HPLP310 – Biblio_35 Fissures radial intern in a thick cylinder under pressure and thermal loading

Summary:

This test is resulting from the validation independent of version 3 in breaking process.

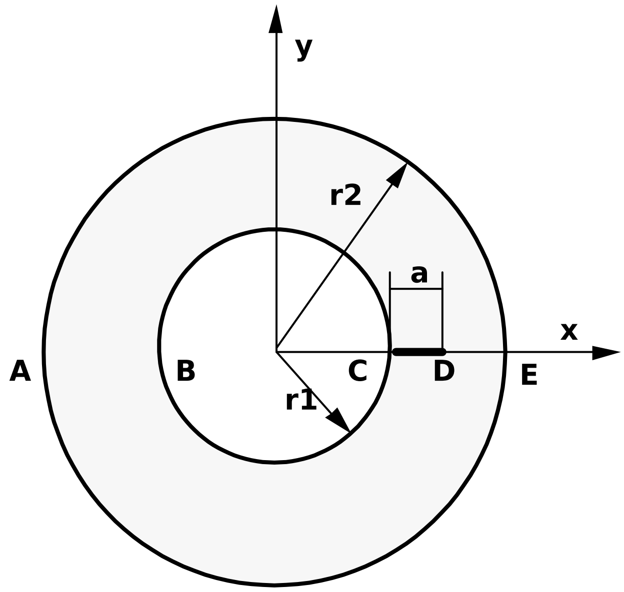
It is about a two-dimensional test in statics in which one models to it not linearity of contact due to refermeture partial of the crack.

The behavior of the structure is thermoelastic linear isotropic.

The case test understands two modelings $2D$ plane for which one studies the influence of the mechanical load factor α. In the first modeling a contact with a material infinimement rigid is used to represent refermeture (symmetrical) crack while in the second a boundary condition of type unilateral connection is put in work.

1 Problem of reference

1.1 Geometry



Cross-section of a thick tube presenting one internal radial crack

Report of the rays $b=r_2/r_1=2\,(r_1=1\,mm,\,r_2=2\,mm)$

Depth of the crack $a/(r_2-r_1)=0{,}05$

1.2 Properties of material

The material is thermoelastic linear isotropic standard.

Young modulus $E=1000\,MPa$
Poisson's ratio $\nu=0{,}3$
Linear dilation coefficient $\alpha_T=1E\text{-}6$
Yield stress $\sigma_0=1\,MPa$ (being used to define the initial stress field created by the process of autofrettage on the assumption of a former behavior of elastoplastic type of Von Mises)

1.3 Boundary conditions and loadings

Boundary conditions (for a half-part in the area $y\geq 0$)

Blocking $UY=0$ on the segment AB and on the ligament DE (symmetry).

Linear relation $UX(A)+UX(E)=0$ (to block the horizontal adjustment)

Loadings

Loading n° 1: radial traction $\sigma_{rr}(r_2)=\sigma_0$ on the external face; this mechanical loading produces the same one K_I that an internal pressure acting simultaneously on the internal ray r_1 and on the lips of the crack, without taking into account of nonthe linearity of contact.

Loading n° 2: thermal loading are equivalent to an autofrettage defined as follows:

$$T_1 = T_\rho + \frac{4\sigma_0}{\sqrt{3}} \bullet \frac{2(1-\nu)}{E\alpha_T} \bullet \ln\left(\frac{\rho}{r_1}\right)$$

$$T = T_1 - \frac{(T_1 - T_\rho)}{\ln\left(\frac{\rho}{r_1}\right)} \bullet \ln\left(\frac{r}{r_1}\right) \qquad r_1 \le r \le \rho$$

$$T = T_\rho \qquad \rho \le r \le r_2$$

In these formulas, ρ indicate the maximum ray of the zone having undergone autofrettage, T_1 the temperature with the ray r_1 and T_ρ the temperature with the ray $r=\rho$ in the thick tube not fissured. In the application concerned here, one takes $\rho=r_2$, which corresponds to the autofrettage of the totality of the section of the thick tube, and one does not take into account it not linearity of contact. One is expected K negative for positive temperatures (put in compression of the tube not fissured).

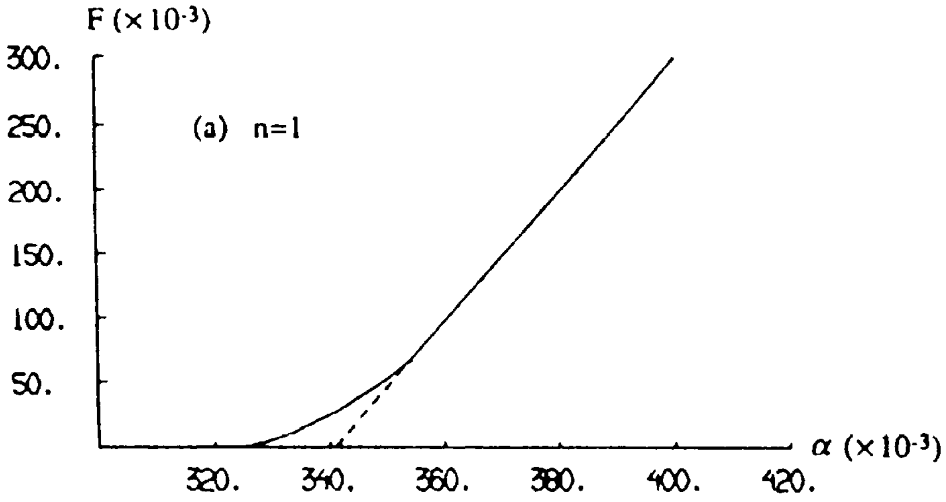
Loading n° 3: linear combination loading n° 2 $+$ α * loading n° 1, α ($\neq \alpha T$!) indicating the mechanical load factor; one takes here into account it not linearity of contact, which supposes an incremental application of the mechanical load.

2 Reference solution

2.1 Method of calculating used for the reference solution

Calculation by finite elements with code ABAQUS. Nonthe linearity of contact is modelled using one-way GAP elements. The factor of intensity of the constraints is calculated starting from the integral J.

2.2 Results of reference



Adimensional factor of intensity of the constraints according to the mechanical factor of loading, in the case of loading n° 3

Notation: $F_L = K_{IL} / \sigma_0 \sqrt{a}$ linear factor of intensity adimensional (obtained by linear combination of the effects of autofrettage and mechanical loading, in stopped feature)

$F_N = K_{IN} / \sigma_0 \sqrt{a}$ nonlinear factor of intensity adimensional (obtained by taking account of nonthe linearity of contact, in full feature).

$$K_I^2 = \frac{EJ}{(1 - \nu^2)}$$

Empirical formula of the factor of intensity of the constraints under external radial tension

$$W = r_2 - r_1$$

$$K_{I0} = P\sqrt{\pi a} \bullet \frac{\dfrac{C_1}{(\ln b)^{0,2}} + \dfrac{C_2}{\ln b}}{\sqrt{1 - 1/b}} \qquad 0,01 \le \frac{a}{W} < 0,8 \quad et \quad 1,5 \le b \le 3,0$$

$$C_1 = 2,397 - 2,705\left(\frac{a}{W}\right)^{0,5} + 0,884\left(\frac{a}{W}\right)^2$$

$$C_2 = -0,244 + 1,447\left(\frac{a}{W}\right)^{0,5} + 0,809\left(\frac{a}{W}\right)^2$$

Empirical formula of the factor of intensity of the constraints in autofrettage in full section

$$K_{Ia} = \sigma_0\sqrt{\pi a} \bullet \frac{C_1 + C_2(\ln b)^{0,75}}{\left(1,8 + \dfrac{1}{b^4}\right)^2} \qquad 0,01 \le \frac{a}{W} < 0,8 \quad et \quad 1,5 \le b \le 3,0$$

$$C_1 = \frac{\left[30,221 - 57,714\left(\frac{a}{W}\right)^{0,05} + 29,954\left(\frac{a}{W}\right)^{0,15} - 2,444\left(\frac{a}{W}\right)^{1,5}\right]}{\sqrt{1 + \left(\frac{a}{W}\right)^{0,25}}}$$

$$C_2 = \frac{\left[-51,522 + 111,027\left(\frac{a}{W}\right)^{0,05} - 63,244\left(\frac{a}{W}\right)^{0,15} + 3,631\left(\frac{a}{W}\right)^{1,5}\right]}{\sqrt{1 - \left(\frac{a}{W}\right)^{0,25}}}$$

2.3 Bibliographical references

1) H.M. SHU, J. SMALL and G. BEZINE: Radial stress intensity factors for aces in thick walled cylinders. I. Symmetrical aces II. Combination of autofrettage and internal presses. Engng.Fract.Mechs., 49, n°4, 611-629, 1994.

3 Modeling A

3.1 Characteristics of modeling

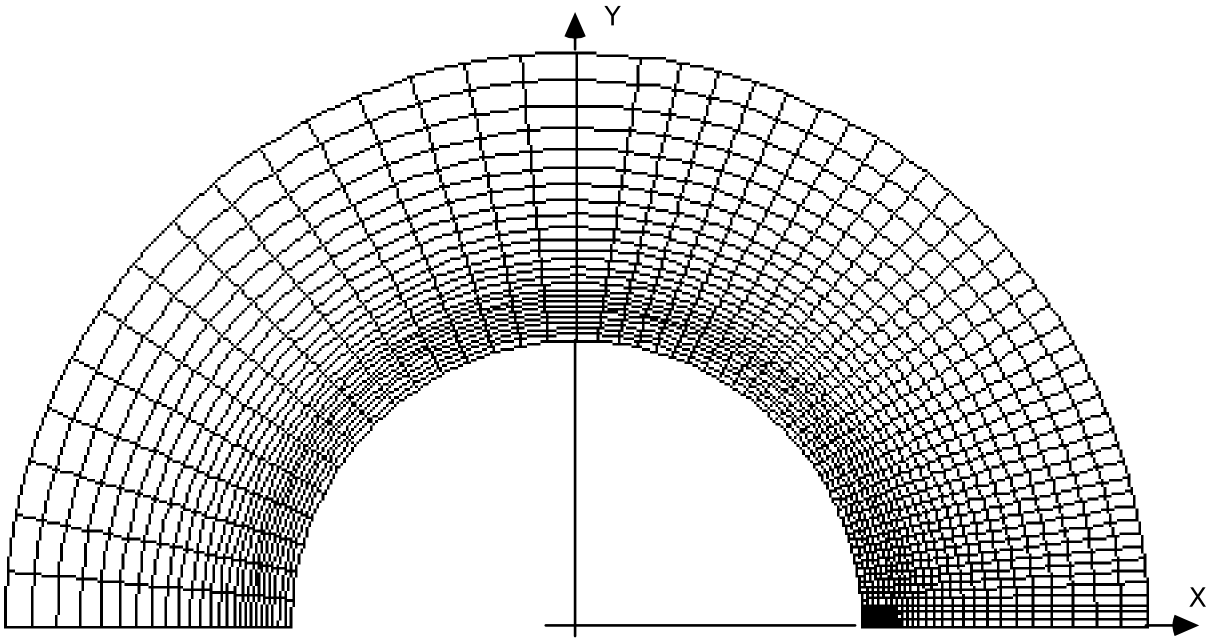
The model consists of quadrangles with 8 nodes and triangles with 6 nodes.
It comprises 4877 nodes and 1598 elements.

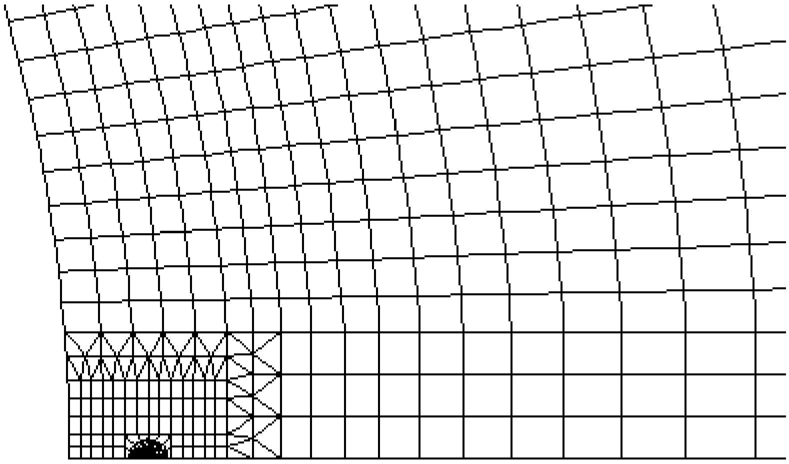
3.2 Characteristics of the grid

Use of the procedure `FISS2D_V1`.

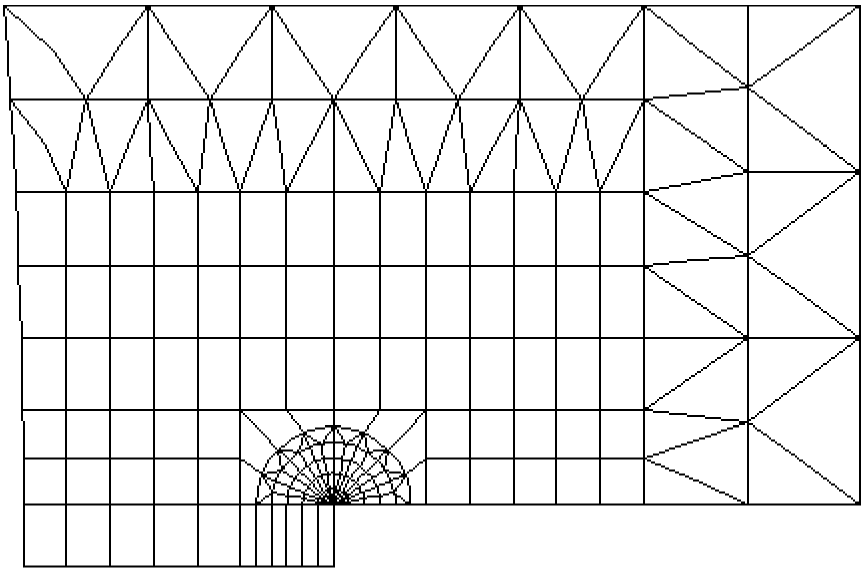
The topological parameters concerning refinement around the bottom of crack are:

- $nc=4$ (many crowns)
- $ns=8$ (many sectors)
- $nbcour=1$ (many crowns of déraffinement)



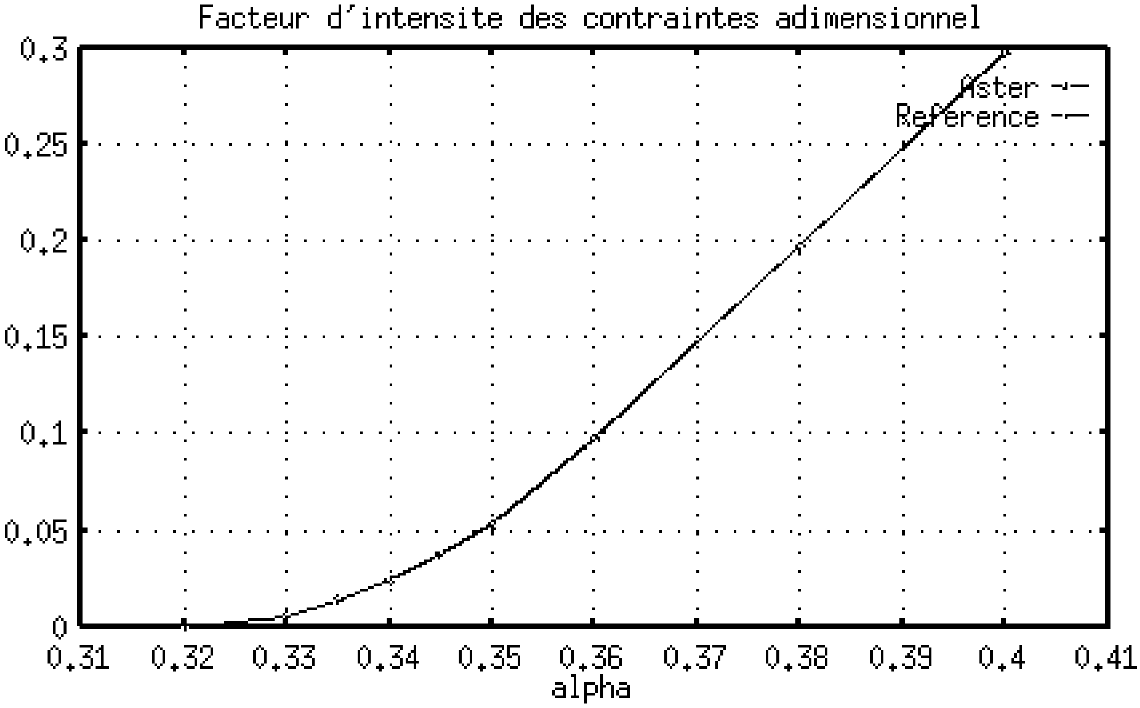


Zoom of the fissured zone



Zoom of the zone fissured with “block of contact”

3.3 Sizes tested and results



Identification	Reference	Type of reference	Tolerance
K_I, loading n°1, crown 0, neglected contact	1.1482	'SOURCE_EXTERNE'	2.0%
K_I, loading n°1, crown 1, neglected contact	1.1482	'SOURCE_EXTERNE'	2.0%
K_I, loading n°1, crown 2, neglected contact	1.1482	'SOURCE_EXTERNE'	2.0%
K_I, loading n°1, crown 3, neglected contact	1.1482	'SOURCE_EXTERNE'	2.0%

Identification	Reference	Type of reference	Tolerance
K_I, loading n°2, crown 0, neglected contact	~0.41237	'SOURCE_EXTERNE'	7.0%
K_I, loading n°2, crown 1, neglected contact	~0.41237	'SOURCE_EXTERNE'	7.0%
K_I, loading n°2, crown 2, neglected contact	~0.41237	'SOURCE_EXTERNE'	7.0%
K_I, loading n°2, crown 3, neglected contact	~0.41237	'SOURCE_EXTERNE'	7.0%

Identification	Reference	Type of reference	Tolerance
K_I, loading n°3, contact, $\alpha=0,33$, crown 0	1,2075E-3	'SOURCE_EXTERNE'	6.0%
K_I, loading n°3, contact, $\alpha=0,335$, crown 0	3,0187E-3	'SOURCE_EXTERNE'	2.5%
K_I, loading n°3, contact, $\alpha=0,34$, crown 0	5,4336E-3	'SOURCE_EXTERNE'	1.0%
K_I, loading n°3, contact, $\alpha=0,345$, crown 0	8,5865E-3	'SOURCE_EXTERNE'	~4.0%
K_I, loading n°3, contact, $\alpha=0,35$, crown 0	1,2075E-2	'SOURCE_EXTERNE'	~4.0%
K_I, loading n°3, contact, $\alpha=0,36$, crown 0	2,1757E-2	'SOURCE_EXTERNE'	1.0%

K_I , loading n°3, contact, $\alpha=0{,}40$, crown 0	6,6478E-2	'SOURCE_EXTERNE'	1.0%

Identification	Reference	Type of reference	Tolerance
K_I, loading n°3, contact, α = 0.33, crown 1	1,2075E-3	'SOURCE_EXTERNE'	2.0%
K_I, loading n°3, contact, α = 0.335, crown 1	3,0187E-3	'SOURCE_EXTERNE'	2.0%
K_I, loading n°3, contact, α = 0.34, crown 1	5,4336E-3	'SOURCE_EXTERNE'	1.0%
K_I, loading n°3, contact, α = 0.345, crown 1	8,5865E-3	'SOURCE_EXTERNE'	~ 4.0%
K_I, loading n°3, contact, α = 0.35, crown 1	1,2075E-2	'SOURCE_EXTERNE'	~ 4.0%
K_I, loading n°3, contact, α = 0.36, crown 1	2,1757E-2	'SOURCE_EXTERNE'	1.0%
K_I, loading n°3, contact, α = 0.40, crown 1	6,6478E-2	'SOURCE_EXTERNE'	1.0%

Identification	Reference	Type of reference	Tolerance
K_I, loading n°3, contact, α = 0.33, crown 2	1,2075E-3	'SOURCE_EXTERNE'	4.5%
K_I, loading n°3, contact, α = 0.335, crown 2	3,0187E-3	'SOURCE_EXTERNE'	2.0%
K_I, loading n°3, contact, α = 0.34, crown 2	5,4336E-3	'SOURCE_EXTERNE'	1.0%
K_I, loading n°3, contact, α = 0.345, crown 2	8,5865E-3	'SOURCE_EXTERNE'	~ 4.0%
K_I, loading n°3, contact, α = 0.35, crown 2	1,2075E-2	'SOURCE_EXTERNE'	~ 4.0%
K_I, loading n°3, contact, α = 0.36, crown 2	2,1757E-2	'SOURCE_EXTERNE'	1.0%
K_I, loading n°3, contact, α = 0.40, crown 2	6,6478E-2	'SOURCE_EXTERNE'	1.0%

Identification	Reference	Type of reference	Tolerance
K_I , loading n°3, contact, $\alpha=0{,}335$, crown 3	3,0187E-3	'SOURCE_EXTERNE'	3.0%
K_I , loading n°3, contact, $\alpha=0{,}34$, crown 3	5,4336E-3	'SOURCE_EXTERNE'	1.0%
K_I , loading n°3, contact, $\alpha=0{,}345$, crown 3	8,5865E-3	'SOURCE_EXTERNE'	~ 4.0%
K_I , loading n°3, contact, $\alpha=0{,}35$, crown 3	1,2075E-3	'SOURCE_EXTERNE'	~ 4.0%
K_I , loading n°3, contact, $\alpha=0{,}36$, crown 3	2,1757E-2	'SOURCE_EXTERNE'	1.0%
K_I , loading n°3, contact, $\alpha=0{,}40$, crown 3	6,6478E-2	'SOURCE_EXTERNE'	1.0%

3.4 Remarks

The tables below give the rate of refund of energy G for two values of the coefficient α who correspond to nonthe separation of the lip of the crack. (There is separation of the lip for $\alpha > 0,32$).

Identification	**Reference**	**G ASTER**
G , loading n°3, contact, $\alpha=0,30$, crown 0	0	~ 8.7941 ~10 ^{16}
G , loading n°3, contact, $\alpha=0,30$, crown 1	0	4.4308 ~10 ^{15}
G , loading n°3, contact, $\alpha=0,30$, crown 2	0	3.3312 ~10 ^{15}
G , loading n°3, contact, $\alpha=0,30$, crown 3	0	4.4794 ~10 ^{13}

Identification	**Reference**	**G ASTER**
G , loading n°3, contact, $\alpha=0,32$, crown 0	0	~ 1,17E-14
G , loading n°3, contact, $\alpha=0,32$, crown 1	0	~ 3,26E-16
G , loading n°3, contact, $\alpha=0,32$, crown 2	0	~ 1,02E-15
G , loading n°3, contact, $\alpha=0,32$, crown 3	0	4,23E-13

4 Modeling B

4.1 Characteristics of modeling

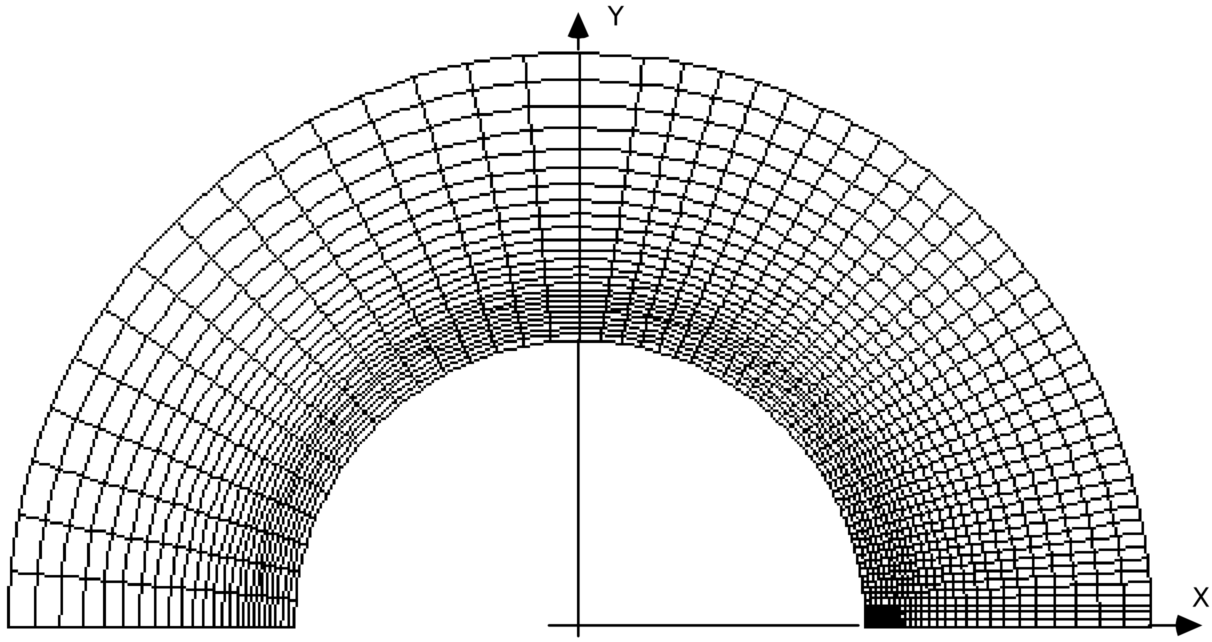
The model consists of quadrangles with 8 nodes and triangles with 6 nodes.
It comprises 4877 nodes and 1598 elements.

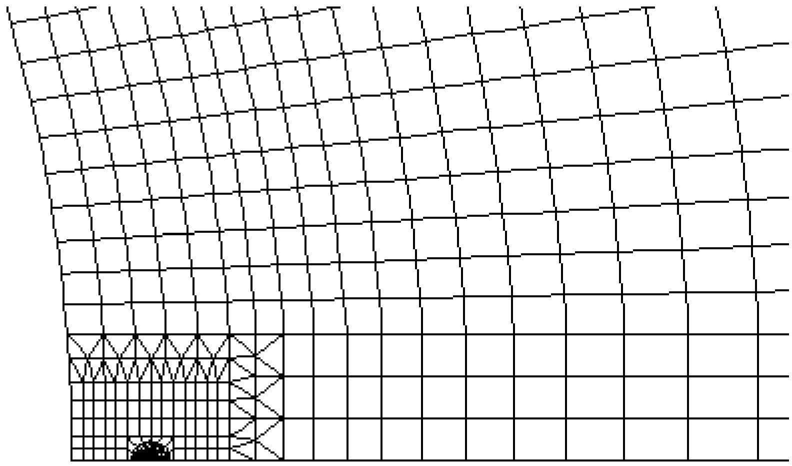
4.2 Characteristics of the grid

Use of the procedure `FISS2D_V1`.

The topological parameters concerning refinement around the bottom of crack are:

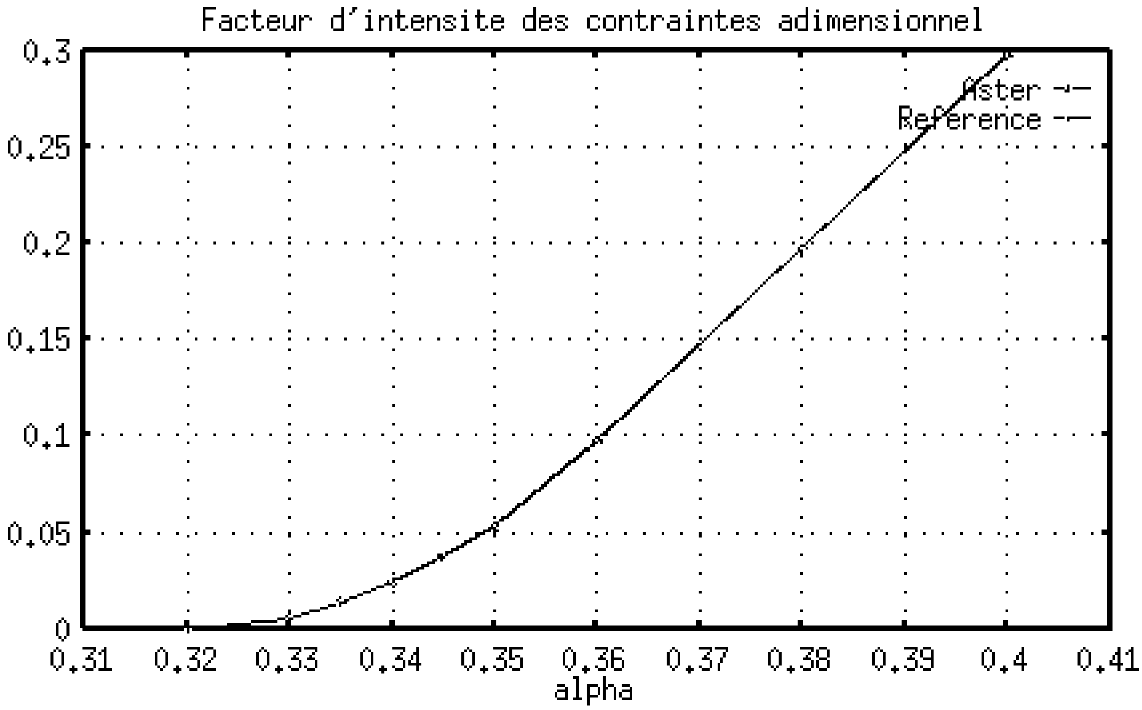
- $nc=4$ (many crowns)
- $ns=8$ (many sectors)
- $nbcour=1$ (many crowns of déraffinement)





Zoom of the fissured zone

4.3 Sizes tested and results



Identification	Reference	Type of reference	Tolerance
K_I , loading n°1, crown 0, neglected contact	1.1482	'SOURCE_EXTERNE'	2.0%
K_I , loading n°1, crown 1, neglected contact	1.1482	'SOURCE_EXTERNE'	2.0%
K_I , loading n°1, crown 2, neglected contact	1.1482	'SOURCE_EXTERNE'	2.0%
K_I , loading n°1, crown 3, neglected contact	1.1482	'SOURCE_EXTERNE'	2.0%

Identification	Reference	Type of reference	Tolerance
K_I , loading n°2, crown 0, neglected contact	~0.41237	'SOURCE_EXTERNE'	7.0%
K_I , loading n°2, crown 1, neglected contact	~0.41237	'SOURCE_EXTERNE'	7.0%

K_I , loading n°2, crown 2, neglected contact	˜ 0.41237	'SOURCE_EXTERNE'	7.0%
K_I , loading n°2, crown 3, neglected contact	˜ 0.41237	'SOURCE_EXTERNE'	7.0%

Identification	Reference	Type of reference	Tolerance
K_I , loading n°3, contact, $\alpha=0{,}33$, crown 0	1,2075E-3	'SOURCE_EXTERNE'	4.5%
K_I , loading n°3, contact, $\alpha=0{,}335$, crown 0	3,0187E-3	'SOURCE_EXTERNE'	3.0%
K_I , loading n°3, contact, $\alpha=0{,}34$, crown 0	5,4336E-3	'SOURCE_EXTERNE'	1.0%
K_I , loading n°3, contact, $\alpha=0{,}345$, crown 0	8,5865E-3	'SOURCE_EXTERNE'	˜ 4.0%
K_I , loading n°3, contact, $\alpha=0{,}35$, crown 0	1,2075E-2	'SOURCE_EXTERNE'	˜ 4.0%
K_I , loading n°3, contact, $\alpha=0{,}36$, crown 0	2,1757E-2	'SOURCE_EXTERNE'	1.0%
K_I , loading n°3, contact, $\alpha=0{,}40$, crown 0	6,6478E-2	'SOURCE_EXTERNE'	1.0%

Identification	Reference	Type of reference	Tolerance
K_I, loading n°3, contact, α = 0.33, crown 1	1,2075E-3	'SOURCE_EXTERNE'	4.5%
K_I, loading n°3, contact, α = 0.335, crown 1	3,0187E-3	'SOURCE_EXTERNE'	3.0%
K_I, loading n°3, contact, α = 0.34, crown 1	5,4336E-3	'SOURCE_EXTERNE'	1.0%
K_I, loading n°3, contact, α = 0.345, crown 1	8,5865E-3	'SOURCE_EXTERNE'	˜ 4.0%
K_I, loading n°3, contact, α = 0.35, crown 1	1,2075E-2	'SOURCE_EXTERNE'	˜ 4.0%
K_I, loading n°3, contact, α = 0.36, crown 1	2,1757E-2	'SOURCE_EXTERNE'	1.0%
K_I, loading n°3, contact, α = 0.40, crown 1	6,6478E-2	'SOURCE_EXTERNE'	1.0%

Identification	Reference	Type of reference	Tolerance
K_I, loading n°3, contact, α = 0.33, crown 2	1,2075E-3	'SOURCE_EXTERNE'	4.5%
K_I, loading n°3, contact, α = 0.335, crown 2	3,0187E-3	'SOURCE_EXTERNE'	3.0%
K_I, loading n°3, contact, α = 0.34, crown 2	5,4336E-3	'SOURCE_EXTERNE'	1.0%
K_I, loading n°3, contact, α = 0.345, crown 2	8,5865E-3	'SOURCE_EXTERNE'	˜ 4.0%
K_I, loading n°3, contact, α = 0.35, crown 2	1,2075E-2	'SOURCE_EXTERNE'	˜ 4.0%
K_I, loading n°3, contact, α = 0.36, crown 2	2,1757E-2	'SOURCE_EXTERNE'	1.0%
K_I, loading n°3, contact, α = 0.40, crown 2	6,6478E-2	'SOURCE_EXTERNE'	1.0%

Identification	Reference	Type of reference	Tolerance
K_I , loading n°3, contact, $\alpha=0{,}335$, crown 3	3,0187E-3	'SOURCE_EXTERNE'	3.0%
K_I , loading n°3, contact, $\alpha=0{,}34$, crown 3	5,4336E-3	'SOURCE_EXTERNE'	1.0%
K_I , loading n°3, contact, $\alpha=0{,}345$, crown 3	8,5865E-3	'SOURCE_EXTERNE'	˜ 4.0%
K_I , loading n°3, contact, $\alpha=0{,}35$, crown 3	1,2075E-3	'SOURCE_EXTERNE'	˜ 4.0%
K_I , loading n°3, contact, $\alpha=0{,}36$, crown 3	2,1757E-2	'SOURCE_EXTERNE'	1.0%
K_I , loading n°3, contact, $\alpha=0{,}40$, crown 3	6,6478E-2	'SOURCE_EXTERNE'	1.0%

4.4 Remarks

The tables below give the rate of refund of energy G for two values of the coefficient α who correspond to nonthe separation of the lip of the crack. (There is separation of the lip for $\alpha>0{,}32$).

Identification	Reference	G ASTER
G , loading n°3, contact, $\alpha=0{,}30$, crown 0	0	˜ 8.7941 $10^{\sim 16}$
G , loading n°3, contact, $\alpha=0{,}30$, crown 1	0	4.4308 $10^{\sim 15}$
G , loading n°3, contact, $\alpha=0{,}30$, crown 2	0	3.3312 $10^{\sim 15}$
G , loading n°3, contact, $\alpha=0{,}30$, crown 3	0	4.4794 $10^{\sim 13}$

Identification	Reference	G ASTER
G , loading n°3, contact, $\alpha=0{,}32$, crown 0	0	˜ 1,17E-14
G , loading n°3, contact, $\alpha=0{,}32$, crown 1	0	˜ 3,26E-16
G , loading n°3, contact, $\alpha=0{,}32$, crown 2	0	˜ 1,02E-15
G , loading n°3, contact, $\alpha=0{,}32$, crown 3	0	4,23E-13

5 Summary of the results

The calculation of G is correct in all the cases, including for a completely closed crack. Modelings with block of contact and unilateral connection give similar results.