

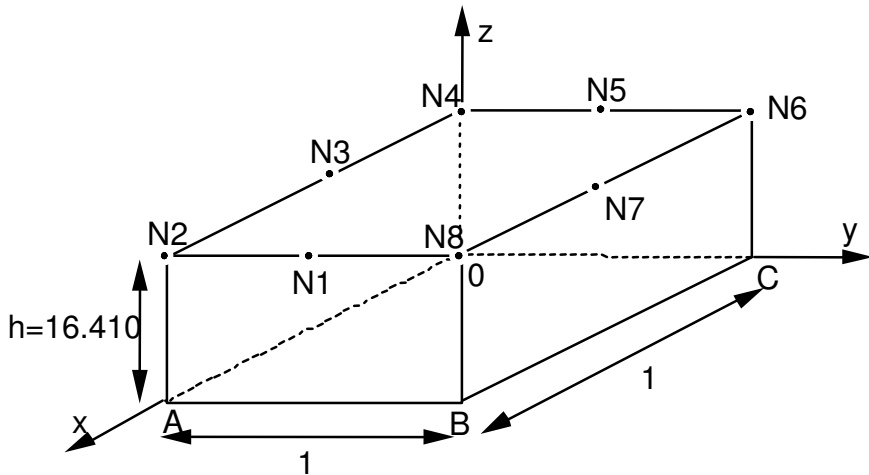
HPLV101 - Homogenisation of a material homogeneous

Summary:

This test tests, in a commonplace situation where the material is homogeneous, the resolution of the thermal and mechanical problems stationary, with loadings corresponding to a variation in temperature and an imposed deformation, close to those corresponding to the elementary problems of the method of periodic homogenisation.

1 Problem of reference

1.1 Geometry



	x	y	z
N4	0	0	16.410
N8	1.	1.	16.410
N3	0.5	0	16.410

1.2 Material properties

Modeling A

$$E = 1.0 \text{ MPa}$$

$$\nu = 0.3$$

$$k = 1.0 \text{ W l(m. } ^\circ\text{C)}$$

$$C_p = 0 \text{ J l(} ^\circ\text{C.m}^3\text{)}$$

Modeling B

$$E_L = 1.0 \text{ MPa}$$

$$E_T = 0.9 \text{ MPa}$$

$$E_N = 0.8 \text{ MPa}$$

$$\nu_{LT} = 0.1$$

$$\nu_{LN} = 0.25$$

$$\nu_{TN} = 0.3333333$$

$$k = 1.0 \text{ W l(m. } ^\circ\text{C)}$$

$$C_p = 0 \text{ J l(} ^\circ\text{C.m}^3\text{)}$$

1.3 Boundary conditions and loadings

- Mechanics 3D:
 - Plan $z=0$: $dz=0$ for the membrane loading;
 $dx=0, dy=0$ for the loading of inflection
 - Plans $y=0, y=1$: $dy=0$
 - Plans $x=0, x=1$: $dx=0$
 - Node: O $dz=0$ (for the only loading of inflection)

Loading:

$$\begin{aligned} \text{déformation membranaire: } \mathbf{E} &= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \text{flexion uniforme imposée: } \mathbf{E} &= \begin{pmatrix} \frac{z}{z_0} & 0 & 0 \\ z_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

$$z_0 = 1 \text{ m}$$

- Mechanics 2D, forced plane:

Axis: $x=0$ $dx=0$ (these conditions do not correspond to the application of the method of homogenisation).
Node: O $dy=0$

$$\text{Loading: deformation } \mathbf{E} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ uniforme imposée}$$

- Thermics 3D and 2D:

Plan $x=0$ $temp=0$ (this condition does not correspond to the application of the method of homogenisation).

Loading: gradient $\mathbf{G} = (-1, 0, 0)$ imposed uniform.

2 Reference solution

2.1 Method of calculating used for the reference solution

- In thermics: the stationary thermal problem is solved:

$$\int_{\Omega} \nabla T \cdot \mathbf{K} \cdot \nabla \theta = \int_{\Omega} \mathbf{G} \cdot \mathbf{K} \cdot \nabla \theta, \quad \forall \theta \in V, \quad \text{avec } \mathbf{G} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

Note:

The boundary conditions chosen here are not those necessary to the method of homogenisation: one would find indeed $T = 0$ everywhere.

The solution is then (checking the conditions defined in [§1.3]): $T(x, y, z) = -x$

The potential energy is then with balance: $W^{th} = -\frac{1}{2} \int_{\Omega} \nabla T \cdot \mathbf{K} \cdot \nabla T = -\frac{1}{2} |\Omega|$ ici

- In mechanics: one solves the problem of elastostatic:

$$\int_{\Omega} \varepsilon(\mathbf{u}) \cdot \mathbf{A} \cdot \varepsilon(\mathbf{v}) = \int_{\Omega} \mathbf{E} \cdot \mathbf{A} \cdot \varepsilon(\mathbf{v}), \quad \forall \mathbf{v} \in \mathbf{W},$$

for the cases:

loading 3D membrane	loading 3D of inflection	loading 2D plane constraints
$\mathbf{E} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\mathbf{E} = \begin{pmatrix} z & 0 & 0 \\ z_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\mathbf{E} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

The solutions are:

- in 3D, loading **membrane** and isotropic elasticity: $\mathbf{u}(x, y, z) = \begin{pmatrix} 0 \\ 0 \\ -\frac{\nu z}{(1-\nu)} \end{pmatrix}$

the potential energy with balance is:

$$W^{pot} = -\frac{1}{2} \int_{\Omega} \varepsilon(\mathbf{u}) \cdot \mathbf{A} \cdot \varepsilon(\mathbf{u}) = -\frac{\nu}{1-\nu} \frac{|\Omega|^2}{2} (\lambda + 2\mu)$$

- in 3D, loading **membrane** and orthotropic elasticity: $\mathbf{u}(x, y, z) = (0, 0, -\beta z)$ with $\beta = \frac{C_{13}}{C_{33}}$ that is

$$\beta = \frac{\frac{E_N}{E_L} (\nu_{LN} + \nu_{LT} \nu_{TN})}{1 - \frac{E_N}{E_T} \nu_{TN}^2}$$

because the local reference mark is not confused with the total

reference mark (nautical angles being worth all 90°).

$$W^{pot} = -\frac{1}{2} \int_{\Omega} \mathbf{u} \cdot \mathbf{A} \cdot \varepsilon(\mathbf{u}) = -\beta^2 \frac{|\Omega|}{2} C_{33}$$

$$\text{where } C_{33} = \frac{1 - \nu_{NT}\nu_{TN}}{E_T E_N \Delta} \text{ and } \Delta = \frac{1 - \nu_{LT}\nu_{TL} - \nu_{LN}\nu_{NL} - \nu_{TN}\nu_{NT} - 2\nu_{LT}\nu_{TN}\nu_{NL}}{E_L E_T E_N}$$

- in 3D, loading of **inflection** : $\mathbf{u}(x, y, z) = \begin{bmatrix} 0 \\ 0 \\ \frac{+ \nu z^2}{2z_0(1-\nu)} \end{bmatrix}$;

$$W^{pot} = - \begin{bmatrix} \nu \\ 1-\nu \end{bmatrix}^2 \cdot \frac{|\Omega|}{2} (\lambda + 2\mu) \cdot \frac{h^2}{3z_0^2}$$

- in 3D, loading **of inflection** and orthotropic elasticity: $\mathbf{u}(x, y, z) = \begin{bmatrix} 0 \\ 0 \\ -\beta \frac{z}{L} \end{bmatrix}$ with $\beta = \frac{C_{13}}{C_{33}}$ that

$$\text{is to say } \beta = \frac{\frac{E_N}{E_L} (\nu_{LN} + \nu_{LT}\nu_{TN})}{1 - \frac{E_N}{E_T} \nu_{TN}^2}$$

because the local reference mark is not confused with the

total reference mark (nautical angles being worth all 90°).

$$W^{pot} = -\frac{1}{2} \int_{\Omega} \varepsilon(\mathbf{u}) \cdot \mathbf{A} \cdot \varepsilon(\mathbf{u}) = -\beta^2 \cdot \frac{|\Omega|}{2} C_{33} \frac{h^2}{3z_0^2}$$

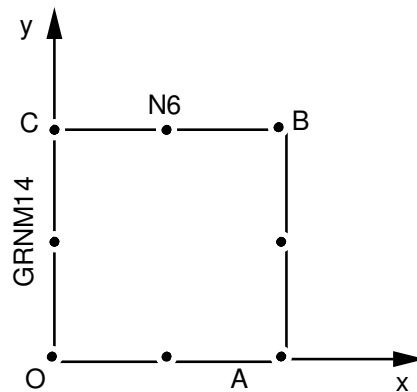
$$\text{where } C_{33} = \frac{1 - \nu_{NT}\nu_{TN}}{E_T E_N \Delta} \text{ and } \Delta = \frac{1 - \nu_{LT}\nu_{TL} - \nu_{LN}\nu_{NL} - \nu_{TN}\nu_{NT} - 2\nu_{LT}\nu_{TN}\nu_{NL}}{E_L E_T E_N}$$

- in 2D, loading plan: $\mathbf{u}(x, y) = (-x, 0)$;

$$W^{pot} = \frac{-|\Omega|}{2(1-\nu^2)}$$

3 Modeling A

3.1 Characteristics of modeling



Boundary conditions and loading:

Thermics: GROUP_NO: GRNM14: TEMP: 0.0
 PRE_GRAD_TEMP: FLUX_X: - 1.0

Mechanics:
(plane constraints) GROUP_NO: GRNM14: DX: 0.0
 NODE: O DY: 0.0
 PRE_EPSI: EPXX: - 1.0

3.2 Characteristics of the grid

Many nodes: 8

Many meshes and types: 1 QUAD8

3.3 Values tested

Not	Size	Reference
<i>CMP</i>		
<i>A</i>	<i>TEMP</i>	- 1.0000
<i>A</i>	<i>DX</i>	- 1.0000
<i>N6</i>	<i>DX</i>	- 0.5000

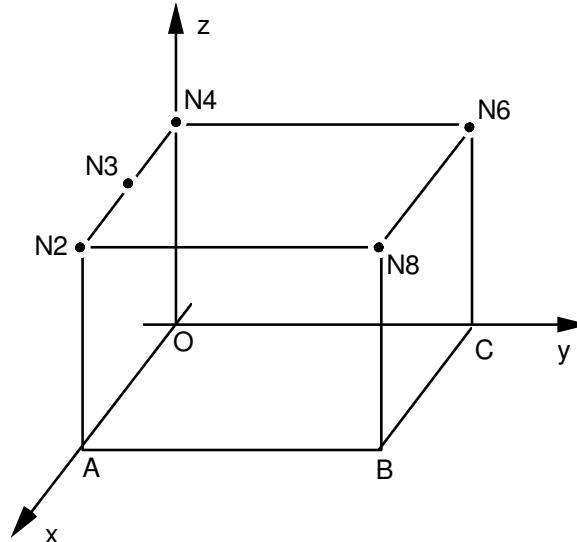
Mesh	Energy potential with balance	Reference
<i>MI</i>	Thermics	- 0.500000000
<i>MI</i>	Mechanics	- 0.549450550

3.4 Remarks

Code_Aster provides the value of the deformation energy, equal contrary to the potential energy to balance (elastic case).

4 Modeling B

4.1 Characteristics of modeling



Name of the meshes of the faces:	ZEGAL0	YEGAL0	YEGAL1	XEGAL0	XEGAL1
Summits:	BCOA	OAN2N4	BCN6N8	CON4N6	ABN8N2

Boundary conditions:

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ZERO: DEFI_CONSTANTE (VALE: 0.0) ;
Thermics: FCT1: DEFI_FONCTION (Nom_para: 'Z', VALE: (0.0
0.0.1.0.1.0));
GROUP_NO: XEGAL0: TEMP: 0.0
PRE_GRAD_TEMP: FLUX_X: -1.0
Mechanics: GROUP_NO: YEGAL0: DY = 0.0
XEGAL1: DX = 0.0
YEGAL1: DY = 0.0
XEGAL0: DZ = 0.0
Membrane case: GROUP_NO: ZEGAL0: DZ = 0.0
PRE_EPSI: EPXX: -1.0
Case inflection: GROUP_NO: ZEGAL0: DX = ZERO, DY = ZERO
NODE: 0 DZ = ZERO
PRE_EPSI: EPXX: FCT1
    
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4.2 Characteristics of the grid

Many nodes: 20
Many meshes and types: 1 HEXA20

4.3 Values tested

In isotropic elasticity

Case	Size	Not	Reference
Thermics	temp	N8	- 1.000000
	temp	N3	- 0.500000
Mechanics	dz	N4	- 7.03285714

membrane	<i>dz</i>	<i>N8</i>	- 7.03285714
Mechanics	<i>dz</i>	<i>N4</i>	57.70459285
inflection	<i>dz</i>	<i>N8</i>	57.70459285

Mesh	Energy potential with balance	Reference
<i>MI</i>	Thermics	- 8.20500
<i>MI</i>	Mechanics Membrane Inflection	- 2.0287088 - 1.8210238 10 ²

In orthotropic elasticity

Case	Size	Not	Reference
Thermics	<i>temp</i>	<i>N8</i>	- 1.000000
	<i>temp</i>	<i>N3</i>	- 0.500000
Mechanics	<i>dz</i>	<i>N4</i>	- 6.63044894
membrane	<i>dz</i>	<i>N8</i>	- 6.63044894
Mechanics	<i>dz</i>	<i>N4</i>	54.40283358
inflection	<i>dz</i>	<i>N8</i>	54.40283358

Mesh	Energy potential with balance	Reference
<i>MI</i>	Thermics	- 8.20500

5 Summary of the results

The results are exact with errors rounding close, since the sought solutions are part of the space of the finite elements selected for modeling.