# HPLA311 - Murakami 11.39. Circular crack in the center of a sphere subjected to a uniform temperature on the lips

**Summary:**

This test is resulting from the validation independent of version 3 in breaking process.

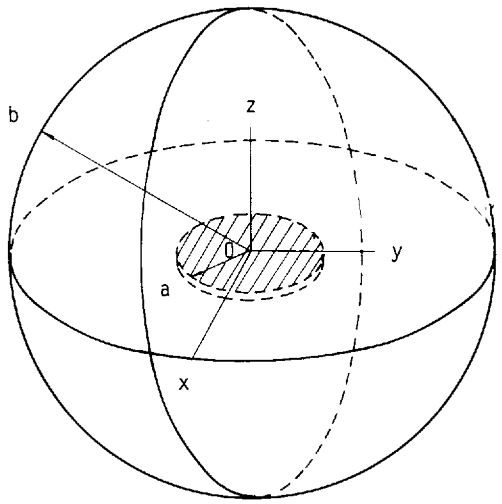
It is about one basic static test in axisymetry under stationary thermal loading calculated by finite elements on the same grid of a limited field.

behavior is thermoelastic linear isotropic.

It understands two axisymmetric modelings for which one varies a/b report, has being the ray of the crack interns circular in the horizontal plane xoy and B ray of the sphere. The factors of intensity of the constraints K and the rate of refund of energy are calculated by the method theta (operator CALC_G).

# 1 Problem of reference

## 1.1 Geometry




a: ray of the crack interns circular in the horizontal plane xoy
b: ray of the sphere, with B = 2,5.$10^{-3}$ Mr.

The ray has varies according to modeling.

## 1.2 Properties of material

| | |
|---|---|
| Young modulus | $E = 2\ 10^{11}$ Pa |
| Poisson's ratio | $\nu = 0.3$ |
| linear dilation coefficient | $\alpha = 1.2\ 10^{-5}$ °$C^{-1}$ |

## 1.3 Boundary conditions and loadings

UX = $U_R$ = 0 on the axis of revolution X = R = 0
UY = $U_Z$ = 0 in the horizontal plane Y = Z = 0, apart from the lips have ≤ R ≤ B

The lips are supposed to be free constraints (not closing partial of the crack).

Worthless temperature on the surface of the sphere.

Temperature of worthless reference (temperature to which the thermal deformations are considered worthless).

Uniform and negative temperature T = - Tf on the lips of the crack, melts of crack understood. The stationary thermal problem (of Dirichlet type) must be solved beforehand by finite elements on the same grid as that intended for mechanical calculation. One takes Tf = 100 °C.

# 2 Reference solution

## 2.1 Method of calculating used for the reference solution

Analytical calculation by transform of Hankel.

## 2.2 Results of reference

For the reference solution, the rays must check the condition a/b < 0.5.

$$\eta=\frac{a}{b}<0,5$$

$$K_I=\frac{E\alpha T_f}{1-\nu}\cdot\sqrt{(\frac{a}{\pi})}\cdot F_I$$

$$F_I=1-0.6366\,\eta-0.4053\,\eta^2+2.0163\,\eta^3-0.6773\,\eta^4-3.8523\,\eta^5+4.1687\,\eta^6+3.2741\,\eta^7$$

## 2.3 Uncertainty on the solution

Badly definite. For the low values of a/b report, the solution must approach asymptotically the reference solution calculated for □ = 0, that is to say $F_I$ = 1, which is then exact (see MURAKAMI 11.23, page 1069).

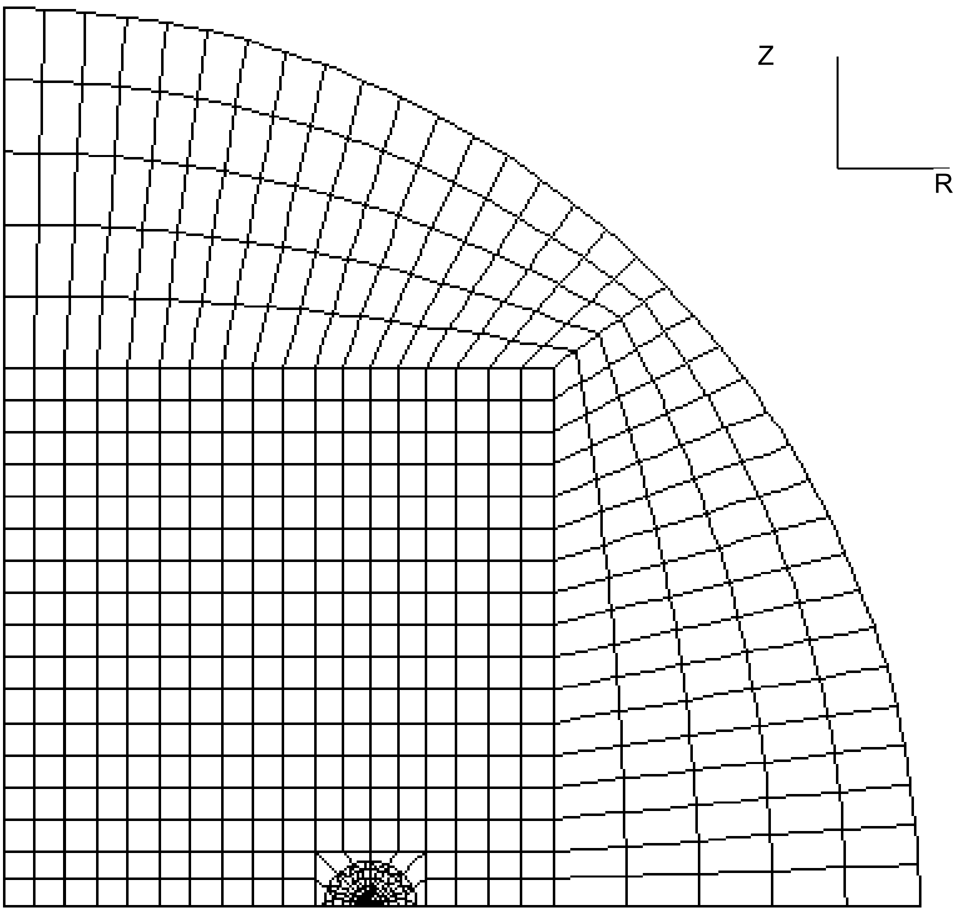
## 2.4 Bibliographical references

[1] Y. MURAKAMI: Stress Intensity Factors Handbook, box 11.39, pages 1089-1090. The Society of Materials Science, Japan, Pergamon Near, 1987.

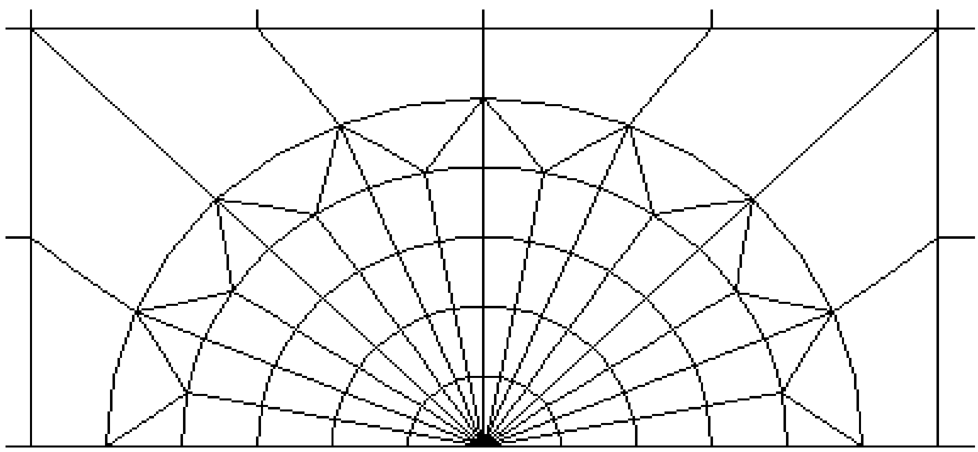
# 3 Modeling A

Modeling A corresponds to the case **a/b = 0.4**.

## 3.1 Characteristics of modeling




**Complete grid**



**Zoom of the point of crack**

## 3.2 Characteristics of the grid

1756 nodes and 569 elements including 529 QUA8 and 40 TRI6

## 3.3 Features tested

| Orders | | |
|---|---|---|
| AFFE_MODELE | THERMICS | AXIS |
| AFFE_CHAR_THER | TEMP_IMPO | |
| AFFE_MODELE | MECHANICS | AXIS |
| AFFE_MATERIAU | AFFE_VARC | NOM_VARC=' TEMP' |
| CALC_THETA | THETA_2D | |
| CALC_G | OPTION | CALC_G |
| CALC_G | OPTION | CALC_K_G |

## 3.4 Definition of the rays of the crowns

Several successive couples of ray for the crowns of lower and higher integration are retained. These rays are to be specified in the order CALC_THETA or in the keyword factor THETA of CALC_G :

| | Crown n°1 | Crown n°2 | Crown n°3 | Crown n°4 |
|---|---|---|---|---|
| rinf | 1.E-6 | 2.5E-5 | 5.E-5 | 7.5E-5 |
| rsup | 2.5E-5 | 5.E-5 | 7.5E-5 | 1.E-4 |

## 3.5 Reference solutions

For a report a/b = 0.4 (and has = $10^{-3}$ m), the reference solution for $K_I$ is:

$$K_I = 4.7419\,\text{MPa}\cdot\sqrt{(\text{m})}$$

To calculate the rate of refund of energy, one uses the formulas of IRWIN in plane deformations:

$$G_{réf} = \frac{1-\nu^2}{E}(K_I^2 + K_{II}^2), \quad \text{with } K_{II}^2 = 0$$

that is to say:

$$G_{réf} = 1.0231.10^2\,\text{J.m}^{-2}$$

**Note:**

1) In the case of axisymmetric calculations, the rate of refund of energy calculated with the option CALC_G of CALC_G corresponds toa total rate of refund of energy $G^{glob}$ for a radian. $G^{glob}$ is equal to the rate of refund of energy room multiplied by the ray of the bottom of crack. One thus has:

$$G_{réf}^{glob} = a \cdot G_{réf} = 1.0231\,\text{J.m}^{-1}$$

2) The rate of refund of energy calculated with the option CALC_K_G of CALC_G corresponds as for him directly to the rate of refund of energy room.

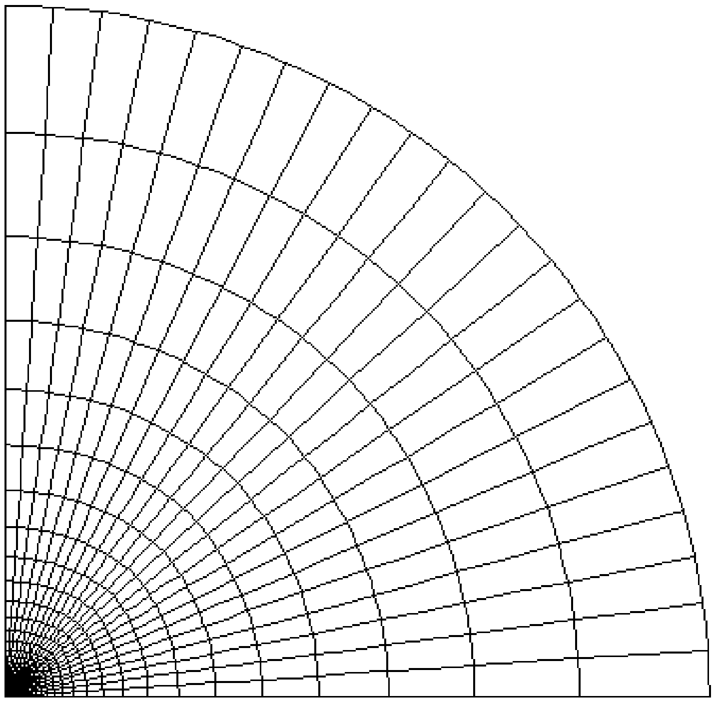
## 3.6 Sizes tested and results

| Parameter | Unit | Option | Crown | Reference | *Aster* | % difference |
|---|---|---|---|---|---|---|
| G | J.m $^{-1}$ | CALC_G | crown n°1 | 1.0231 | 0.9701 | -5,18 |
| G | J.m $^{-1}$ | CALC_G | crown n°2 | 1.0231 | 1.0051 | -1,74 |
| G | J.m $^{-1}$ | CALC_G | crown n°3 | 1.0231 | 1.0055 | - 1.72 |
| G | J.m $^{-1}$ | CALC_G | crown n°4 | 1.0231 | 1,01 | -1,71 |
| G | J.m$^{-2}$ | CALC_K_G | crown n°2 | 1,0231.10 $^{2}$ | 1,0052.10 $^{2}$ | - 1.75 |
| K1 | MPa. m $^{-2}$ | CALC_K_G | crown n°1 | 4.7419 | 4.4145 | - 6.89 |
| K1 | MPa. m $^{-2}$ | CALC_K_G | crown n°2 | 4.7419 | 4.7571 | 0.30 |
| K1 | MPa. m $^{-2}$ | CALC_K_G | crown n°3 | 4.7419 | 4.7913 | 1.04 |
| K1 | MPa. m $^{-2}$ | CALC_K_G | crown n°4 | 4.7419 | 4.8244 | 1.74 |

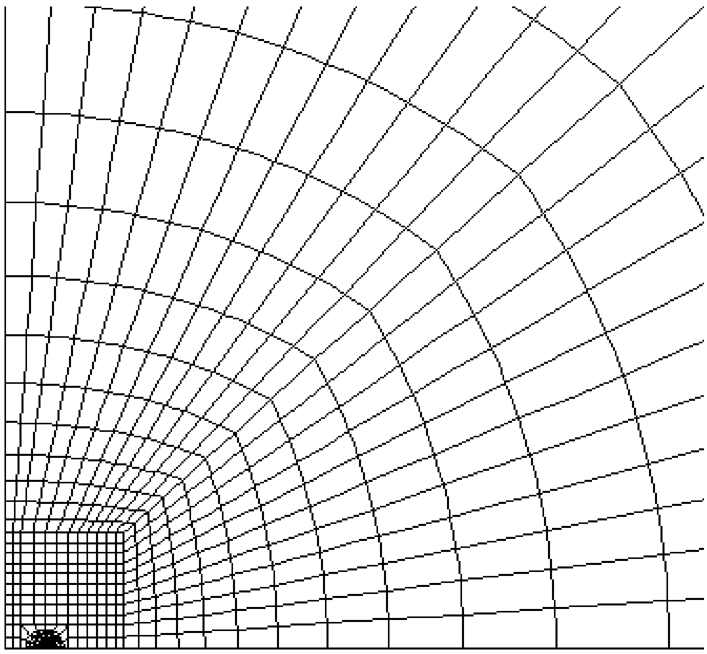
# 4 Modeling B

Modeling B corresponds to the case **a/b = 0.01**.

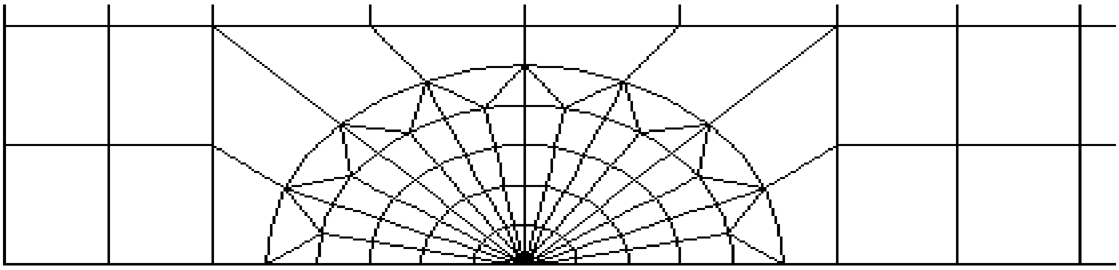
## 4.1 Characteristics of modeling



**Complete grid**



**Zoom**



**Zoom of the point of crack**

## 4.2 Characteristics of the grid

2095 nodes and 680 elements including 640 QUA8 and 40 TRI6

## 4.3 Features tested

**Orders**

| | | |
|---|---|---|
| AFFE_MODELE | THERMICS | AXIS |
| AFFE_CHAR_THER | TEMP_IMPO | |
| AFFE_MODELE | MECHANICS | AXIS |
| AFFE_MATERIAU | AFFE_VARC | NOM_VARC=' TEMP' |
| CALC_THETA | THETA_2D | |
| CALC_G | OPTION | CALC_G |
| CALC_G | OPTION | CALC_K_G |

## 4.4 Definition of the rays of the crowns

Several successive couples of ray for the crowns of lower and higher integration are retained. These rays are to be specified in the order CALC_THETA or in the keyword factor THETA of CALC_G :

| | Crown n°0 | Crown n°1 | Crown n°2 | Crown n°3 | Crown n°4 |
|---|---|---|---|---|---|
| rinf | 1.E-6 | 2.5E-5 | 2.75E-5 | 3.E-5 | 3.25E-5 |
| rsup | 2.5E-5 | 2.75E-5 | 3.E-5 | 3.25E-5 | 3.5E-5 |

***Note:*** For the calculation of the factors of intensity of the constraints with the option CALC_K_G of CALC_G, one should not which the ray of the crowns is larger than the ray of the bottom of crack. The ray of the bottom of crack being here equal to 2.5E-5, only the crown n°0 can be used.

## 4.5 Reference solutions

For a report a/b = 0.01 (and has = 2,5.10$^{-5}$ m), the reference solution for $K_I$ is:

$$K_I = 0.9609\ \mathrm{MPa}\cdot\sqrt{(\mathrm{m})}$$

To calculate the rate of refund of energy, one uses the formulas of IRWIN in plane deformations:

$$G_{réf} = \frac{1-\nu^2}{E}\left(K_I^2 + K_{II}^2\right), \quad \text{with } K_{II}^2 = 0$$

that is to say:

$$G_{réf} = 4.2019\ \mathrm{J.m^{-2}}$$

**Note:**

3) In the case of axisymmetric calculations, the rate of refund of energy calculated with the option `CALC_G` of `CALC_G` corresponds toa total rate of refund of energy $G^{glob}$ for a radian. $G^{glob}$ is equal to the rate of refund of energy room multiplied by the ray of the bottom of crack. One thus has:

$$G_{réf}^{glob} = a\cdot G_{réf} = 1.0505\cdot 10^{-4}\ \mathrm{J.m^{-1}}$$

4) The rate of refund of energy calculated with the option `CALC_K_G` of `CALC_G` corresponds as for him directly to the rate of refund of energy room.

## 4.6 Sizes tested and results

| Parameter | Unit | Option | Crown | Reference | *Aster* | % difference |
|---|---|---|---|---|---|---|
| G | J.m $^{-1}$ | `CALC_G` | crown n°1 | 1,0505.10$^{-4}$ | 1,0387.10$^{-4}$ | -5,18 |
| G | J.m $^{-1}$ | `CALC_G` | crown n°2 | 1,0505.10$^{-4}$ | 1,0388.10$^{-4}$ | -1,74 |
| G | J.m $^{-1}$ | `CALC_G` | crown n°3 | 1,0505.10$^{-4}$ | 1,0388.10$^{-4}$ | -1,72 |
| G | J.m $^{-1}$ | `CALC_G` | crown n°4 | 1,0505.10$^{-4}$ | 1,0388.10$^{-4}$ | -1,71 |
| G | J.m $^{-2}$ | `CALC_K_G` | crown n°0 | 4.2019 | 4.1653 | -0,87 |
| K1 | MPa. m $^{-2}$ | `CALC_K_G` | crown n°0 | 0.9609 | 0.9653 | -0,46 |

# 5 Summary of the results

- The calculation of K and G in axisymetry in the presence of a stationary thermal loading, gives good performances since the maximum change for G is of 1.75% (out first crown) for $\nu$ = 0.4.

- Results of K and G for $\nu$ =0,01 (modeling B) are better than for $\nu$ = 0.4 (modeling A).

- The calculation of G is slightly less less sensitive to the choice of the crowns of integration than the calculation of K.