HPLP311 - Murakami 11.17 Fissures in the center of a rectangular thin section which prevents a uniform heat flow in isotropic medium

Summary:

It is about an isotropic linear thermoelastic calculation static.

It is a basic test in $2D$ plan for a stationary thermal loading calculated by finite elements on the same grid with an isotropic material in mode II.

Objective:

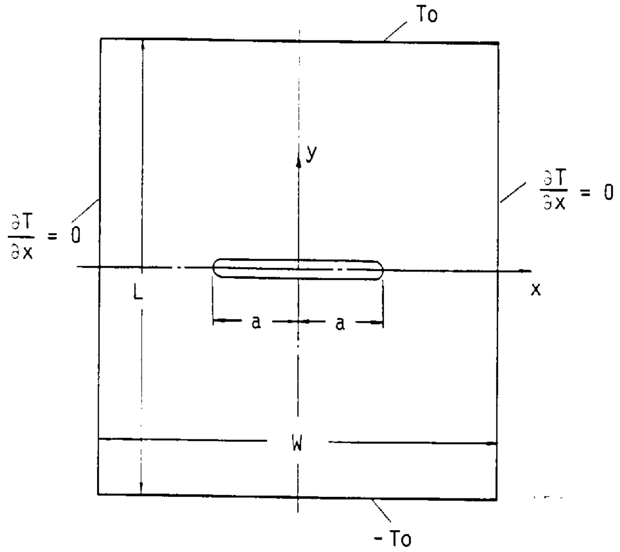
- basic test in $2D$ plan, for a stationary thermal loading calculated by finite elements on the same grid, with isotropic material, in mode II,
- validation of the calculation of K_{II},
- variability of G according to the topology (sectors, crowns) of the radiant grid. Checking of the invariance of the results in breaking process, at an end of crack, the grid of the other end of the same crack.

Calculation is tested on a complete grid and a half-grid. Parameters L/W and $2A/W$ being fixed.

One measures a relative variation on K_{II}, the precision is nevertheless badly defined.

1 Problem of reference

1.1 Geometry



Width of the plate: $W=0{,}6\,m$
Length of the plate: $L=0{,}3\,m$
Length of the crack: $2a=0{,}3\,m$

1.2 Properties of material

Notation for thermoelastic properties:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} + \begin{Bmatrix} \alpha_{11} \\ \alpha_{22} \\ 0 \end{Bmatrix} (T - T_{ref})$$

$$S_{11} = 1/E_x$$

$$S_{22} = 1/E_y$$

$$S_{12} = -\nu_x/E_x = -\nu_y/E_y$$

$$S_{66} = 1/G_{xy}$$

$$\alpha_{11} = \alpha_x$$

$$\alpha_{22} = \alpha_y$$

One limits oneself to isotropic material, as well from the thermal point of view as mechanical:

$$E_x = E_y = 2.10^5\,MPa$$

$$\nu_x = \nu_y = 0{,}3$$

$$\alpha_x = \alpha_y = 1{,}2\,10^{-5}\,{}^\circ C^{-1}$$

$$\lambda_x = \lambda_y = 54\,W/m\,{}^\circ C$$

1.3 Boundary conditions and loading

Two models are considered:

- the half-model $x \geq 0$
- the complete model

Boundary conditions mechanical:

- half-model

  $UX=0$ along the axis of symmetry $X=0$
  $UY=0$ at the point $(W/2,0)$

- complete model

  $UX=0$ at the point $(0, L/2)$
  $UY=0$ at the points $(-L/2,0)$ and $(L/2,0)$

Boundary conditions thermal:

- half-model

  $T=100°C$ on the higher edge $Y=L/2$
  $T=-100°C$ on the lower edge $Y=-L/2$
  null flow on the axis of symmetry, the free edge $X=W/2$ and on the edge of the crack

- complete model

  $T=100°C$ on the higher edge $Y=L/2$
  $T=-100°C$ on the lower edge $Y=-L/2$
  null flow on the free edges $X=\pm W/2$ and on the edge of the crack

2 Reference solution

2.1 Method of calculating used for the reference solution

Complex potential.

2.2 Results of reference

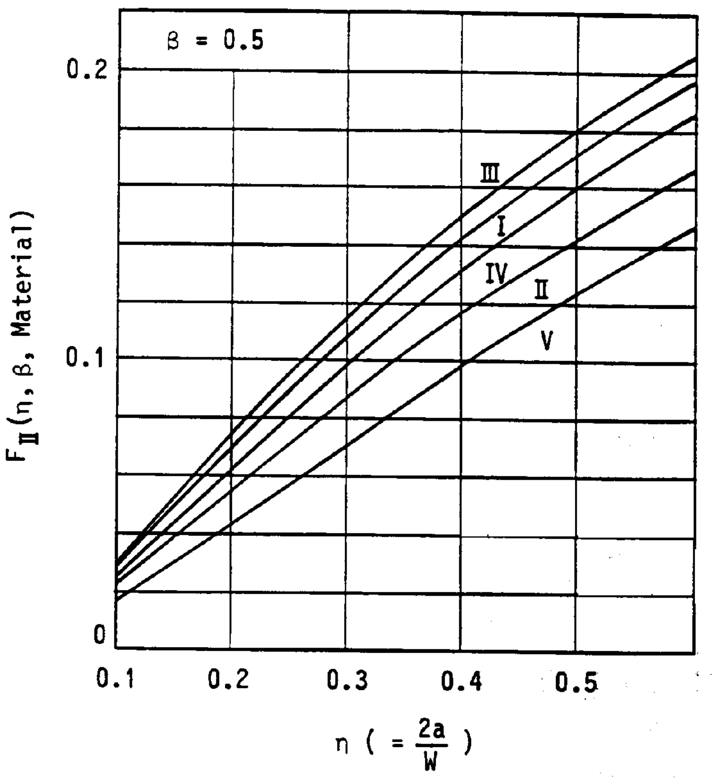
$$\eta = \frac{2a}{W}$$

$$\beta = \frac{L}{W}$$

$$K_{II} = \frac{\alpha_{11} T_0}{S_{11}} \bullet \sqrt{\frac{W}{2}} \bullet F_{II}$$

where the geometrical factor of correction F_{II} is given according to η for each material, in the typical case $\beta = 0{,}5$ on the curves below.

The isotropic material being represented by the curve I



2.3 Uncertainty on the solution

Nondefinite precision.

2.4 Bibliographical references

1) Y. MURAKAMI: Stress Intensity Factors Handbook, box 11.17, pages 1045-1047. The Society of Materials Science, Japan, Pergamon Near, 1987.

3 Modelings A, B, C, D, E and F

3.1 Characteristics of modeling

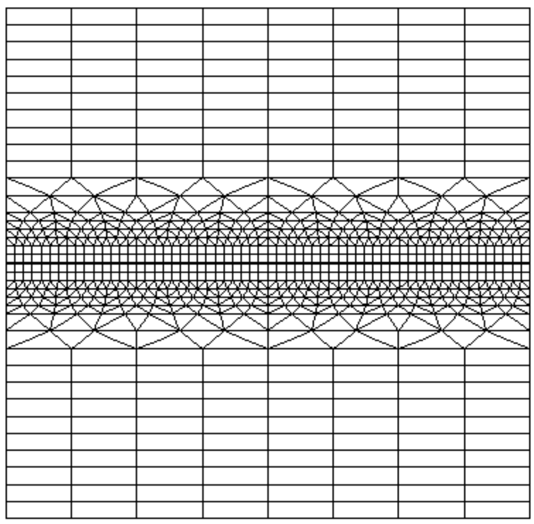
These 6 modelings correspond to 6 grids where one varies 3 topological parameters. The table below summarizes the various studied cases:

	$NS=8$, $NC=4$	$NS=4$, $NC=3$
$rt=0{,}001*a$	With	B
$rt=0{,}01*a$	C	D
$rt=0{,}1*a$	E	F

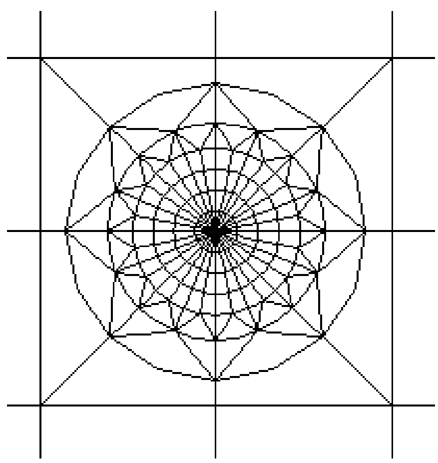
The topological parameters which vary are:

NS : many sectors on 90°
NC : many crowns
rt : the ray of the largest crown (with half a: length of the crack)

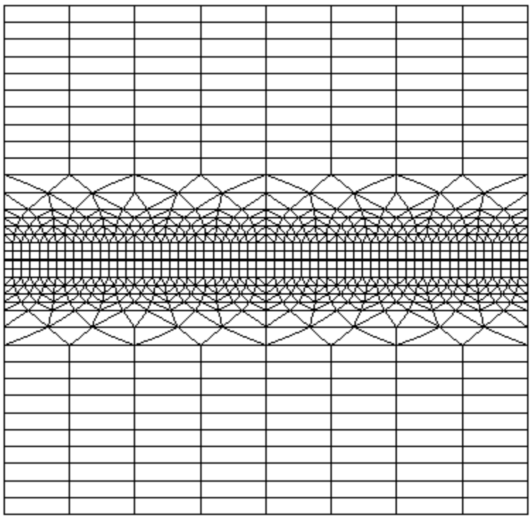
3.1.1 Modelings A and B



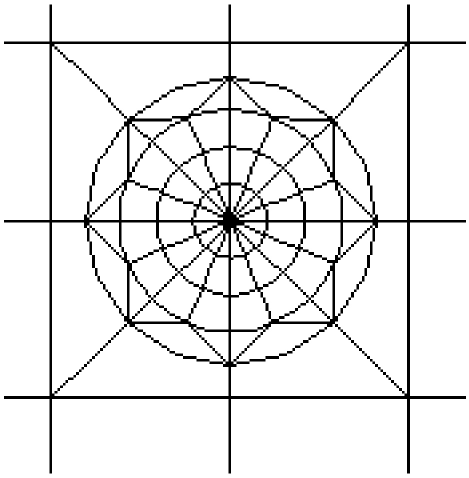
Half grid - Modeling A



Zoom of the point of crack - Modeling A

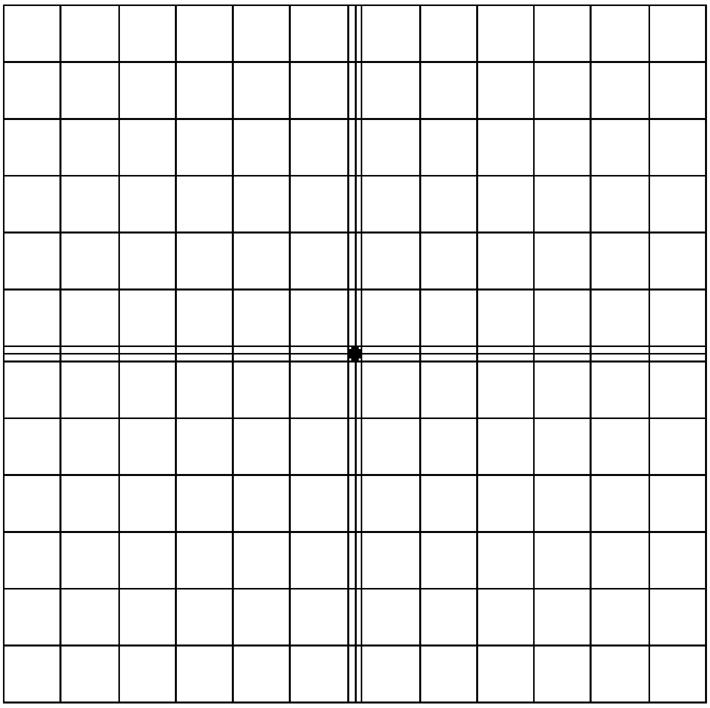


Half grid - Modeling B

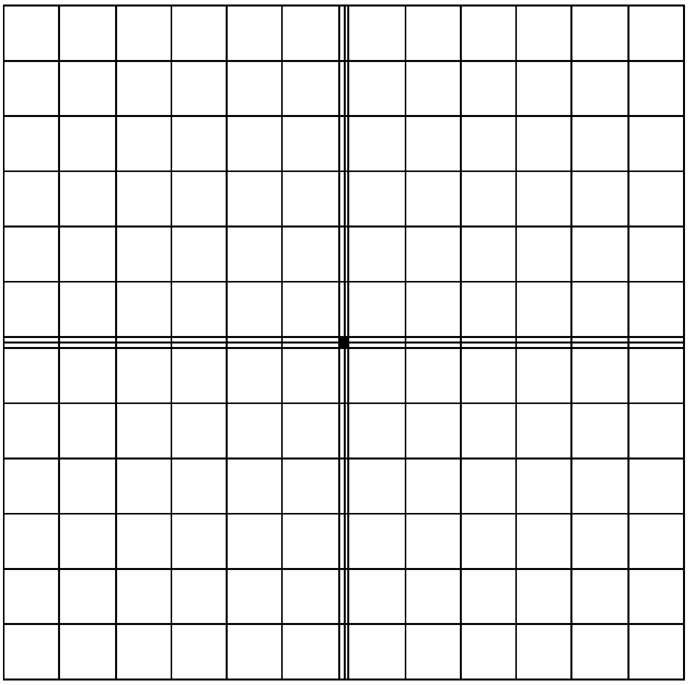


Zoom of the point of crack - Modeling B

3.1.2 Modelings C and D

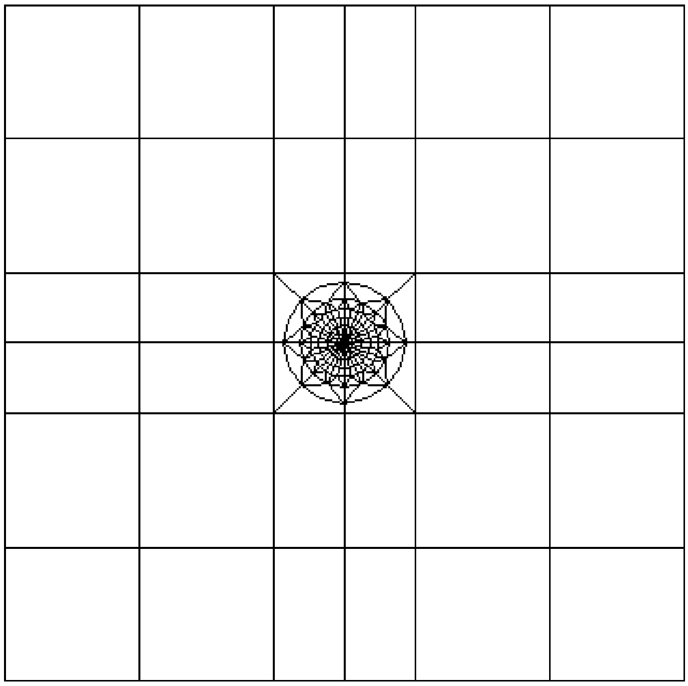


Complete grid - Modeling C

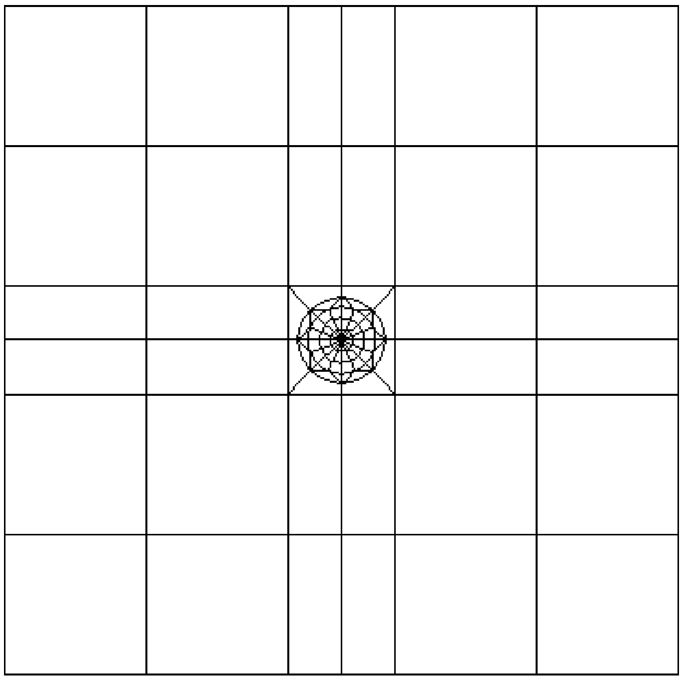


Complete grid - Modeling D

3.1.3 Modelings E and F



Complete grid - Modeling E



Complete grid - Modeling F

3.1.4 Definition of the rays of the crowns

For these various cases, we define the values of the higher and lower rays, to specify in the order CALC_THETA :

Modeling A

	1st crown	2nd crown	3rd crown	4th crown
$rinf(m)$	3,75E-5	7,500E-5	1,125E-4	1,500E–4
$\mathrm{rsup}(m)$	7,50E-5	1,125E-4	1,500E-4	1,875E-4

Modeling B

	1st crown	2nd crown	3rd crown
$rinf(m)$	5,00E-5	1,00E-4	1,50E-4
$\mathrm{rsup}(m)$	1,00E-4	1,50E-4	2,00E-4

Modeling C

	1st crown	2nd crown	3rd crown	4th crown
$rinf(m)$	3,75E-4	7,500E-4	1,125E-3	1,500E–3
$\mathrm{rsup}(m)$	7,50E-4	1,125E-3	1,500E-3	1,875E-3

Modeling D

	1st crown	2nd crown	3rd crown
$rinf(m)$	5,00E-4	1,00E-3	1,50E-3
$\mathrm{rsup}(m)$	1,00E-3	1,50E-3	2,00E-3

Modeling E

	1st crown	2nd crown	3rd crown	4th crown
$rinf(m)$	3,75E-3	7,500E-3	1,125E-2	1,500E–2
$\mathrm{rsup}(m)$	7,50E-3	1,125E-2	1,500E-2	1,875E-2

Modeling F

	1st crown	2nd crown	3rd crown
$rinf(m)$	5,00E-3	1,00E-2	1,50E-2
$\mathrm{rsup}(m)$	1,00E-2	1,50E-2	2,00E-2

3.2 Characteristics of the grid

Half-grid; grid radiating at the right end of the crack.

The table below gives the constitution of the studied grids:

	$NS=8$, $NC=4$	$NS=4$, $NC=3$
$rt=0{,}001*a$	3831 nodes, 1516 elements, 884 `TRI6`, 632 `QUA8`.	3507 nodes, 1388 elements, 820 `TRI6`, 568 `QUA8`.
$rt=0{,}01*a$	1179 nodes, 400 elements, 104 `TRI6`, 296 `QUA8`.	855 nodes, 272 elements, 40 `TRI6`, 232 `QUA8`.
$rt=0{,}1*a$	659 nodes, 240 elements, 104 `TRI6`, 136 `QUA8`.	335 nodes, 112 elements, 40 `TRI6`, 72 `QUA8`.

4 Results of modelings A, B, C, D, E and F

4.1 Values tested

Identification	Reference	Aster	% difference
Diameter crowns external = $0,001*a$			
Radiant grid	$NS=8$	$NC=4$	**Modeling A**
K_{II} , crown n°1	2,2347E+7	2,2814E7	2.09
K_{II} , crown n°2	2,2347E+7	2,2813E7	2.08
K_{II} , crown n°3	2,2347E+7	2,2814E7	2.09
K_{II} , crown n°4	2,2347E+7	2,2814E7	2.09
Radiant grid	$NS=4$	$NC=3$	**Modeling B**
K_{II} , crown n°1	2,2347E+7	2,282E7	2.10
K_{II} , crown n°2	2,2347E+7	2,282E7	2.10
K_{II} , crown n°3	2,2347E+7	2,281E7	2.09
Diameter crowns external = $0,01*a$			
Radiant grid	$NS=8$	$NC=4$	**Modeling C**
K_{II} , crown n°1	2,2347E+7	$2.166\ 10^{7}$	3.058
K_{II} , crown n°2	2,2347E+7	$2.214\ 10^{7}$	0.919
K_{II} , crown n°3	2,2347E+7	$2.214\ 10^{7}$	0.919
K_{II} , crown n°4	2,2347E+7	$2.214\ 10^{7}$	0.919
Radiant grid	NS= 4	NC= 3	Modeling D
K_{II} , crown n°1	2,2347E+7	$2.214\ 10^{7}$	0.919
K_{II} , crown n°2	2,2347E+7	$2.214\ 10^{7}$	0.919
K_{II} , crown n°3	2,2347E+7	$2.214\ 10^{7}$	0.919
Diameter crowns external = $0,1*a$			
Radiant grid	$NS=8$	$NC=4$	**Modeling E**
K_{II} , crown n°1	2,2347E+7	$2.2632\ 10^{7}$	1.276
K_{II} , crown n°2	2,2347E+7	$2.2572\ 10^{7}$	1.009
K_{II} , crown n°3	2,2347E+7	$2.2572\ 10^{7}$	1.008
K_{II} , crown n°4	2,2347E+7	$2.2564\ 10^{7}$	0.972
Radiant grid	$NS=4$	$NC=3$	**Modeling F**
K_{II} , crown n°1	2,2347E+7	2,255E7	0.932
K_{II} , crown n°2	2,2347E+7	2,2568E7	0.988
K_{II} , crown n°3	2,2347E+7	2,2568E7	0.987

Identification	Reference	*Aster*	$\%$ difference
Diameter crowns external = $0{,}001*a$			
Radiant grid	$NS=8$	$NC=4$	**Modeling A**
G , crown n°1	2,4969E+3	2,5984E+3	4.07
G , crown n°2	2,4969E+3	2,5990E+3	4.09
G , crown n°3	2,4969E+3	2,5992E+3	4.10
G , crown n°4	2,4969E+3	2,5993E+3	4.10
Radiant grid	$NS=4$	$NC=3$	**Modeling B**
G , crown n°1	2,4969E+3	2.600 10^3	4.134
G , crown n°2	2,4969E+3	2.5996 10^3	4.114
G , crown n°3	2,4969E+3	2.5996 10^3	4.111
Diameter crowns external = $0{,}01*a$			
Radiant grid	$NS=8$	$NC=4$	**Modeling C**
G , crown n°1	2,4969E+3	2.451 10^3	1.842
G , crown n°2	2,4969E+3	2.475 10^3	0.858
G , crown n°3	2,4969E+3	2.475 10^3	0.858
G , crown n°4	2,4969E+3	2.475 10^3	0.858
Radiant grid	**NS= 4**	**NC= 3**	**Modeling D**
G , crown n°1	2,4969E+3	2.475 10^3	0.858
G , crown n°2	2,4969E+3	2.475 10^3	0.858
G , crown n°3	2,4969E+3	2.475 10^3	0.858
Diameter crowns external = $0{,}1*a$			
Radiant grid	**NS= 8**	**NC= 4**	**Modeling E**
G , crown n°1	2,4969E+3	2,5624E3	2.627
G , crown n°2	2,4969E+3	2,5503E3	2.139
G , crown n°3	2,4969E+3	2,5499E3	2.124
G , crown n°4	2,4969E+3	2.5489 E3	2.084
Radiant grid	**NS= 4**	**NC= 3**	**Modeling F**
G , crown n°1	2,4969E+3	2.5470 E3	2.006
G , crown n°2	2,4969E+3	2.5497 E3	2.117
G , crown n°3	2,4969E+3	2.5491 E3	2.094

4.2 Remarks

In the reference, the author supposes that $KI=0$, but it does not check it a posteriori.

With regard to the rate of refund of energy G , if we suppose that $KI=0$, we draw the value of reference starting from the formula from IRWIN in plane constraints:

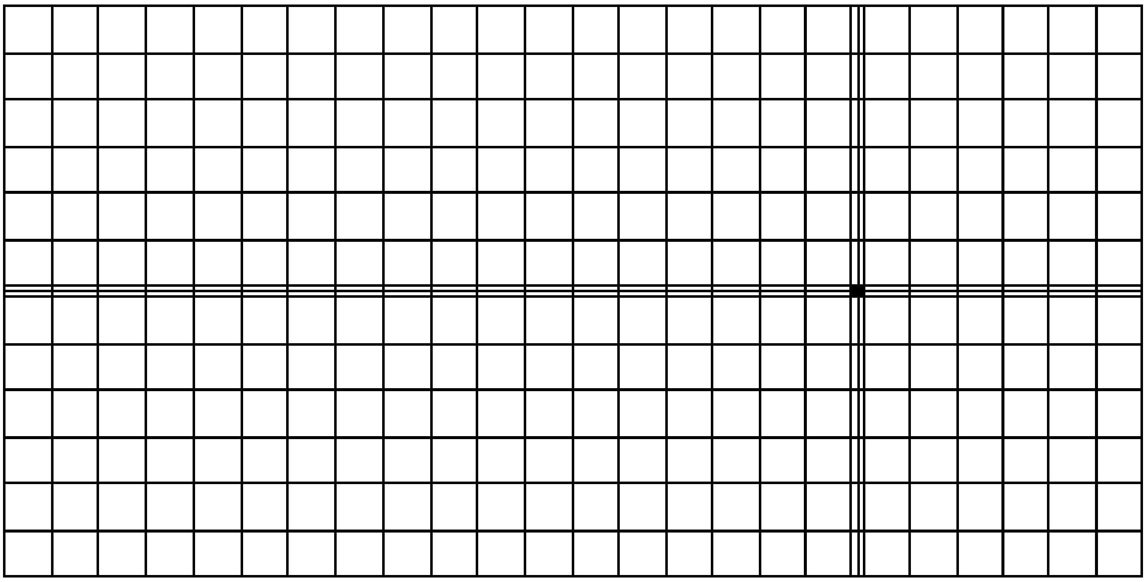
$$G_{ref}=(1/E)*KII^2$$

5 Modeling G

5.1 Characteristics of modeling

For this modeling, we use the complete model with the best parameters NS , NC and rt calculated in preceding modelings. We thus used the following values:

- $NS=8$,
- $NC=4$,
- $rt=0{,}01*a$.



Complete grid

5.2 Characteristics of the grid

Complete model, with grid radiating only at the right end of the crack and regular grid, not refined, at the left end.

The grid consists of 1718 nodes and 568 elements, including 464 elements QUA8 and 104 elements TRI6.

5.3 Sizes tested and results

Identification	Reference	Aster	% difference
Diameter crowns external = $0,01*a$			
Radiant grid	$NS=8$	$NC=4$	
K_{II} , crown n°1	2,2347E+7	2,2640E7	1.31
K_{II} , crown n°2	2,2347E+7	2,2640E7	1.31
K_{II} , crown n°3	2,2347E+7	2,2640E7	1.31
K_{II} , crown n°4	2,2347E+7	2,2641E7	1.31

Identification	Reference	Aster	% difference
Diameter crowns external = $0,01*a$			
Radiant grid	$NS=8$	$NC=4$	
G , crown n°1	2,4969E+3	2,5620E3	2.610
G , crown n°2	2,4969E+3	2,5626E3	2.631
G , crown n°3	2,4969E+3	2,5627E3	2.635
G , crown n°4	2,4969E+3	2.5628E3	2.640

6 Summary of the results

Differences between the reference solution and the results of *Code_Aster* do not exceed 3% on the coefficients of intensity of constraints and 4% for the rate of refund of energy. One checks the invariance of the results compared to the various crowns of integration.