

## HPLA100 - Heavy thermoelastic hollow roll in uniform rotation

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### Summary

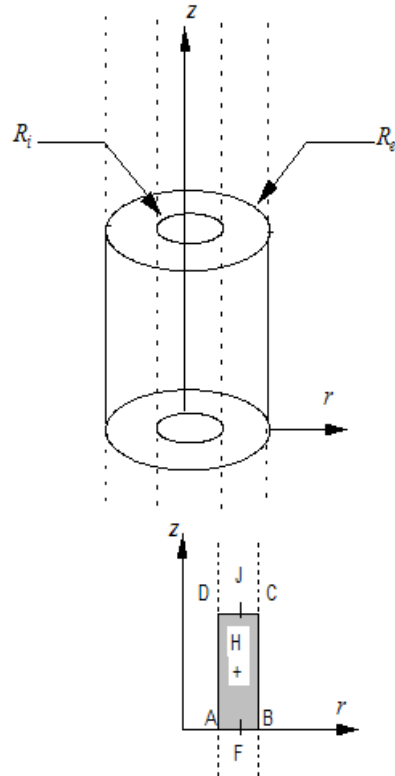
The purpose of this test is to test the second members corresponding to the effects of gravity, a thermal dilation and acceleration due to a uniform rotation. For modelings *C* and *D* (hull 3D), a chained thermoelastic calculation and a thermoelastoplastic calculation without plastic evolution were carried out.

One has the results for modelings:

- 2D axisymmetric: axisymmetric isoparametric finite elements on meshes QUAD8,
- axisymmetric hulls: axisymmetric isoparametric finite elements on meshes SEG3 (linear grid of the meridian section),
- hulls 3D : finite elements MEC3QU9H, MEC3TR7H on meshes QUAD9 and TRIA7, respectively,
- plates *DKT* : finite elements plans DKQ, DKT on meshes QUAD4 and TRIA3, respectively. One tests also the orthotropic hulls of DEFI\_COMPOSITE for two layers of the same isotropic material and the keyword ELAS\_COQU of DEFI\_MATERIAU for the assignment of characteristics of homogenized plates.

## 1 Problem of reference

### 1.1 Geometry



Geometrical of the cylinder ( $m$ ) :

Interior ray	$R_i = 19.5$
External ray	$R_e = 20.5$
Not $F$	$R = 20.0$
Thickness	$h = 1.0$
Height	$L = 10.0$

### 1.2 Properties of material

The material is homogeneous isotropic, thermoelastic linear, the initial state is virgin.

Young modulus	$E = 2.0 E - 5 N.mm^{-2}$
Poisson's ratio	$\nu = 0.3$
Voluminal density	$\rho = 8.0 E^{-6} kg.mm^{-3}$
Dilation coefficient	$\alpha = 1.0 E^{-5} \text{ } ^\circ C^{-1}$

## 1.3 Boundary conditions and loadings

- Imposed displacement:
  - $\Omega = 1.0 s^{-1}$  according to the axis  $OZ$
- Imposed loading:
  - gravity,  $g = 10.0 m.s^{-2}$  according to the axis  $OZ$
  - force of traction on the higher face:  $-160.0 E^{-4} N$  equivalent with a force distributed on  $CD$  of  $-8.0 E^{-4} N.mm^{-1}$
- Thermal dilation:  $T(\rho) - T_{ref}(\rho) = \frac{(T_s + T_i)}{2} + \frac{(T_s - T_i) \cdot (r - R)}{h}$  with:
  - case 1:  $T_s = 0.5^\circ C, T_i = -0.5^\circ C, T_{ref} = 0.0^\circ C$
  - case 2:  $T_s = 0.1^\circ C, T_i = 0.1^\circ C, T_{ref} = 0.0^\circ C$

These fields of temperature are calculated with `THER_LINEAIRE`, using a stationary calculation on the same grid, but with a model `PLAN` in order to have a solution refines in the thickness.

The boundary conditions in displacement (and rotation) being different according to modeling considered, they will be described later on (in the paragraphs relating to modelings).

## 2 Reference solution

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### 2.1 Method of calculating used for the reference solution

The method of calculating used for the reference solution was determined by F. Voldoire (EDF R & D/AMA) and is presented in the appendix.

The analytical results of reference are:

- displacements and rotations,
- axial stress, generalized efforts (in theory of hulls),

in skins internal and external on the sections  $AB$  and  $CD$ .

In 2D axisymmetric, the total solutions given in appendix are such as  $\varepsilon_{rz}=0$  where  $r$  and  $z$  are the directions radial and axial cylinder, respectively. For the loadings of uniform rotation and thermal dilation, the boundary conditions are selected so that the solutions do not depend on  $z$  (one has in particular  $\varepsilon_{zz}=0$ ).

For the hulls, with boundary conditions equivalent, rotation  $\theta_\theta$  around the axis orthoradial is worthless for the loadings of uniform rotation and thermal dilation, which is not the case of the loading of gravity where rotation is constant (the cylinder is formatted conical then). On the other hand in all the cases, the transverse distortion is worthless; thus the theories of Coils - Kirchhoff and those of Hencky-Mindlin provide the same reference solution.

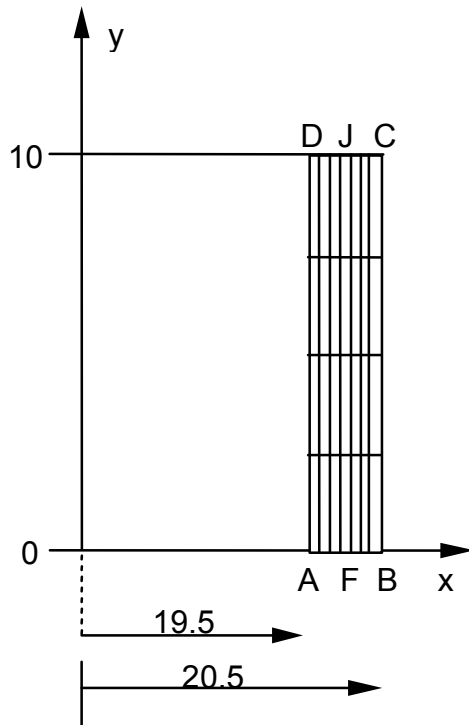
### 2.2 Reference variable

- $DX$  : displacement according to the axis  $OX$ ,
- $DY$  : displacement according to the axis  $OY$ ,
- $DZ$  : displacement according to the axis  $OZ$ ,
- $NXX$  : normal effort according to the axis  $OX$ ,
- $NYY$  : normal effort according to the axis  $OY$ ,
- $MXX$  : moment around the axis  $OX$ ,
- $MYY$  : moment around the axis  $OY$ ,
- $SIXX$  : constraint according to the axis  $OX$ ,
- $SIYY$  : constraint according to the axis  $OY$ .

## 3 Modeling A

### 3.1 Characteristics of modeling

Finite elements 2D axisymmetric



The discretized geometry is represented above:

<i>Bord</i>	<i>group_no</i>
<i>BC</i>	<i>BC</i>
<i>DA</i>	<i>DA</i>
<i>AB</i>	<i>BAS</i>
<i>CD</i>	<i>HAUT</i>

### 3.2 Characteristics of the grid

The grid is regular: 4 elements in the height, 8 in the thickness.

Many nodes: 121

Number of meshes and type: 32 QUAD8

### 3.3 Boundary conditions in displacement

#### 3.3.1 Gravity

Displacement  $DY$  is blocked at the point  $F$  only.

#### 3.3.2 Rotation

Displacement  $DY$  is blocked on the sides  $[AB]$  (GROUP\_NO = 'LOW') and on  $[CD]$  (GROUP\_NO = 'HIGH').

### 3.3.3 Thermal case of dilation n°1

Displacement  $DY$  is blocked on all the structure.

### 3.3.4 Thermal case of dilation n°2

Displacement  $DY$  is blocked on the sides  $[AB]$  (GROUP\_NO = 'LOW') and  $[CD]$  (GROUP\_NO = 'HIGH').

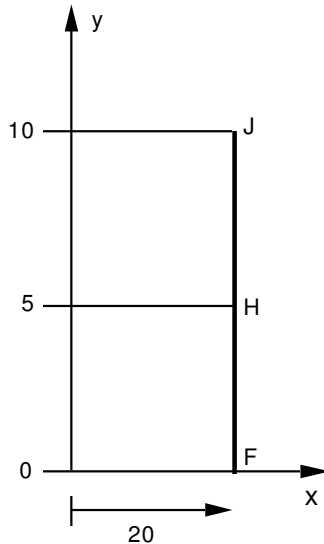
## 3.4 Results of modeling A

Identification	Node (Mesh)	Value tested	Reference
Gravity	$N78$	$DX$ (mm)	- 2.34000 10 <sup>-8</sup>
	$N120$	$DY$ (mm)	- 1,185 10 <sup>-9</sup>
	$N13$	$DY$ (mm)	1.2150 10 <sup>-9</sup>
	$N78(M13)$	$SIYY$ (MPa)	8.0000 10 <sup>-4</sup>
Uniform rotation - centrifugal force	$N120$	$DX$ (mm)	- 2.94240 10 <sup>-7</sup>
	$N13$	$DX$ (mm)	2.88010 10 <sup>-7</sup>
	$N120(M1)$	$SIYY$ (MPa)	9.94880 10 <sup>-4</sup>
	$N13(M32)$	$SIYY$ (MPa)	9.26310 10 <sup>-4</sup>
Dilation case 1	$N120$	$DX$ (mm)	1.056145 10 <sup>-6</sup>
	$N13$	$DX$ (mm)	1.110317 10 <sup>-6</sup>
	$N120(M1)$	$SIYY$ (MPa)	1.4321427
Dilation case 2	$N120$	$DX$ (mm)	2.53500 10 <sup>-5</sup>
	$N120(M1)$	$SIYY$ (MPa)	- 2.00000 10 <sup>-1</sup>

## 4 Modeling B

### 4.1 Characteristics of modeling

Elements of axisymmetric hull



The discretized geometry is represented above. One chooses the theory of hulls of Coils - Kirchhoff (for that one takes a transverse coefficient of shearing of  $10^6$ ). One neglects the correction of metric in the thickness. The thickness is of  $1\text{ mm}$ .

node	GROUP_NO
<i>J</i>	<i>GRNO13</i>
<i>H</i>	<i>GRNO14</i>
<i>F</i>	<i>GRNO6</i>

### 4.2 Characteristics of the grid

Many nodes: 21

Number of meshes and type: 10 SEG3

### 4.3 Boundary conditions in displacement and rotation

#### 4.3.1 Gravity

Displacement  $DY$  is blocked at the point  $F$  only (*GRNO6*).

#### 4.3.2 Rotation

Displacement  $DY$  is blocked at the point  $F$  (*GRNO6*) at the point  $J$  (*GRNO13*).  
Rotation around the axis  $Z$  is worthless in these two points.

#### 4.3.3 Thermal case of dilation n°1

Displacement  $DY$  as well as rotation around the axis  $Z$  are blocked on all the structure ( $\text{GROUP\_NO} = \text{'GRNO15'}$ ).

## 4.3.4 Thermal case of dilation n°2

Displacement  $DY$  is blocked at the point  $F$  (GROUP\_NO = 'GRN006') and at the point  $J$  (GROUP\_NO = 'GRN013'). Rotation around the axis  $Z$  is worthless on the same groups of nodes.

## 4.4 Results of modeling B

Identification	Node (Mesh)	Value tested	Reference
<b>Gravity</b>	$J$	$DX (mm)$	$-2.40000 \cdot 10^{-8}$
	$H$	$DY (mm)$	$5.00000 \cdot 10^{-9}$
	$H$	$DRZ$	$2.40000 \cdot 10^{-9}$
	$J (MI0)$	$NXX (N)$	$8.00000 \cdot 10^{-4}$
	$J (MI0)$ Internal skin	$SIXX (MPa)$	$8.00000 \cdot 10^{-4}$
<b>Rotation - centrifugal force</b>	$F$	$DX (mm)$	$2.91200 \cdot 10^{-7}$
	$F (MI)$	$NXX (N)$	$9.60000 \cdot 10^{-4}$
	$F (MI)$ Internal skin	$SIXX (MPa)$	$9.60000 \cdot 10^{-4}$
<b>Dilation case 1</b>	$F (MI)$	$MXX (N.mm)$	$-2.38095 \cdot 10^{-1}$
	$F (MI)$ Internal skin	$SIXX (Mpa)$	1.428571
<b>Dilation case 2</b>	$F$	$DX (mm)$	$26.0000 \cdot 10^{-6}$
	$F (MI)$	$NXX (N)$	$-2.00000 \cdot 10^{-1}$
	$F (MI)$ Internal skin	$SIXX (MPa)$	$-2.00000 \cdot 10^{-1}$

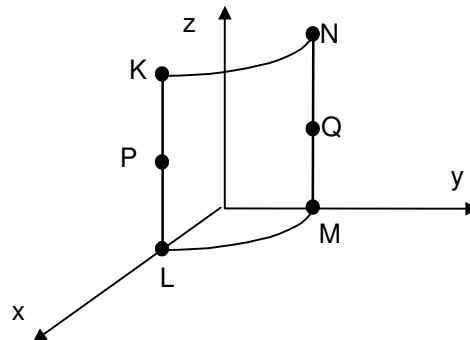


## 5 Modeling C

### 5.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of hull 3D: QUAD9



The discretized geometry is represented above.

<i>point</i>	<i>nœud</i>
<i>K</i>	<i>NO72</i>
<i>L</i>	<i>NO1</i>
<i>M</i>	<i>NO33</i>
<i>N</i>	<i>NO39</i>
<i>P</i>	<i>NO186</i>
<i>Q</i>	<i>NO190</i>

The factor of correction of shearing  $A_{CIS}$  is worth  $5/6$  (theory of hulls of Reissner).

### 5.2 Characteristics of the grid

Many external nodes: 121

Many meshes and types: 32 QUAD9 + 8 SEG3

### 5.3 Boundary conditions in displacement and rotation

#### 5.3.1 Gravity

Displacement  $DZ$  is blocked on the group of nodes  $LM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

## 5.3.2 Rotation

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

## 5.3.3 Thermal case of dilation n°1

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

## 5.3.4 Thermal case of dilation n°2

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

## 5.4 Results of modeling C

Identification	Node (mesh)	Value tested	Reference
Gravity	<i>K</i>	<i>DX</i> (mm)	- 2.40000 10 <sup>-8</sup>
	<i>N</i>	<i>DY</i> (mm)	- 2.40000 10 <sup>-8</sup>
	<i>P</i>	<i>DZ</i> (mm)	5.00000 10 <sup>-9</sup>
	<i>Q</i>	<i>DZ</i> (mm)	5.00000 10 <sup>-9</sup>
	<i>P</i>	<i>DRY</i>	2.40000 10 <sup>-9</sup>
	<i>Q</i>	<i>DRX</i>	2.40000 10 <sup>-9</sup>
	<i>K</i> ( <i>M4</i> )	<i>NYY</i> (N)	8.00000 10 <sup>-4</sup>
	<i>N</i> ( <i>M32</i> )	<i>NYY</i> (N)	8.00000 10 <sup>-4</sup>
	<i>K</i> ( <i>M4</i> ) Internal skin	<i>SIYY</i> (MPa)	8.00000 10 <sup>-4</sup>
	<i>N</i> ( <i>M32</i> ) Internal skin	<i>SIYY</i> (MPa)	8.00000 10 <sup>-4</sup>
Rotation – centrifugal force	<i>L</i>	<i>DX</i> (mm)	2.91200 10 <sup>-7</sup>
	<i>M</i>	<i>DY</i> (mm)	2.91200 10 <sup>-7</sup>
	<i>L</i> ( <i>M1</i> )	<i>NYY</i> (N)	9.60000 10 <sup>-4</sup>
	<i>M</i> ( <i>M29</i> )	<i>NYY</i> (N)	9.60000 10 <sup>-4</sup>
	<i>L</i> ( <i>M1</i> ) Internal skin	<i>SIYY</i> (MPa)	9.84600 10 <sup>-4</sup>
	<i>M</i> ( <i>M29</i> ) Internal skin	<i>SIYY</i> (MPa)	9.84600 10 <sup>-4</sup>
Dilation case 1	<i>L</i> ( <i>M1</i> )	<i>MYY</i> (N.mm)	- 2.38095 10 <sup>-1</sup>
	<i>M</i> ( <i>M29</i> )	<i>MYY</i> (N.mm)	- 2.38095 10 <sup>-1</sup>
	<i>L</i> ( <i>M1</i> ) Internal skin	<i>SIYY</i> (MPa)	1.428571
	<i>M</i> ( <i>M29</i> ) Internal skin	<i>SIYY</i> (MPa)	1.428571
Dilation case 2	<i>L</i>	<i>DX</i> (mm)	25.9946 10 <sup>-6</sup>
	<i>M</i>	<i>DY</i> (mm)	25.9946 10 <sup>-6</sup>
	<i>L</i> ( <i>M1</i> )	<i>NYY</i> (N)	- 2.00000 10 <sup>-1</sup>
	<i>M</i> ( <i>M29</i> )	<i>NYY</i> (N)	- 2.00000 10 <sup>-1</sup>
	<i>L</i> ( <i>M1</i> ) Internal skin	<i>SIYY</i> (MPa)	- 1.97800 10 <sup>-1</sup>
	<i>M</i> ( <i>M29</i> ) Internal skin	<i>SIYY</i> (MPa)	- 2.97800 10 <sup>-1</sup>

## 5.5 Remarks

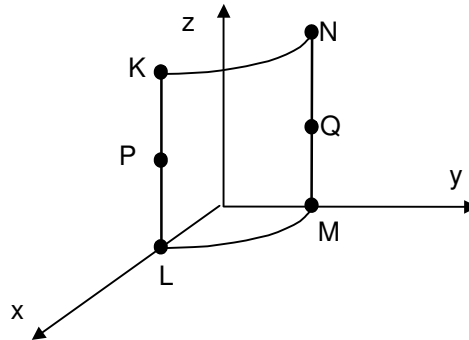
Satisfactory and identical results for calculations with MECA\_STATIQUE and STAT\_NON\_LINE.

## 6 Modeling D

### 6.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of hull 3D: TRIA7



The discretized geometry is represented above.

<i>point</i>	<i>nœud</i>
<i>K</i>	<i>NO72</i>
<i>L</i>	<i>NO1</i>
<i>M</i>	<i>NO33</i>
<i>N</i>	<i>NO39</i>
<i>P</i>	<i>NO186</i>
<i>Q</i>	<i>NO190</i>

The factor of correction of shearing  $A_{CIS}$  is worth  $5/6$  (theory of Reissner hulls).

### 6.2 Characteristics of the grid

Many external nodes: 153

Many meshes and types: 64 TRIA7 + 8 SEG3

### 6.3 Boundary conditions in displacement and rotation

#### 6.3.1 Gravity

Displacement  $DZ$  is blocked on the group of nodes  $LM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

#### 6.3.2 Rotation

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

### 6.3.3 Thermal case of dilation n°1

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

### 6.3.4 Thermal case of dilation n°2

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

## 6.4 Values tested

Identification	Node (mesh)	Value tested	Reference
<b>Gravity</b>	<i>K</i>	<i>DX (mm)</i>	$-2.40000 \cdot 10^{-8}$
	<i>N</i>	<i>DY (mm)</i>	$-2.40000 \cdot 10^{-8}$
	<i>P</i>	<i>DZ (mm)</i>	$5.00000 \cdot 10^{-9}$
	<i>Q</i>	<i>DZ (mm)</i>	$5.00000 \cdot 10^{-9}$
	<i>P</i>	<i>-DRY</i>	$2.40000 \cdot 10^{-9}$
	<i>Q</i>	<i>DRX</i>	$2.40000 \cdot 10^{-9}$
	<i>K (M60)</i>	<i>NYI (N)</i>	$8.00000 \cdot 10^{-4}$
	<i>N (M56)</i>	<i>NYI (N)</i>	$8.00000 \cdot 10^{-4}$
	<i>K (M60) Internal skin</i>	<i>SIYY (MPa)</i>	$8.00000 \cdot 10^{-4}$
	<i>N (M56) Internal skin</i>	<i>SIYY (MPa)</i>	$8.00000 \cdot 10^{-4}$
<b>Rotation – centrifugal force</b>	<i>L</i>	<i>DX (mm)</i>	$2.91200 \cdot 10^{-7}$
	<i>M</i>	<i>DY (mm)</i>	$2.91200 \cdot 10^{-7}$
	<i>L (M25)</i>	<i>NYI (N)</i>	$9.60000 \cdot 10^{-4}$
	<i>M (M53)</i>	<i>NYI (N)</i>	$9.60000 \cdot 10^{-4}$
	<i>L (M25) Internal skin</i>	<i>SIYY (MPa)</i>	$9.84600 \cdot 10^{-4}$
	<i>M (M53) Internal skin</i>	<i>SIYY (MPa)</i>	$9.84600 \cdot 10^{-4}$
<b>Dilation case 1</b>	<i>L (M25)</i>	<i>MYY (N.mm)</i>	$-2.38095 \cdot 10^{-1}$
	<i>M (M53)</i>	<i>MYY (N.mm)</i>	$-2.38095 \cdot 10^{-1}$
	<i>L (M25) Internal skin</i>	<i>SIYY (MPa)</i>	1.428571
	<i>M (M53) Internal skin</i>	<i>SIYY (MPa)</i>	1.428571
<b>Dilation case 2</b>	<i>L</i>	<i>DX (mm)</i>	$26.0000 \cdot 10^{-6}$
	<i>M</i>	<i>DY (mm)</i>	$26.0000 \cdot 10^{-6}$
	<i>L (M25)</i>	<i>NYI (N)</i>	$-2.00000 \cdot 10^{-1}$
	<i>M (M53)</i>	<i>NYI (N)</i>	$-2.00000 \cdot 10^{-1}$
	<i>L (M25) Internal skin</i>	<i>SIYY (MPa)</i>	$-1.97800 \cdot 10^{-1}$
	<i>M (M53) Internal skin</i>	<i>SIYY (MPa)</i>	$-1.97800 \cdot 10^{-1}$

## 6.5 Remarks

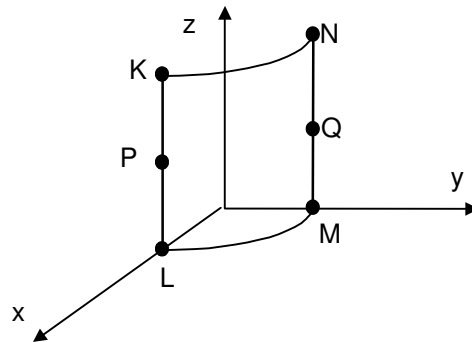
Satisfactory and identical results for calculations with MECA\_STATIQUE and STAT\_NON\_LINE.

## 7 Modeling E

### 7.1 Characteristics of modeling

Modeling of one quarter of cylinder.

Elements of plate DKQ : QUAD4



The discretized geometry is represented above.

not	node
<i>K</i>	<i>NO160</i>
<i>L</i>	<i>NO203</i>
<i>M</i>	<i>NO11</i>
<i>N</i>	<i>NO1</i>
<i>P</i>	<i>NO226</i>
<i>Q</i>	<i>NO6</i>

### 7.2 Characteristics of the grid

Many nodes: 231

Many meshes and types: 200 QUAD4 + 80 SEG2

### 7.3 Boundary conditions in displacement and rotation

#### 7.3.1 Gravity

Displacement  $DZ$  is blocked on the group of nodes  $LM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

#### 7.3.2 Thermal case of dilation n°1

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

### 7.3.3 Thermal case of dilation n°2

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

## 7.4 Results of modeling E

Identification	Node (mesh)	Value tested	Reference
Gravity	$K$	$DX (mm)$	$-2.40000 \cdot 10^{-8}$
	$N$	$DY (mm)$	$-2.40000 \cdot 10^{-8}$
	$P$	$DZ (mm)$	$5.00000 \cdot 10^{-9}$
	$Q$	$DZ (mm)$	$5.00000 \cdot 10^{-9}$
	$P$	$-DRY$	$2.40000 \cdot 10^{-9}$
	$Q$	$DRX$	$2.40000 \cdot 10^{-9}$
	$K (181)$	$NYY (N)$	$8.00000 \cdot 10^{-4}$
	$N (M200)$	$NYY (N)$	$8.00000 \cdot 10^{-4}$
	$K (M181)$ Internal skin	$SIYY (MPa)$	$8.00000 \cdot 10^{-4}$
	$N (M200)$ Internal skin	$SIYY (MPa)$	$8.00000 \cdot 10^{-4}$
Dilation case 1	$L (MI)$	$MYY (N.mm)$	$-2.38095 \cdot 10^{-1}$
	$M (M20)$	$MYY (N.mm)$	$-2.38095 \cdot 10^{-1}$
	$L (MI)$ Internal skin	$SIYY (MPa)$	1.428571
	$M (M20)$ Internal skin	$SIYY (MPa)$	1.428571
Dilation case 2	$L$	$DX (mm)$	$26.0000 \cdot 10^{-6}$
	$M$	$DY (mm)$	$26.0000 \cdot 10^{-6}$
	$L (MI)$	$NYY (N)$	$-2.00000 \cdot 10^{-1}$
	$M (M20)$	$NYY (N)$	$-2.00000 \cdot 10^{-1}$
	$L (MI)$ Internal skin	$SIYY (MPa)$	$-2.00000 \cdot 10^{-1}$
	$M (M20)$ Internal skin	$SIYY (MPa)$	$-2.00000 \cdot 10^{-1}$

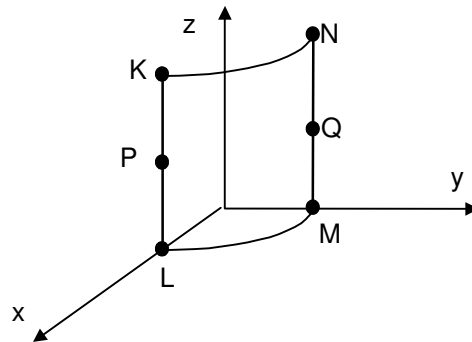


## 8 Modeling F

### 8.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of plate DKT : TRIA3



The discretized geometry is represented above.

not	node
<i>K</i>	<i>NO1</i>
<i>L</i>	<i>NO11</i>
<i>M</i>	<i>NO161</i>
<i>N</i>	<i>NO227</i>
<i>P</i>	<i>NO6</i>
<i>Q</i>	<i>NO215</i>

### 8.2 Characteristics of the grid

Many nodes: 231

Number of meshes and type: 400 TRIA3 + 80 SEG2

### 8.3 Boundary conditions in displacement and rotation

#### 8.3.1 Gravity

Displacement  $DZ$  is blocked on the group of nodes  $LM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

#### 8.3.2 Thermal case of dilation n°1

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$  .  
Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$

### 8.3.3 Thermal case of dilation n°2

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$  .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$  .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$  .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$  .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$

## 8.4 Results of modeling F

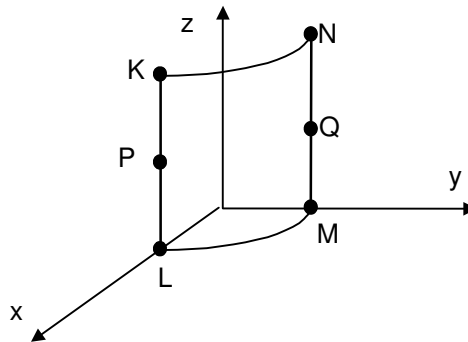
Identification	Node (mesh)	Value tested	Reference
Gravity	$K$	$DX (mm)$	$-2.40000 \cdot 10^{-8}$
	$N$	$DY (mm)$	$-2.40000 \cdot 10^{-8}$
	$P$	$DZ (mm)$	$5.00000 \cdot 10^{-9}$
	$Q$	$DZ (mm)$	$5.00000 \cdot 10^{-9}$
	$-DRX$	$-DRY$	$2.40000 \cdot 10^{-9}$
	$DRX$	$DRX$	$2.40000 \cdot 10^{-9}$
	$K (M362)$	$NYI (N)$	$8.00000 \cdot 10^{-4}$
	$N (M400)$	$NYI (N)$	$8.00000 \cdot 10^{-4}$
	$L (M362)$ Internal skin	$SIYY (MPa)$	$8.00000 \cdot 10^{-4}$
	$N (M400)$ Internal skin	$SIYY (MPa)$	$8.00000 \cdot 10^{-4}$
Dilation case 1	$L (MI)$	$MYY (N.mm)$	$-2.38095 \cdot 10^{-1}$
	$M (M39)$	$MYY (N.mm)$	$-2.38095 \cdot 10^{-1}$
	$L (MI)$ Internal skin	$SIXX (MPa)$	1.428571
	$M (M39)$ Internal skin	$SIXX (MPa)$	1.428571
Dilation case 2	$L$	$DX (mm)$	$26.0000 \cdot 10^{-6}$
	$M$	$DY (mm)$	$26.0000 \cdot 10^{-6}$
	$L (MI)$	$NYI (N)$	$-2.00000 \cdot 10^{-1}$
	$M (M39)$	$NYI (N)$	$-2.00000 \cdot 10^{-1}$
	$L (MI)$ Internal skin	$SIYY (MPa)$	$-2.00000 \cdot 10^{-1}$
	$M (M39)$ Internal skin	$SIYY (MPa)$	$-2.00000 \cdot 10^{-1}$

## 9 Modeling G

### 9.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of plate DKT : TRIA3. Modeling uses a double-layered plate whose characteristics of orthotropy are those of material defined in § 1.2 .



The discretized geometry is represented above.

not	node
<i>K</i>	<i>NO1</i>
<i>L</i>	<i>NO11</i>
<i>M</i>	<i>NO161</i>
<i>N</i>	<i>NO227</i>
<i>P</i>	<i>NO6</i>
<i>Q</i>	<i>NO215</i>

### 9.2 Characteristics of the grid

Many nodes: 231

Many meshes and types: 400 TRIA3 + 80 SEG2

### 9.3 Boundary conditions in displacement and rotation

#### 9.3.1 Gravity

Displacement  $DZ$  is blocked on the group of nodes  $LM$  .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$  .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$  .

#### 9.3.2 Thermal case of dilation n°1

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$  .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$  .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

### 9.3.3 Thermal case of dilation n°2

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

## 9.4 Results of modeling G

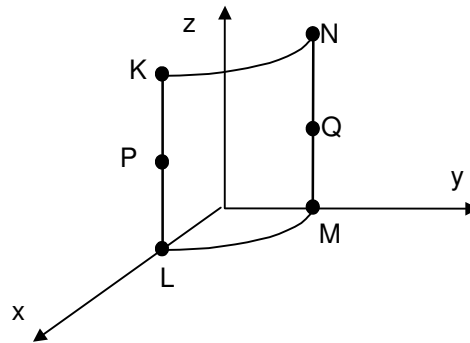
Identification	Node (mesh)	Value tested	Reference
Gravity	$K$	$DX$	$-2.40000 \cdot 10^{-8}$
	$N$	$DY$	$-2.40000 \cdot 10^{-8}$
	$P$	$DZ$	$5.0 \cdot 10^{-9}$
	$Q$	$DZ$	$5.0 \cdot 10^{-9}$
	$P$	$-DRY$	$2.40000 \cdot 10^{-9}$
	$Q$	$DRX$	$2.40000 \cdot 10^{-9}$
	$K(M362)$	$NYY$	$8.00000 \cdot 10^{-4}$
	$N(M400)$	$NYY$	$8.00000 \cdot 10^{-4}$
	$K(M362)$	$SIYY$	$8.00000 \cdot 10^{-4}$
	$N(M400)$	$SIYY$	$8.00000 \cdot 10^{-4}$
Dilation case 1	$L(M1)$	$MYY$	$-2.38095 \cdot 10^{-1}$
	$M(M39)$	$MYY$	$-2.38095 \cdot 10^{-1}$
	$L(M1)$ , internal skin	$SIXX$	1.428571
	$L(M1)$ , external skin	$SIXX$	$-1.428571$
	$M(M39)$ , internal skin	$SIXX$	1.428571
	$M(M39)$ , external skin	$SIXX$	$-1.428571$
Dilation case 2	$L$	$DX$	$26.0 \cdot 10^{-6}$
	$M$	$DY$	$26.0 \cdot 10^{-6}$
	$L(M1)$	$NYY$	$-2.00000 \cdot 10^{-1}$
	$M(M39)$	$NYY$	$-2.00000 \cdot 10^{-1}$
	$L(M1)$	$SIYY$	$-2.00000 \cdot 10^{-1}$
	$M(M39)$	$SIYY$	$-2.00000 \cdot 10^{-1}$

## 10 Modeling H

### 10.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of plate  $DKQ$  : QUAD4. Modeling uses a double-layered plate whose characteristics of orthotropy are those of material defined in § 1.2 .



The discretized geometry is represented above.

not	node
<i>K</i>	<i>NO160</i>
<i>L</i>	<i>NO203</i>
<i>M</i>	<i>NO11</i>
<i>N</i>	<i>NO1</i>
<i>P</i>	<i>NO226</i>
<i>Q</i>	<i>NO6</i>

## 10.2 Characteristics of the grid

Many external nodes: 231

Many meshes and types: 200 QUAD4 + 80 SEG2

## 10.3 Boundary conditions in displacement and rotation

### 10.3.1 Gravity

Displacement  $DZ$  is blocked on the group of nodes  $LM$  .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$  .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$  .

### 10.3.2 Thermal case of dilation n°1

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$  .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$  .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$  .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$  .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$  .

## 10.3.3 Thermal case of dilation n°2

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

## 10.4 Results of modeling H

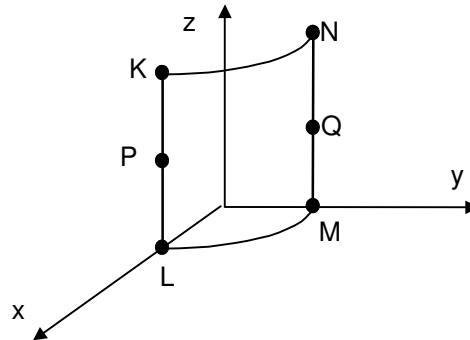
Identification	Node (mesh)	Value tested	Reference
Gravity	$K$	$DX (mm)$	$-2.40000 \cdot 10^{-8}$
	$N$	$DY (mm)$	$-2.40000 \cdot 10^{-8}$
	$P$	$DZ (mm)$	$5.00000 \cdot 10^{-9}$
	$Q$	$DZ (mm)$	$5.00000 \cdot 10^{-9}$
	$P$	$-DRX$	$2.40000 \cdot 10^{-9}$
	$Q$	$DRX$	$2.40000 \cdot 10^{-9}$
	$K (M181)$	$NYY (N)$	$8.00000 \cdot 10^{-4}$
	$N (M200)$	$NYY (N)$	$8.00000 \cdot 10^{-4}$
	$K (M181)$	$SIYY (Pa)$	$8.00000 \cdot 10^{-4}$
	$N (M200)$	$SIYY (Pa)$	$8.00000 \cdot 10^{-4}$
Dilation case 1	$L (M1)$	$MYY (N.mm)$	$-2.38095 \cdot 10^{-1}$
	$M (M20)$	$MYY (N.mm)$	$-2.38095 \cdot 10^{-1}$
	$L (M1)$ , internal skin	$SIYY (Pa)$	1.428571
	$L (M1)$ , external skin	$SIYY (Pa)$	$-1.428571$
	$M (M20)$ , internal skin	$SIYY (Pa)$	1.428571
	$M (M20)$ , external skin	$SIYY (Pa)$	$-1.428571$
Dilation case 2	$L$	$DX (mm)$	$26.0000 \cdot 10^{-6}$
	$M$	$DY (mm)$	$26.0000 \cdot 10^{-6}$
	$L (M1)$	$NYY (N)$	$-2.00000 \cdot 10^{-1}$
	$M (M20)$	$NYY (N)$	$-2.00000 \cdot 10^{-1}$
	$L (M1)$	$SIYY (Pa)$	$-2.00000 \cdot 10^{-1}$
	$M (M20)$	$SIYY (Pa)$	$-2.00000 \cdot 10^{-1}$

## 11 Modeling I

### 11.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of plate  $DKQ$  : QUAD4. Assignment of characteristics of homogenized plates corresponding to material of § 1.2 .



The discretized geometry is represented above.

not	node
$K$	NO160
$L$	NO203
$M$	NO11
$N$	NO1
$P$	NO226
$Q$	NO6

### 11.2 Characteristics of the grid

Many external nodes: 231

Many meshes and types: 200 QUAD4 + 80 SEG2

### 11.3 Boundary conditions in displacement and rotation

#### 11.3.1 Gravity

Displacement  $DZ$  is blocked on the group of nodes  $LM$  .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$  .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$  .

#### 11.3.2 Thermal case of dilation n°1

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$  .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$  .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

### 11.3.3 Thermal case of dilation n°2

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

## 11.4 Results of modeling I

Identification	Node (mesh)	Value tested	Reference
Gravity	$K$	$DX (mm)$	$-2.40000 \cdot 10^{-8}$
	$N$	$DY (mm)$	$-2.40000 \cdot 10^{-8}$
	$P$	$DZ (mm)$	$5.00000 \cdot 10^{-9}$
	$Q$	$DZ (mm)$	$5.00000 \cdot 10^{-9}$
	$P$	$-DRX$	$2.40000 \cdot 10^{-9}$
	$Q$	$DRX$	$2.40000 \cdot 10^{-9}$
	$K (M181)$	$NY (N)$	$8.00000 \cdot 10^{-4}$
	$N (M200)$	$NY (N)$	$8.00000 \cdot 10^{-4}$
	Dilation case 1	$L (M1)$	$MYY (N.mm)$
$M (M20)$		$MYY (N.mm)$	$-2.38095 \cdot 10^{-1}$
Dilation case 2	$L$	$DX (mm)$	$26.0000 \cdot 10^{-6}$
	$M$	$DY (mm)$	$26.0000 \cdot 10^{-6}$
	$L (M1)$	$NY (N)$	$-2.00000 \cdot 10^{-1}$
	$M (M20)$	$NY (N)$	$-2.00000 \cdot 10^{-1}$

## 11.5 Remarks

The local constraints are not calculated since the characteristics material of the plate are supposed to come from a calculation of homogenisation.

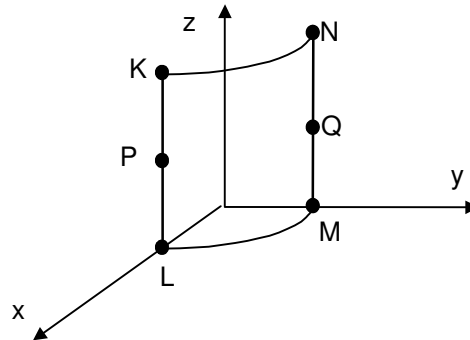


## 12 Modeling J

### 12.1 Characteristics of modeling

Modeling of a quarter of cylinder.

Elements of plate DKT : TRIA3. Assignment of characteristics of homogenized plates corresponding to material of § 1.2 .



The discretized geometry is represented above.

not	node
<i>K</i>	<i>NO1</i>
<i>L</i>	<i>NO11</i>
<i>M</i>	<i>NO161</i>
<i>N</i>	<i>NO227</i>
<i>P</i>	<i>NO6</i>
<i>Q</i>	<i>NO215</i>

### 12.2 Characteristics of the grid

Many nodes: 231

Many meshes and types: 400 TRIA3 + 80 SEG2

### 12.3 Boundary conditions in displacement and rotation

#### 12.3.1 Gravity

Displacement  $DZ$  is blocked on the group of nodes  $LM$  .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$  .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$  .

#### 12.3.2 Thermal case of dilation n°1

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$  .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$  .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

### 12.3.3 Thermal case of dilation n°2

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

## 12.4 Results of modeling J

Identification	Node (mesh)	Value tested	Reference
Gravity	$K$	$DX (mm)$	$-2.40000 \cdot 10^{-8}$
	$N$	$DY (mm)$	$-2.40000 \cdot 10^{-8}$
	$P$	$DZ (mm)$	$5.0 \cdot 10^{-9}$
	$Q$	$DZ (mm)$	$5.0 \cdot 10^{-9}$
	$P$	$-DRY$	$2.40000 \cdot 10^{-9}$
	$Q$	$DRX$	$2.40000 \cdot 10^{-9}$
	$K (M362)$	$NYY (N)$	$8.00000 \cdot 10^{-4}$
	$N (M400)$	$NYY (N)$	$8.00000 \cdot 10^{-4}$
	Dilation case 1	$L (M1)$	$MYY (N.mm)$
$M (M39)$		$MYY (N.mm)$	$-2.38095 \cdot 10^{-1}$
Dilation case 2	$L$	$DX (mm)$	$26.0 \cdot 10^{-6}$
	$M$	$DY (mm)$	$26.0 \cdot 10^{-6}$
	$L (M1)$	$NYY (N)$	$-2.00000 \cdot 10^{-1}$
	$M (M39)$	$NYY (N)$	$-2.00000 \cdot 10^{-1}$

## 12.5 Remarks

The local constraints are not calculated since the characteristics material of the plate are supposed to come from a calculation of homogenisation.

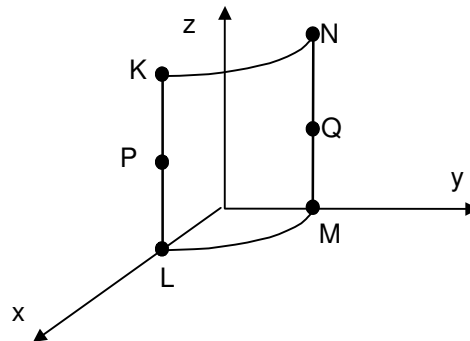
## 13 Modeling K

### 13.1 Characteristics of modeling

Identical to modeling E with elements `DKQG` (modeling `DKTG`).

The purpose of this modeling is to test the taking into account of the temperature for the elements `DKQG`.

Elements of plate `DKQG` : `QUAD4`



The discretized geometry is represented above.

not	node
<i>K</i>	<i>NO160</i>
<i>L</i>	<i>NO203</i>
<i>M</i>	<i>NO11</i>
<i>N</i>	<i>NO1</i>
<i>P</i>	<i>NO226</i>
<i>Q</i>	<i>NO6</i>

### 13.2 Characteristics of the grid

Many nodes: 231

Many meshes and types: 200 `QUAD4` + 80 `SEG2`

### 13.3 Boundary conditions in displacement and rotation

#### 13.3.1 Gravity

Displacement  $DZ$  is blocked on the group of nodes  $LM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

#### 13.3.2 Thermal case of dilation n°1

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

### 13.3.3 Thermal case of dilation n°2

Displacement  $DZ$  as well as rotations around the axes  $X$  and  $Y$  are blocked on the groups of nodes  $KNSANSKN$  and  $LMSANSLM$ .

Displacement  $DY$  as well as rotations around the axes  $X$  and  $Z$  are blocked on the group of nodes  $KL$ .

Displacement  $DX$  as well as rotations around the axes  $Y$  and  $Z$  are blocked on the group of nodes  $MN$ .

Displacement  $DZ$  as well as rotation around the axis  $Y$  are blocked on the group of nodes  $KETL$ .

Displacement  $DZ$  as well as rotation around the axis  $X$  are blocked on the group of nodes  $METN$ .

## 13.4 Results of modeling K

Identification	Node (mesh)	Value tested	Reference
Gravity	$K$	$DX (mm)$	$-2.40000 \cdot 10^{-8}$
	$N$	$DY (mm)$	$-2.40000 \cdot 10^{-8}$
	$P$	$DZ (mm)$	$5.00000 \cdot 10^{-9}$
	$Q$	$DZ (mm)$	$5.00000 \cdot 10^{-9}$
	$P$	$-DRY$	$2.40000 \cdot 10^{-9}$
	$Q$	$DRX$	$2.40000 \cdot 10^{-9}$
	$K (181)$	$NYY (N)$	$8.00000 \cdot 10^{-4}$
	$N (M200)$	$NYY (N)$	$8.00000 \cdot 10^{-4}$
Dilation case 1	$L (M1)$	$MYY (N.mm)$	$-2.38095 \cdot 10^{-1}$
	$M (M20)$	$MYY (N.mm)$	$-2.38095 \cdot 10^{-1}$
Dilation case 2	$L$	$DX (mm)$	$26.0000 \cdot 10^{-6}$
	$M$	$DY (mm)$	$26.0000 \cdot 10^{-6}$
	$L (M1)$	$NYY (N)$	$-2.00000 \cdot 10^{-1}$
	$M (M20)$	$NYY (N)$	$-2.00000 \cdot 10^{-1}$

## 14 Summary of the results

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Very good performances obtained for modelings  $A$  and  $B$  are explained by the fact that the reference solutions belong to the space generated by the selected finite elements. Only remain the digital rounding errors.

Satisfactory results were got for modelings of plates and hulls in space  $C, D, E, F, G, H, I, J$  and  $K$ . For these last, a chained thermoelastic calculation was carried out. Results with modeling  $DKT$  ( $E, F, G, H, I, J$ ) show that the elements quadrangle have a better behavior than the elements triangle. It is necessary to have a sufficiently fine discretization with these elements plans in order to be able to correctly model the circular geometry of the cylindrical hull. Indeed, to discretize the geometry of the cylinder by plane or parabolic facets is not in conformity and induces a parasitic inflection which decreases with the smoothness of grid. Thus a multiplication amongst elements by two on the height of the structure makes fall the maximum error relative of 5.48% (case presented here) to 2.8%. Results with modeling  $COQUE\_3D$  ( $C$  and  $D$ ) are very good except for gravity with the element triangle.

Calculations with modeling  $DKTG$  ( $K$ ) the same results as with modeling give  $DKT$ .

## 15 Appendix

### 15.1 Uniform loading of rotation around $OZ$

#### 15.1.1 Axisymmetric model 2D

The density of centrifugal force is:  $\rho \Omega^2 r e_r$ .

One considers the boundary conditions following:

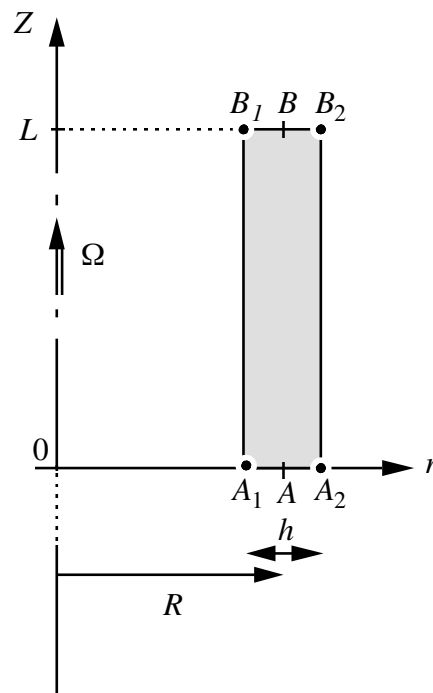
$$u_z(r, z) = 0 \text{ in } z = 0 \text{ and } z = L$$

One applies displacement in the form:

$$\begin{aligned} u_r &= u(r) \\ u_z &= u_\theta = 0 \end{aligned}$$

As follows:

$$\varepsilon_{rr} = u'; \quad \varepsilon_{\theta\theta} = \frac{u}{r}; \quad \varepsilon_{zz} = \varepsilon_{rz} = \varepsilon_{\theta z} = \varepsilon_{r\theta} = 0$$



Geometry of the hollow roll:

$$\begin{aligned} R &= 20 \text{ mm} \\ h &= 1 \text{ mm} \end{aligned}$$

The elastic constraints are expressed:

$$\begin{aligned}\sigma_{rr} &= \frac{E}{(1+\nu)(1-2\nu)} \left[ (1-\nu)u' + \nu \frac{u}{r} \right] \\ \sigma_{\theta\theta} &= \frac{E}{(1+\nu)(1-2\nu)} \left[ (1-\nu) \frac{u}{r} + \nu u' \right] \\ \sigma_{zz} &= \frac{\nu E}{(1+\nu)(1-2\nu)} \left[ \frac{u}{r} + u' \right]\end{aligned}$$

The radial equilibrium equation is written:

$$(r \sigma_{rr})_{,r} - \sigma_{\theta\theta} = -\rho \Omega^2 r^2$$

As follows:

$$\left( \frac{ru}{r} \right)' = \frac{-(1+\nu)(1-2\nu)}{(1-\nu)E} \rho \Omega^2 r \quad \text{éq 1.1-1}$$

**Note:**

$$\frac{u}{r} + u' = \frac{(ru)'}{r}$$

From where the general solution:

$$u(r) = \frac{-(1+\nu)(1-2\nu)}{(1-\nu)E} \rho \Omega^2 \frac{r^3}{8} + Ar + \frac{B}{r} \quad \text{éq 1.1-2}$$

The constraints are then:

$$\begin{aligned}\sigma_{rr}(r) &= \frac{-3-2\nu}{1-\nu} \rho \Omega^2 \frac{r^2}{8} + \frac{E}{(1+\nu)(1-2\nu)} \left( A - (1-2\nu) \frac{B}{r^2} \right) \\ \sigma_{\theta\theta}(r) &= \frac{-1+2\nu}{1-\nu} \rho \Omega^2 \frac{r^2}{8} + \frac{E}{(1+\nu)(1-2\nu)} \left( A - (1-2\nu) \frac{B}{r^2} \right) \\ \sigma_{zz}(r) &= \frac{-\nu}{1-\nu} \rho \Omega^2 \frac{r^2}{2} + \frac{2\nu E}{(1+\nu)(1-2\nu)} A\end{aligned} \quad \text{éq 1.1-3}$$

The boundary conditions in constraints are:

$$\sigma_{rr} = 0 \quad \text{in } r = R \pm \frac{h}{2}$$

One notes:

$$x = \frac{h}{2R}$$

One obtains thanks to [éq 1.1-3] :

$$B = \frac{(3-2\nu)(1+\nu)}{8(1-\nu)E} \rho \Omega^2 R^4 (1-x^2)^2$$

then:

$$A = \frac{(3-2\nu)(1+\nu)(1-2\nu)}{4(1-\nu)E} \rho \Omega^2 R^2 (1-x^2)$$

**Digital application:**

$$\begin{aligned}\rho &= 8.10^{-6} \text{ kg/mm}^3 \\ \Omega &= 1 \text{ s}^{-1} \\ E &= 2.105 \text{ N/mm}^2 \\ \nu &= 0.3\end{aligned}$$

$$\begin{aligned}\text{From where: } A &= 7.13588.10^{-9} \text{ mm}^2 \\ B &= 3.561258.10^{-6} \text{ mm}^2\end{aligned}$$

**Note:**

$$\begin{aligned}\frac{(1+\nu)(1-2\nu)}{(1-\nu)E} \rho \frac{\Omega^2}{8} &= 3.714286 \text{ E-12 mm}^2 \\ \frac{\nu}{1-\nu} \rho \frac{\Omega^2}{2} &= 1.714286 \text{ E-6 MPa.mm}^2\end{aligned}$$

As follows:

$$\begin{aligned}\bullet \text{ in internal skin: } & \begin{cases} u_r = 2.9424 \text{ E}^{-7} \text{ mm} \\ \sigma_{zz} = 0.99488 \text{ E}^{-3} \text{ Mpa} \end{cases} \\ \bullet \text{ in external skin: } & \begin{cases} u_r = 2.8801 \text{ E}^{-7} \text{ mm} \\ \sigma_{zz} = 0.92631 \text{ E}^{-3} \text{ Mpa} \end{cases}\end{aligned}$$

## 15.1.2 Axisymmetric model hull

The centrifugal force is equivalent to one **pressure distributed** :

$$p = \rho \Omega^2 hR \left(1 + \frac{h^2}{12R^2}\right)$$

The solution is membranous, normal balance is written:

$$N_{\theta\theta} = pR$$



The membrane deformation is:  $E_{\theta\theta} = \frac{w}{R}$ , whereas  $E_{\theta\theta} = 0 = K_{\theta\theta} = K_{zz}$ .

In elasticity:

$$N_{\theta\theta} = \frac{Eh}{1-\nu^2} E_{\theta\theta} ; N_{zz} = \nu N_{\theta\theta} ; M_{\alpha\beta} = 0$$

From where the solution (marks with arrows and circumferential normal effort):

$$w = \frac{(1-\nu^2)\rho\Omega^2}{E} R^3 \left(1 + \frac{h^2}{12R^2}\right) ; N_{\theta\theta} = \rho\Omega^2 R^2 h \left(1 + \frac{h^2}{12R^2}\right)$$

Axial stress is worth:

$$\sigma_{zz} = \nu \rho \Omega^2 R^2 \left(1 + \frac{h^2}{12R^2}\right) \quad (\text{constant in the thickness})$$

If one does not take account of the correction of metric, the term should be removed  $\left(1 + \frac{h^2}{12R^2}\right)$  in the preceding expressions.

**Digital application** (without correction of metric) :

$$\begin{aligned} p &= 1,600000 \cdot 10^{-4} \text{ MPa} \\ w &= 2,912000 \cdot 10^{-7} \text{ mm} \\ N_{zz} &= 0,96000 \cdot 10^{-3} \text{ N/mm} \\ \sigma_{zz} &= 0,96000 \cdot 10^{-3} \text{ MPa} \end{aligned}$$

## 15.2 Loading of gravity

### 15.2.1 Axisymmetric model 2D

The density of force is:  $-\rho g e_z$  (vertical gravity).

One considers the boundary conditions following:

$$U_z(r, z) = 0 \text{ in } r = R \text{ and } z = 0 \text{ (circle of support)}$$

with uniform traction:  $\sigma_{zz}(r, z) = \rho g L$  in  $z = L$ , balancing the weight.

One applies the elastic solution of the type:

$$\sigma = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

so that:

$$\varepsilon_{rr} = \varepsilon_{\theta\theta} = -\nu \varepsilon_{zz} = -\nu u_{z,z} = -\nu \frac{\sigma_{zz}}{E} ; \varepsilon_{rz} = 0 = \varepsilon_{r\theta} = \varepsilon_{\theta z}$$

One observes as follows:

$$u_{r,r} = \frac{u_r}{r} \Leftrightarrow u_r(r, z) = -\nu A'(z)r$$

Then:

$$\begin{aligned} -\nu A'(z) &= \varepsilon_{rr} = -\nu \varepsilon_{zz} \Leftrightarrow u_{z,z}(r, z) = A'(z) \\ &\Leftrightarrow u_r(r, z) = A(z) + B \end{aligned}$$

Of  $\varepsilon_{rz} = 0$ , one draws:

$$B'(r) - \nu r A''(z) = 0$$

that is to say:

$$A''(z) = cste = \alpha ; \quad B'(r) = \alpha \nu r$$

Boundary conditions in effort, one obtains:

$$A(z) = \frac{\rho g z^2}{2E} + \beta ; \quad B(r) = \nu \frac{\rho g r^2}{2E}$$

Lastly,  $\beta$  check:  $\beta = -\nu \rho g \frac{R^2}{2E}$

As follows:

$$u_r(r, z) = \frac{-\nu \rho g z r}{E} \quad ; \quad u_z(r, z) = \frac{\rho g}{2E} (z^2 + \nu (r^2 - R^2)) \quad \text{éq 2.1-1}$$

$$\sigma_{zz}(r, z) = \rho g z$$

### Digital application

$$g = 10 \text{ N/kg}$$

$$\rho = 8.10^{-6} \text{ kg/mm}^3$$

$$R = 20 \text{ mm}$$

$$L = 10 \text{ mm}$$

$$E = 2.10^5 \text{ N/mm}^2$$

$$\nu = 0.3$$

$$h = 1 \text{ mm}$$

- in internal skin: 
$$\begin{cases} u_r(L) = -2.34000 E^{-8} \text{ mm} \\ \sigma_{zz}(L) = 8.0000 E^{-4} \text{ Mpa} \\ u_z(O) = -1.185000 E^{-9} \text{ mm} \end{cases}$$
- in external skin: 
$$\begin{cases} u_r(L) = -2.46000 E^{-8} \text{ mm} \\ \sigma_{zz}(L) = 8.0000 E^{-4} \text{ Mpa} \\ u_z(O) = 1.215000 E^{-9} \text{ mm} \end{cases}$$

## 15.2.2 Axisymmetric model hull

A vertical traction is exerted in  $z = L$  :

$$F = \rho g h L$$

Gravity leads to a vertical force:

$$f = -\rho g h e_z$$

The boundary condition on the circle of support is:  $u_z(z) = 0$  in  $z = 0$

The solution is membranous, vertical balance is written:

$$N_{zz,z} = \rho g h$$

Moreover:  $N_{\theta\theta} = 0$  . In elasticity, one deduces then:

$$E_{\theta\theta} = \frac{w}{R} = \frac{-\nu N_{zz}}{Eh} = \frac{-\nu \rho g z}{E}$$

$$E_{zz} = u_{z,z} = \frac{N_{zz}}{Eh} \Rightarrow u_z(z) = \frac{\rho g}{2E} z^2$$

Axial stress is:

$$\sigma_{zz} = \rho g z \quad (\text{constant in the thickness})$$

**Digital application:**

$$F = 8.10^{-4} N.mm^{-1}$$

$$w(L) = -2.4000.10^{-8} mm$$

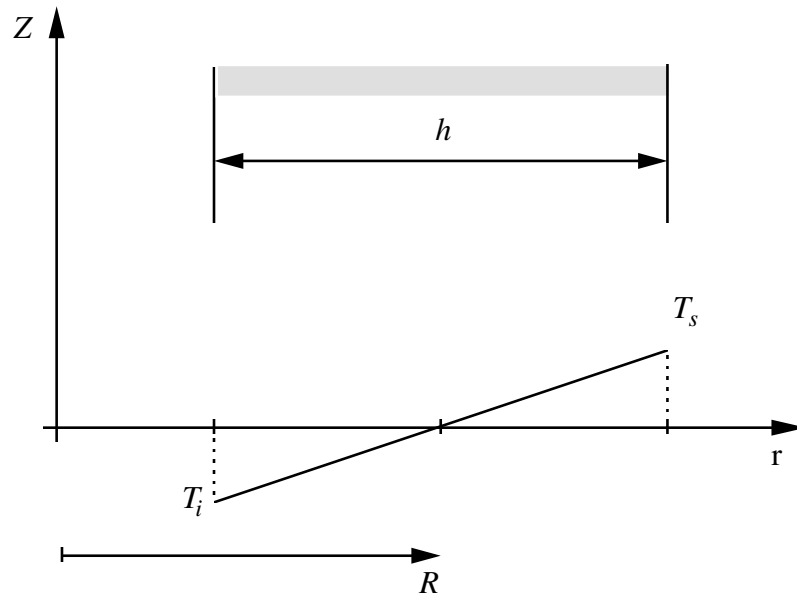
$$N_{zz}(L) = 8.0000.10^{-4} N.mm^{-1}$$

$$\sigma_{zz}(L) = 8.0000.10^{-4} N.mm^{-1}$$

## 15.3 Thermomechanical loading

### 15.3.1 Axisymmetric model 2D

$$T(r) - T_{réf}(r) = \frac{T_s + T_i}{2} + \frac{(T_s - T_i)}{h}(r - R) \quad \text{éq 3.1.-1}$$



One applies displacement in the form:

$$u_r = u(r) \quad ; \quad u_z = u_\theta = 0$$

with the boundary conditions suitable. Thus, the elastic constraints are expressed:

$$\begin{aligned} \sigma_{rr} &= \frac{E}{(1+\nu)(1-2\nu)} \left[ (1-\nu)u' + \nu \frac{u}{r} \right] - \frac{\alpha E}{1-2\nu} (T - T_{réf}) \\ \sigma_{\theta\theta} &= \frac{E}{(1+\nu)(1-2\nu)} \left[ (1-\nu) \frac{u}{r} + \nu u' \right] - \frac{\alpha E}{1-2\nu} (T - T_{réf}) \\ \sigma_{zz} &= \frac{\nu E}{(1+\nu)(1-2\nu)} \left[ \frac{u}{r} + u' \right] - \frac{\alpha E}{1-2\nu} (T - T_{réf}) \end{aligned} \quad \text{éq 1.1-3}$$

The radial equilibrium equation  $(r, \sigma_{rr})_{,r} - \sigma_{\theta\theta} = 0$  give:

$$\left( \frac{ru}{r} \right)' = \frac{\alpha(1+\nu)'}{(1-\nu)} (T - T_{réf})' \quad \text{éq 3.1-2}$$

From where the general solution:

$$u(r) = \frac{\alpha(1+\nu)}{(1-\nu)} \frac{(T_s - T_i)}{h} \frac{r^2}{3} + Ar + \frac{B}{r} \quad \text{éq 3.1-3}$$

The constraints are then:

$$\begin{aligned}\sigma_{rr}(r) &= \frac{\alpha E(T_s - T_i)}{h} \left( \frac{R}{1-2\nu} - \frac{r}{3(1-\nu)} \right) - \frac{\alpha E}{1-2\nu} (T_s - T_i) + \frac{E}{(1+\nu)(1-2\nu)} \left( A - (1-2\nu) \frac{B}{r^2} \right) \\ \sigma_{\theta\theta}(r) &= \frac{\alpha E(T_s - T_i)}{h} \left( \frac{R}{1-2\nu} - \frac{2r}{3(1-\nu)} \right) - \frac{\alpha E}{1-2\nu} (T_s - T_i) + \frac{E}{(1+\nu)(1-2\nu)} \left( A - (1-2\nu) \frac{B}{r^2} \right) \\ \sigma_{zz}(r) &= \frac{\alpha E(T_s - T_i)}{h} \left( \frac{R}{1-2\nu} - \frac{r}{3(1-\nu)} \right) - \frac{\alpha E}{1-2\nu} (T_s - T_i) + \frac{2\nu E}{(1+\nu)(1-2\nu)} A\end{aligned}\quad \text{éq 3.1-4}$$

The boundary conditions in efforts are: in  $r = R \pm \frac{h}{2}$ ,  $\sigma_{rr} = 0$ . One notes:  $x = \frac{h}{2R}$ . One obtains thanks to [éq 3.1-4]:

$$B = \frac{\alpha(T_s - T_i)}{6h(1-\nu)} (1+\nu) R^3 (1-x^2)^2$$

then:

$$A = \alpha(1+\nu) \left[ \frac{-(T_s - T_i)R}{6h(1-\nu)} (3 - (1-2\nu)x^2) + \frac{(T_s + T_i)}{2} \right]$$

### Digital application:

$$F = 8.10^{-4} \text{ N.mm}^{-1}$$

$$w(L) = -2.4000.10^{-8} \text{ mm}$$

$$N_{zz}(L) = 8.0000.10^{-4} \text{ N.mm}^{-1}$$

$$\sigma_{zz}(L) = 8.0000.10^{-4} \text{ N.mm}^{-1}$$

$$\text{From where: } A = -0.18569881.10^{-3} \text{ mm}^2$$

$$B = 0.02473096 \text{ mm}^2$$

### Note:

$$\frac{\alpha(1+\nu)}{1-\nu} \frac{T_s - T_i}{3h} = 0.61904762 \text{ E-5}$$

- in internal skin:  $\begin{cases} u_r = 1.056145 \text{ E}^{-6} \text{ mm} \\ \sigma_{zz} = 1.4321427 \text{ Mpa} \end{cases}$
  - in external skin:  $\begin{cases} u_r = 1.110317 \text{ E}^{-6} \text{ mm} \\ \sigma_{zz} = -1.4250001 \text{ Mpa} \end{cases}$
- $\sigma_{zz} = 1.449319 \text{ Mpa}$   
ou  
 $\sigma_{zz} = 1.428671 \text{ Mpa}$  ( sans correction métrique )

If one takes  $T_s = T_i = 0.1^\circ C$  :

$$A = 0,00130000 \cdot 10^{-3}$$

$$B = 0,0 \text{ mm}^2$$

As follows:

- in internal skin: 
$$\left. \begin{aligned} u_r &= 25.350000 E^{-6} \text{ mm} \\ \sigma_{zz} &= -0.200000 \text{ Mpa} \end{aligned} \right\}$$
- in external skin: 
$$\left. \begin{aligned} u_r &= 26.650000 E^{-6} \text{ mm} \\ \sigma_{zz} &= -0.200000 \text{ Mpa} \end{aligned} \right\}$$

## 15.3.2 Axisymmetric model hull

For the field of temperature in the thickness given by [éq 3.1-1], one obtains the following expression of the law of behavior:

$$N_{\theta\theta} = \frac{Eh}{1-\nu^2} (E_{\theta\theta} + \nu E_{zz}) - \frac{\alpha E h}{1-\nu} \left[ \frac{T_s + T_i}{2} + \frac{T_s - T_i}{12} \frac{h}{R} \right]$$

$$N_{zz} = \frac{Eh}{1-\nu^2} (\nu E_{\theta\theta} + E_{zz}) - \frac{\alpha E h}{1-\nu} \left[ \frac{T_s + T_i}{2} + \frac{T_s - T_i}{12} \frac{h}{R} \right]$$

éq 3.2-1

and:

$$M_{\theta\theta} = \frac{Eh^3}{12(1-\nu^2)} (K_{\theta\theta} + \nu K_{zz}) - \frac{\alpha E h^2}{12(1-\nu)} \left[ \frac{T_s + T_i}{2} \frac{h}{R} + T_s - T_i \right]$$

$$M_{zz} = \frac{Eh^3}{12(1-\nu^2)} (\nu K_{\theta\theta} + K_{zz}) - \frac{\alpha E h^2}{12(1-\nu)} \left[ \frac{T_s + T_i}{2} \frac{h}{R} + T_s - T_i \right]$$

éq 3.2-2

According to these expressions, thermal terms in  $\frac{h}{R}$  are to be neglected if one does not consider the correction of metric in the thickness, i.e. the usual models.

In our situation:

$$E_{\theta\theta} = \frac{w}{R}$$

$$E_{zz} = 0$$

$$K_{\theta\theta} = K_{zz} = 0$$

Normal balance with the hull is written:

$$N_{\theta\theta} = 0$$

from where the arrow:

$$w = \alpha(1+\nu) \left[ \frac{T_s + T_i}{2} + \frac{T_s + T_i}{12} \frac{h}{R} \right] R$$

and:

$$N_{zz} = \alpha Eh \left[ \frac{T_s + T_i}{2} + \frac{T_s + T_i}{12} \frac{h}{R} \right]$$
$$M_{zz} = \frac{-\alpha Eh^2}{12(1-\nu)} \left[ (T_s - T_i) + \frac{T_s + T_i}{2} \frac{h}{R} \right]$$

As the second member of dilation does not take account of the correction of metric, terms in  $h/R$  above are neglected.

## Digital application

$$R = 20 \text{ mm}$$

$$h = 1 \text{ mm}$$

$$\alpha = 10^{-5} \text{ } ^\circ\text{C}^{-1}$$

$$T_s = T_i = 0.5 \text{ } ^\circ\text{C}$$

$$\nu = 0.3$$

$$E = 2.10^5 \text{ N/mm}^2$$

From where:

$$M_{zz} = -0.2380952 \text{ N}$$

$$\sigma_{zz} = 1.449319 \text{ Mpa}$$

• in internal skin:

ou

$$\sigma_{zz} = 1.428671 \text{ Mpa ( sans correction métrique )}$$

If one takes  $T_s = T_i = 0,1 \text{ } ^\circ\text{C}$  :

$$w = 26.00000.10^{-6} \text{ mm}$$

$$N_{zz} = -0.2 \text{ N.mm}^{-1}$$

$$M_{zz} = -0.001190476 \text{ N}$$

$$\sigma_{zz} = -0.2122466 \text{ Mpa}$$

• in internal skin:

ou

$$\sigma_{zz} = -0.200000 \text{ Mpa ( sans correction métrique )}$$

\* the constraints in the thickness with correction of metric are given by:

$$\sigma_{zz}(x_3) = \frac{N_{zz} - \frac{M_{zz}}{R}}{h \left( 1 - \frac{h^2}{12R^2} \right)} + \left( M_{zz} - N_{zz} \frac{h^2}{12R} \right) \frac{12x_3}{h^3 \left( 1 - \frac{h^2}{12R^2} \right)}$$