

SSND105 - Law of behavior visco-élasto-plastic with effect of memory

Summary:

The problem is quasi-static non-linear in mechanics of the structures. Laws tested, `VMIS_CIN2_MEMO` and `VISC_CIN2_MEMO`, are laws with non-linear kinematic work hardening, isotropic work hardening, and memory of maximum work hardening. One analyzes the answer in a material point, with a pre-work hardening, then a cyclic loading.

Modeling A makes it possible to validate the effect of memory with `VMIS_CIN2_MEMO` in a case where work hardening is purely isotropic, for a simple traction. The reference solution for this modeling is analytical.

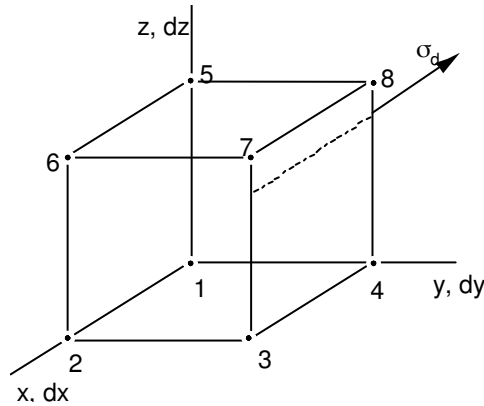
Modeling B compares the results got with effect of memory, and without effect of memory between the laws `VISC_CIN2_MEMO` and `VISCOCHAB`, for a cyclic loading with pre-work hardening.

Modeling C is similar to modeling B, and makes it possible to validate the two models into axisymmetric.

Modeling D is similar to modeling C, and makes it possible to check that the models being able to take into account the effect of nonproportionality give in this case results identical to the preceding models.

1 Problem of reference

1.1 Geometry



Face YZ : (1, 4, 5, 8)

Face XZ : (1, 2, 5, 6)

Face 1YZ : (2, 3, 6, 7)

Face 1XZ : (4, 3, 8, 7)

σ_d : pression

1.2 Properties of materials

Isotropic elasticity $E=145\,000\text{ MPa}$ $\nu=0.3$

Elastoplasticity with effect of memory (modeling A): model VISC_CIN2_MEMO

Isotropic work hardening

R_0 35 MPa B 12

Memory

DRIVEN 19 Q_0 140 MPa

ETA 0.5 Q_M 460 MPa

Kinematic work hardening (modeling A)

C1 0 G1_0 0

C2 0 G2_0 0

Viscoplasticity with effect of memory (modelings B and C): model VISC_CIN2_MEMO

Parameters identical to the preceding values, except:

LEMAITRE

UN_SUR_K $1/70(\text{MPa S}^{1/N})^{-1}=0.0142857$ NR 24

Kinematic work hardening (modeling B)

C1 1950 MPa G1_0 50

C2 65000 MPa G2_0 1300

Model viscoplasticity VISCOCHAB (modelings B and C)

k	35 MPa	B	12	ETA	0.5	C2	65000 MPa
A_K	0	M_R	1	C1	1950 MPa	M_2	1
A_R	1	G_R	0	M_1	1	D2	1
K_0	$70\text{ MPa S}^{1/N}$	DRIVE N	19	D1	1	G_X2	0
NR	24	Q_M	460	G_X1	0	G2_0	1300 MPa
ALP	0 MPa	Q_0	40 MPa	G1_0	50 MPa	A_I	1

QR_0	200 MPa	
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1.3 Boundary conditions and loadings

$$\begin{aligned} N6 & \quad dy = dz = 0 \\ N2 & \quad dy = 0 \\ FACE1YZ & \quad dx = 0 \end{aligned}$$

Traction (modeling A): $FACEYZ \quad F_x = -0.25 \times coef \quad Coef = 120 \text{ for } t = 8s .$

Pre-work hardening (modeling B) $FACEYZ \quad S_{xx} = 250 \text{ MPa} \times coef2 \quad S_{xx} = 250 \text{ Mpa} \times coef2$
 $coef2 = 1 \text{ for } t = 10s , \text{ then discharge } (coef2 = 0) \text{ for } t = 11s .$

From 11s , 20 cycles in imposed deformation (+ 0.5%)

2 Reference solution

2.1 Method of calculating used for the reference solution

One can calculate the analytical solution corresponding to pre-work hardening (traction monotonous, modeling A):

The system of equations of the problem with effect of memory is written (20 equations) [R5.03.04]:

Elasticity : $\tilde{\sigma} = 2 \mu (\tilde{\xi} - \varepsilon^p)$

$$\text{Criterion of plasticity} \quad \left\| \tilde{\sigma} - \frac{2}{3} C_1 \alpha_0 - \frac{2}{3} C_2 \alpha_2 \right\|_{eq} = R_0 + R(p)$$

$$\text{Plastic flow: } \dot{\varepsilon}^p = \dot{p} \mathbf{n} \quad \text{with} \quad \mathbf{n} = \frac{3}{2} \frac{\tilde{\sigma} - \frac{2}{3} C_1 \alpha_0 - \frac{2}{3} C_2 \alpha_0}{\left\| \tilde{\sigma} - \frac{2}{3} C_1 \alpha_0 - \frac{2}{3} C_2 \alpha_0 \right\|_{eq}}$$

Isotropic work hardening : $\dot{R} = b(Q - R)\dot{p}$

Memory of maximum work hardening: $Q = Q_0 + (Q_m - Q_0)(1 - e^{-2\mu q})$

where Q is determined by:

• a field $F(\varepsilon^p, \xi, q) = \frac{2}{3} J_2(\varepsilon^p - \xi) - q \leq 0$ characterizing the maximum plastic deformations, whose

Q measures the ray and ξ the center

• ξ is calculated according to a law of normality i.e.: $\dot{\xi} = \dot{q} \mathbf{n}^*$, with $\mathbf{n}^* = \frac{3}{2} \frac{\varepsilon^p - \xi}{J_2(\varepsilon^p - \xi)}$

On the surface of the field of maximum work hardening, one has $F = 0$. By applying the condition

$$dF = 0, \text{ one obtains the expression of speed: } \dot{q} = \eta \langle \mathbf{n} : \mathbf{n}^* \rangle \dot{p}$$

For a material point in uniaxial load, the fields (uniform) have as components:

$$\sigma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \varepsilon^p = p \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

In this case, at the time of the first uniaxial load in direction X:

$$\begin{aligned} \xi^- &= 0 \\ q^- &= 0 \\ \Delta q &= \eta \varepsilon_x^p \end{aligned}$$

In this case, $q = \frac{1}{2} \Delta \varepsilon_{\max}^p$, implies that $\eta = \frac{1}{2}$. In this case, $\Delta \xi = \frac{1}{2} (\varepsilon^p)$

Moreover, in the case of a cycle of symmetrical traction compression (in plastic deformation), one obtains, during the first symmetrical discharge (with $\eta = \frac{1}{2}$):

$$\begin{aligned} \xi^- &= \frac{1}{2} \varepsilon_{\max}^p \\ q^- &= \frac{1}{2} \varepsilon_{xx \max}^p \\ \Delta q &= \eta \left[\frac{2}{3} J_2(\varepsilon^p) - q^- \right] = \eta \left[\varepsilon_{xx \min}^p - \xi^- \right] - \frac{1}{2} \varepsilon_{xx \max}^p = \frac{1}{2} \left[\varepsilon_{xx \min}^p \right] \end{aligned}$$

$$\begin{aligned} q &= q^- + \Delta q = \varepsilon_{xx \max}^p = \frac{1}{2} \Delta \varepsilon_{xx}^p \\ \Delta \xi &= \frac{(1 - \eta) \Delta q (\varepsilon^p - \xi^-)}{\eta q^- + \Delta q} = -\frac{1}{2} \Delta \varepsilon_{xx \max}^p \end{aligned}$$

$\xi = \xi^0 + \Delta \xi = 0$ what corresponds well to the expected result (cf [bib2]): field $F = 0$ centered on the origin, and of half the amplitude of plastic deformation.

In the case of an increasing traction, and if kinematic work hardening is neglected, the equations to be solved become:

$$\sigma \leq R_0 + R(p)$$

The function thus should be calculated $R(p)$, such as:

$$dR = b(Q - R)dp \quad \text{with} \quad Q = Q_0 + (Q_m - Q_0)(1 - e^{-2\mu q})$$

Moreover, it is considered that one is in load, therefore $F(\varepsilon^p, \xi, q) = 0$

$$dq = \eta dp$$

The differential equation thus should be integrated:

$$dR = b(Q_0 + (Q_m - Q_0)(1 - e^{-2\mu\eta}) - R)dp$$

what is integrated in the following way:

$$dR + bR = 0 \Rightarrow R = \lambda e^{-bp}$$

Method of variation of the constant: $R = \lambda(p)e^{-bp}$

$$d\lambda e^{-bp} = b(Q_m - (Q_m - Q_0)e^{-2\mu\eta p})dp$$

$$d\lambda = bQ_m e^{bp} dp + b(Q_0 - Q_m)e^{(b-2\mu\eta)p} dp$$

while integrating:

$$\lambda = Q_m e^{bp} + \frac{b(Q_0 - Q_m)}{(b - 2\mu\eta)} e^{(b-2\mu\eta)p} + K$$

from where

$$R(p) = Q_m + \frac{b(Q_0 - Q_m)}{(b - 2\mu\eta)} e^{-2\mu\eta p} + K e^{-bp}$$

The constant K is defined by the initial conditions: for $p=0$, $R=0$

$$0 = Q_m + \frac{b(Q_0 - Q_m)}{(b - 2\mu\eta)} + K \text{ that is to say } K = \frac{b(Q_m - Q_0)}{(b - 2\mu\eta)} - Q_m = \frac{-bQ_0 + 2\mu\eta Q_m}{(b - 2\mu\eta)}$$

Finally:

$$R(p) = Q_m + \frac{b(Q_0 - Q_m)}{(b - 2\mu\eta)} e^{-2\mu\eta p} + \frac{2\mu\eta Q_m - bQ_0}{(b - 2\mu\eta)} e^{-bp}$$

One thus has in load: $\sigma = R_0 + R(p)$

2.2 Results of reference

Modeling a:

Value of $SIXX$ at the final moment: $\sigma = R_0 + R(p)$

$$\text{with } R(p) = Q_m + \frac{b(Q_0 - Q_m)}{(b - 2\mu\eta)} e^{-2\mu\eta p} + \frac{2\mu\eta Q_m - bQ_0}{(b - 2\mu\eta)} e^{-bp}$$

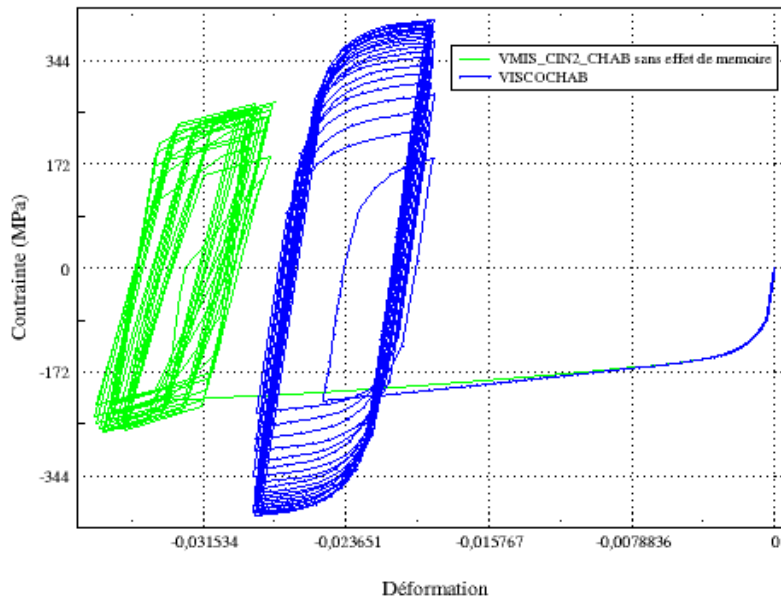
$t=8s$, one must find $SIXX = 120 \text{ Mpa}$.

For that one calculation $R(p)$ starting from the value of p at the moment $t=8s$.

Modeling b:

One will compare the results obtained with `VISC_CIN2_MEMO` with those obtained with `VISCOCHAB`, at the end of pre-work hardening and at the end of 10 cycles. The curves below highlight of the effect of memory (per comparison with `VISC_CIN2_CHAB` who does not model it): after a pre-work hardening, the cycles with imposed deformation are stabilized with an amplitude of constraints higher than that obtained without effect of memory:

Essai cyclique DEPS= $\pm 0.5\%$



2.3 Uncertainty on the solution

- Analytical modeling a:
- Modeling b: intercomparison enters `VISCOCHAB` and `VISC_CIN2_MEMO` : precision of the digital integration, estimated at less 1% .
- Modeling C: validation ddes behaviors in `2D AXIS` ; the results must be identical to those of modeling B.

2.4 Bibliographical references

- [1] R5.03.04 "Behaviors élasto-visco-plastics of J.L.Chaboche".
- [2] J.M.PROIX " Comportement viscoplastique fascinant in account not proportionality of the loading" EDF R & D - CR-AMA12-284, 12/12/12

3 Modeling A

3.1 Characteristics of modeling

Modeling 3D, 1 hexa8. Simple traction.

3.2 Sizes tested and results

Identification	Reference	tolerance
σ_{xx}	120	0,20%
p	$3.70925 E - 2$	0,10%

4 Modeling B

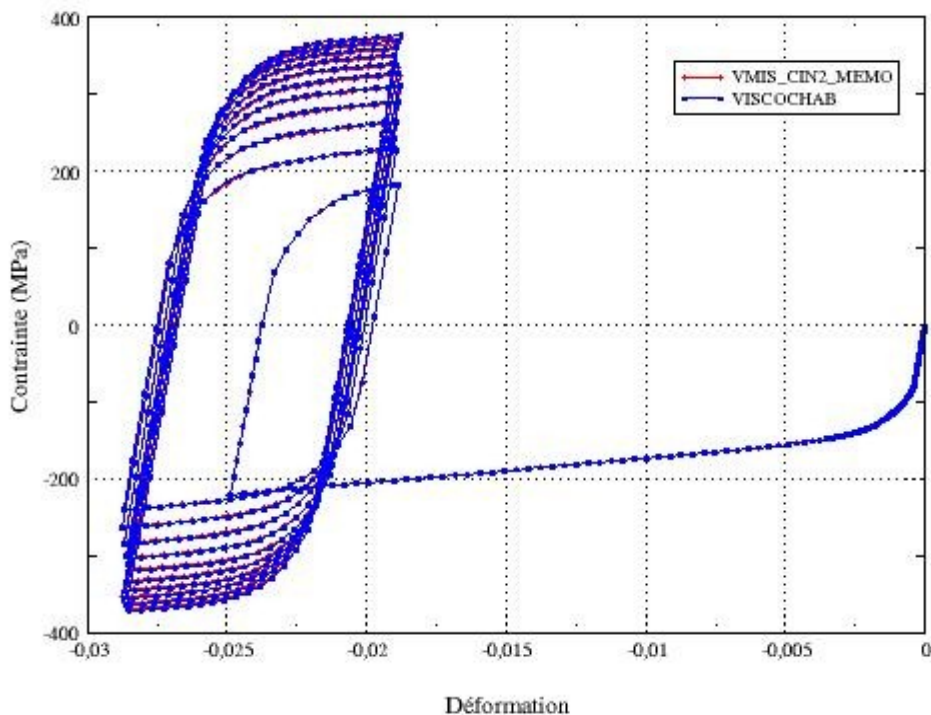
4.1 Characteristics of modeling

Pre-work hardening in traction then cycles with imposed deformation, comparison VISCOCHAB and VISC_CIN2_MEMO. 250 pas de time for 10 cycles.

4.2 Sizes tested and results

Identification	Moment	VISCOCHAB	VISC_CIN2_MEMO	% difference
σ_{xx}	10	220	220	0
σ_{xx}	11	0	0	0
σ_{xx}	113.5	$3.75459E + 02$	$3.72353E + 02$	-0.8
ϵ_{xx}	113.5	$-1.87638E - 02$	$-1.87638E - 02$	0

Essai cyclique DEPS=+/-0.5%



4.3 Remarks

The difference of 0.8% on the constraints at the final moment grows blurred if the step of time is refined: with a step of time 2 times finer, the variation become 0.4% .

5 Modeling C

5.1 Characteristics of modeling

Pre-work hardening in traction then cycles with imposed deformation, comparison VISCOCHAB and VISC_CIN2_MEMO. 250 pas de time for 10 cycles. Modeling 2D AXIS .

5.2 Sizes tested and results

Identification	Momen t	VISCOCHAB	VISC_CIN2_MEMO	% difference
σ_{xx}	113.5	3.75459E+02	3.72353E+02	-0.8
ε_{xx}	113.5	-1.87638E-02	-1.87638E-02	

6 Modeling D

6.1 Characteristics of modeling

This modeling is identical to modeling C, with models of the type NRAD (not radially). Results of the models VISC_MEMO_NRAD and VISC_CIN2_NRAD can be compared with those of modeling C, since the effect of nonradiality must be inoperative here. Tests of VMIS_MEMO_NRAD, VMIS_CIN2_NRAD (without viscosity) are of nonregression.

6.2 Sizes tested and results

Behavior VISC_MEMO_NRAD

Identification	Momen t	Reference VISC_CIN2_MEMO
σ_{xx}	113.5	369.679
ε_{xx}	113.5	-1.8773E-02

Behavior VISC_CIN2_NRAD

Identification	Momen t	Reference VISC_CIN2_CHAB
σ_{xx}	113.5	269.6
σ_{xx}	10	220

Behavior VMIS_MEMO_NRAD

Identification	Momen t	Reference
σ_{xx}	113.5	372.2 (not regression)
σ_{xx}	10	220 (analytical)

Behavior VMIS_CIN2_NRAD

Identification	Momen t	Reference VISC_CIN2_MEMO
σ_{xx}	113.5	225.254 (not regression)

σ_{xx}	10	220 (analytical)
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7 Summary of the results

Four modelings make it possible to validate, on a material point, the behaviors of the kinematic type nonlinear for purpose of memory, in plasticity and viscoplasticity.