

HPLP100 - Calculation of the rate of refund of the energy of a plate fissured in thermoelasticity

Summary

It is about a test in thermoelasticity for a two-dimensional problem. A fissured rectangular plate is considered and one places oneself on the assumption of the plane deformations.

In **modeling A**, the rate of refund of energy is calculated in postprocessing by two different methods:

- classical calculation by the method theta,
- calculation by the formula of IRWIN starting from the coefficients of intensity of constraints KI and KII .

These two calculations are carried out on 4 different crowns of integration. Their interest is to compare the values of G and of $G(IRWIN)$ compared to the reference solution and to test the invariance of calculations compared to the various crowns of integration.

1 Problem of reference

1.1 Geometry

It is about a fissured rectangular plate (one represents only the quarter of the structure):

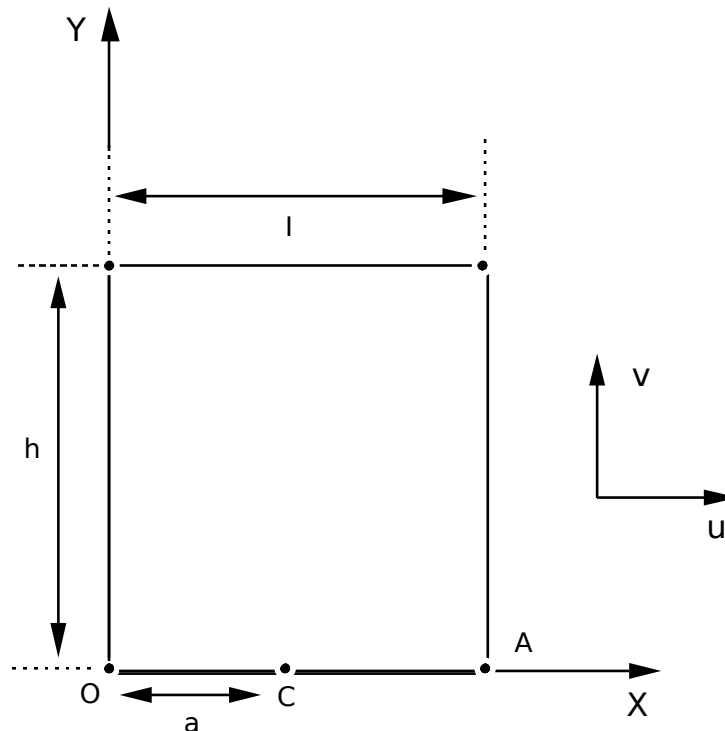


Figure 1.1-a: Fissured rectangular plate

Dimensions of this plate are the following ones:

Half-height of the plate: $h = 200.0 \text{ mm}$
 Half-width of the plate: $I = 100.0 \text{ mm}$
 Half-length of the crack: $a = 50.0 \text{ mm}$

1.2 Properties of material

Thermal properties: $C_p = 0$
 $\lambda = 1.0 \text{ W/m}^\circ\text{C}$
 Mechanical properties: $E = 200000 \text{ MPa}$
 $\nu = 0.3$
 $\alpha = 5.10^{-6} / ^\circ\text{C}$

We are on the assumption of the plane deformations

1.3 Boundary conditions and loadings

- Temperature imposed in $X=0$: $T = -100.0^\circ\text{C}$
- Temperature imposed in $X=100$: $T = +100.0^\circ\text{C}$
- Displacement for $a < X < I$, $Y=0$: $v=0$.
- Displacement for $0 < X < I$, $Y=H$: $v=0$.

- Displacement for $X=0.$, $Y=H$: $u=0.$

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is resulting from WILSON and YU [bib1]:

$$K_I = \frac{E \alpha T_0}{1 - \nu} F \sqrt{\pi a} \quad F = 0.154$$

a en mm
 E en N/mm²

$$K_I = 92.0291$$

In plane deformations, the formula of IRWIN gives: $G = \frac{(1 - \nu^2)}{E} (K_I^2 + K_{II}^2)$

that is to say numerically: $G = 3.8535 \cdot 10^{-1}$

2.2 Results of reference

The results of reference are those resulting from the reference solution from WILSON and YU [bib1]:

$$G = 3.8535 \cdot 10^{-1}$$
$$K_I = 92.0291$$
$$K_{II} = 0.$$

2.3 Bibliographical references

- 1) The Uses of J-Integrals in thermal stress ace problems - International Newspaper of Fracture (1979) WILSON and YU.
- 2) Qualification complementary to codes INCA/MAYA in linear thermoelasticity. Technical note DRE/STRE/LMA 84/598

3 Modeling A

3.1 Characteristics of modeling

There are 4 crowns defined by the order CALC_THETA :

Crown 1:	$R_{inf} = 10.$	$R_{sup} = 40.$
Crown 2:	$R_{inf} = 15.$	$R_{sup} = 45.$
Crown 3:	$R_{inf} = 5.$	$R_{sup} = 47.$
Crown 4:	$R_{inf} = 3.$	$R_{sup} = 48.$

The bottom of crack is defined by DEFI_FOND_FISS, and for each crown one carries out:

- a calculation of G classic (option CALC_G of CALC_G),
- a calculation of G by the formula of IRWIN starting from the coefficients of intensity of constraints K_I and K_{II} (option CALC_K_G of CALC_G).

3.2 Characteristics of the grid

Many nodes: 853

Many meshes and types: 359 meshes TRIA6 and 27 meshes QUAD8

3.3 Values tested and results of modeling A

The values tested are those of G obtained by the classical method and that of G_{IRWIN} obtained by the formula of IRWIN starting from the coefficients of intensity of constraints:

Identification	Reference	Tolerance
Crown 1 G	$3.8535 \cdot 10^{-1}$	8,00%
Crown 1 G_{IRWIN}	$3.8535 \cdot 10^{-1}$	8,00%
Crown 2 G	$3.8535 \cdot 10^{-1}$	8,00%
Crown 2 G_{IRWIN}	$3.8535 \cdot 10^{-1}$	8,00%
Crown 3 G	$3.8535 \cdot 10^{-1}$	8,00%
Crown 3 G_{IRWIN}	$3.8535 \cdot 10^{-1}$	8,00%
Crown 4 G	$3.8535 \cdot 10^{-1}$	8,00%
Crown 4 G_{IRWIN}	$3.8535 \cdot 10^{-1}$	8,00%

3.4 Remarks

The digital values are stable compared to the various crowns of integration and almost identical for the two methods of calculating. Nevertheless the variation with the values of reference is about 6 with 7%, which seems high.

4 Summaries of the results

At the time of **the first modeling**, the variation with the values of reference is from 6 to 7%. The validation independent of the breaking process batch should bring brief replies on the validity of G in thermoelasticity.