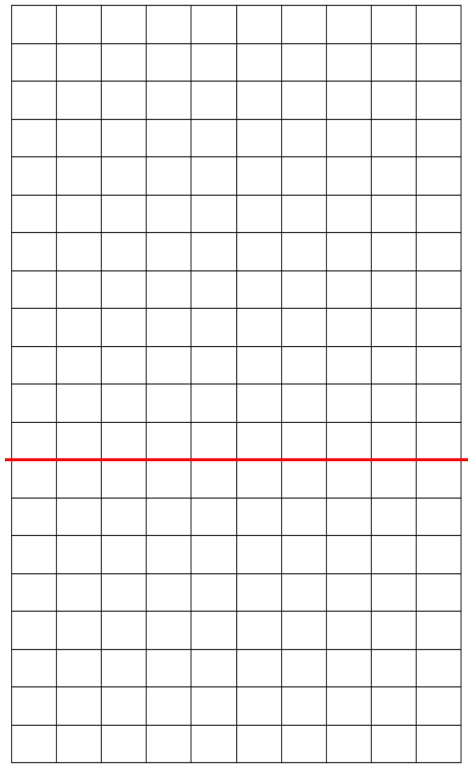

Figure 3.2 -a: Grid 2D quadrangle and position of the interface

3.3 Sizes tested and results

Displacements resulting from the operator `STAT_NON_LINE` are post-treaties so as to recover the values with the nodes of the crack resulting from the new grid.

Identification	Reference	Aster	tolerance
DX at the point A	1.10^{-5}	Analytical	1.10^{-12}
DX at the point B	-1.10^{-5}	Analytical	1.10^{-12}
DY at the point A	$5,52.10^{-6}$	Analytical	1.10^{-12}
DY at the point B	$5,52.10^{-6}$	Analytical	1.10^{-12}
DY at the point O	$8,02.10^{-6}$	Analytical	1.10^{-12}

3.4 Comments

This valid test:

- the calculation of the matrix of rigidity and the vectors second members (taken into account of the pressure distributed on quadratic elements of edges),
- postprocessing X-FEM elements $P2$.

4 Modeling B

4.1 Characteristics of modeling

Modeling: `C_PLAN` .

The structure is a healthy rectangle, into which an interface is introduced directly into the command file using the operator `DEFI_FISS_XFEM` [U4.82.08]. The interface is present at a distance $L_Y=1,8\,m$ higher edge of the plate.

4.2 Characteristics of the grid

Many nodes: 597
Many meshs and types: 180 `QUAD8` for the plate and 46 `SEG3` for the edges.

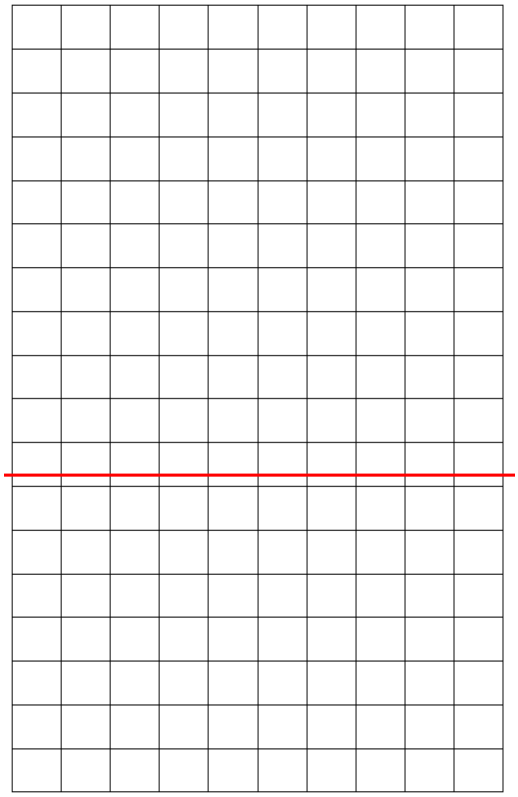

Figure 4.2 -a: Grid 2D quadrangle and position of the interface

4.3 Sizes tested and results

Displacements resulting from the operator `STAT_NON_LINE` are post-taities in order to manner to recover the values with the nodes of the crack resulting from the new grid.

Identification	Reference	Aster	tolerance
DX at the point A	1.10^{-5}	Analytical	1.10^{-12}
DX at the point B	-1.10^{-5}	Analytical	1.10^{-12}
DY at the point	$5,52.10^{-6}$	Analytical	1.10^{-12}

A			
DY at the point B	$5,52.10^{-6}$	Analytical	1.10^{-12}
DY at the point O	$8,02.10^{-6}$	Analytical	1.10^{-12}

4.4 Comments

This valid test:

- the calculation of the matrix of rigidity and the vectors second members (taken into account of the pressure distributed on quadratic elements of edges),
- under cutting (configuration in right interface and elements on right board),
- postprocessing X-FEM elements $P2$.

5 Modeling C

5.1 Characteristics of modeling

Modeling: 3D .

The structure is parallelepipedic rectangular healthy, into which an interface is introduced directly into the command file using the operator DEFI_FISS_XFEM [U4.82.08]. The interface is present at a distance $L_Y=1{,}8\,m$ higher edge of the plate.

5.2 Characteristics of the grid

Many nodes: 8644
Many meshs and types: 6989
of which TRIA6: 2600
of which TETRA10 :4389

5.3 Sizes tested and results

Displacements resulting from the operator STAT_NON_LINE are post-treaties so as to recover the values with the nodes of the crack resulting from the new grid.

Identification	Reference	Aster	tolerance
DX on the line AA'	1.10^{-5}	Analytical	1.10^{-10}
DX on the line BB'	-1.10^{-5}	Analytical	1.10^{-10}
DY at the point A'	$5{,}52.10^{-6}$	Analytical	1.10^{-10}
DY at the point B'	$5{,}52.10^{-6}$	Analytical	1.10^{-10}
DY at the point O	$8{,}02.10^{-6}$	Analytical	1.10^{-10}
DZ on the line $A'B'$	$-2{,}0.10^{-8}$	Analytical	1.10^{-10}

6 Modeling D

6.1 Characteristics of modeling

Modeling: 3D .

The structure is parallelepipedic rectangular healthy, into which an interface is introduced directly into the command file using the operator DEFI_FISS_XFEM [U4.82.08]. The interface is present at a distance $L_Y=1{,}8\,m$ higher edge of the plate.

6.2 Characteristics of the grid

Many nodes: 5653
Many meshs: 3800
of which SEG3: 100
of which TRIA6: 2400
of which QUAD8: 100
of which PENTA15:1200

6.3 Sizes tested and results

Displacements resulting from the operator STAT_NON_LINE are post-treaties so as to recover the values with the nodes of the crack resulting from the new grid.

Identification	Reference	Aster	tolerance
DX on the line AA'	1.10^{-5}	Analytical	1.10^{-10}
DX on the line BB'	-1.10^{-5}	Analytical	1.10^{-10}
DY at the point A'	$5{,}52.10^{-6}$	Analytical	1.10^{-10}
DY at the point B'	$5{,}52.10^{-6}$	Analytical	1.10^{-10}
DY at the point O	$8{,}02.10^{-6}$	Analytical	1.10^{-10}
DZ on the line $A'B'$	$-2{,}0.10^{-8}$	Analytical	1.10^{-10}

7 Modeling E

7.1 Characteristics of modeling

Modeling: `3D` .

The structure is parallelepipedic rectangular healthy, into which an interface is introduced directly into the command file using the operator `DEFI_FISS_XFEM` [U4.82.08]. The interface is present at a distance $L_Y=1,8\,m$ higher edge of the plate.

7.2 Characteristics of the grid

Many nodes: 4453
Many meshs: 2000
of which `SEG3:` 100
of which `QUAD:` 1300
of which `HEXA20`:600

7.3 Sizes tested and results

Displacements resulting from the operator `STAT_NON_LINE` are post-treaties so as to recover the values with the nodes of the crack resulting from the new grid.

Identification	Reference	Aster	tolerance
DX on the line AA'	1.10^{-5}	Analytical	1.10^{-10}
DX on the line BB'	-1.10^{-5}	Analytical	1.10^{-10}
DY at the point A'	$5{,}52.10^{-6}$	Analytical	1.10^{-10}
DY at the point B'	$5{,}52.10^{-6}$	Analytical	1.10^{-10}
DY at the point O	$8{,}02.10^{-6}$	Analytical	1.10^{-10}
DZ on the line $A'B'$	$-2{,}0.10^{-8}$	Analytical	1.10^{-10}

8 Summary of the results of modeling

The goals of this test are achieved.

- It was a question of showing the feasibility of the taking into account of enrichment by the Heaviside function of the classical functions of form on quadratic elements. Only the case of a crack crossing the structure completely was considered (interface).
- The method is validated in $2D$, $P2$ on a grid quadrangle.
- The method is validated in $3D$, $P2$ on a rectangular parallelepipedic grid.
- One obtains a better solution with modeling C_PLAN that modeling 3D.