HPLV103 - Calculation of K_I and of G thermoelastic 3D for a circular crack

Summary

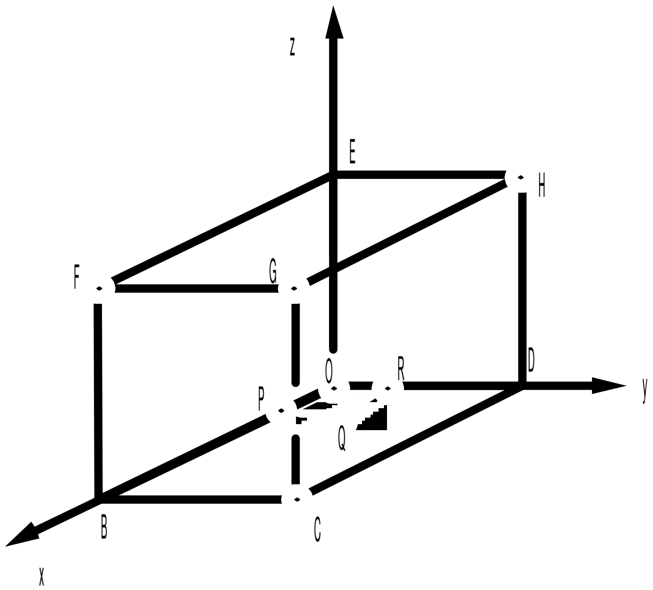
It is about a test of breaking process into thermomechanical for a three-dimensional problem. One considers a circular crack plunged in a thermoelastic medium. One imposes a uniform temperature on the lips of the crack. This test makes it possible to calculate the total rate of refund of energy G and the factor of intensity of the constraints room K_I in various points of the bottom of crack.

The interest of the test is the invariance of G and of K_I according to various crowns and the comparison with an analytical solution.

1 Problem of reference

1.1 Geometry

One considers a circular crack plunged in a thermoelastic medium. Taking into account symmetries of the problem, only a eighth of the structure is represented:



Dimensions of the crack are the following ones:

$$OP = \mathrm{OR} = 1.0$$

The medium is modelled by a parallelepiped of dimensions:

$$OB = OD = OC = 30.0$$

1.2 Material properties

Thermal conductivity:	$\lambda = 1.$
Thermal dilation coefficient:	$\alpha = 10^{-6} / {}^\circ C$
Young modulus:	$E = 2.10^5\ MPa$
Poisson's ratio:	$\nu = 0.3$

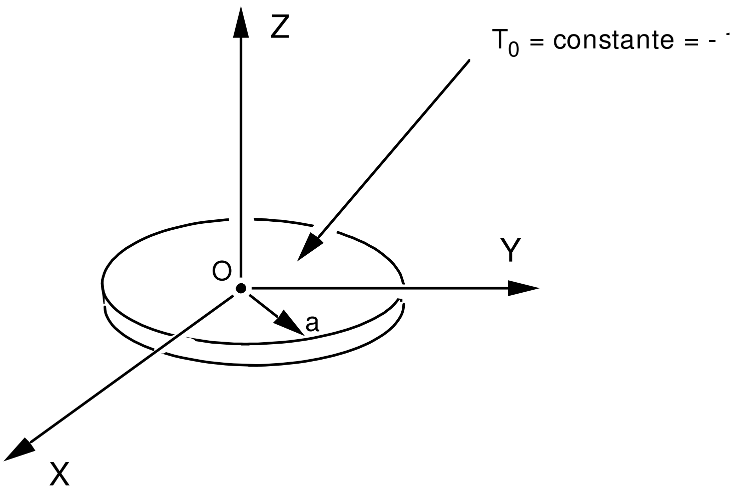
1.3 Boundary conditions and loadings

- Mechanics: imposed displacements (`DDL_IMPO`) on the following groups of meshs:
  - $DX = 0$ on $ODHE$;
  - $DY = 0$ on $OEFB$;
  - $DZ = 0$ on $PBCDRQ$ (i.e lower face of the parallelepiped, without the lip of the crack).
- Thermics: imposed temperature (`TEMP_IMPO`) on the following groups of meshs:
  - $TEMP = 0$ on $BCGH$, $CDHG$ and $EFGH$ (outsides of the parallelepiped);
  - $TEMP = -1$ on $OPQR$.

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is resulting from the collection of MURAKAMI [bib1]:



The expression of the rate of refund of energy is the following one:

$$G = \frac{(1-\nu^2)}{E} K_1^2 \text{ avec} K_1 = \frac{\alpha E}{\pi(1-\nu)} |T_0| \sqrt{\pi a} F(\eta)$$, with $\eta = a/b$ and,

$$F(\eta) = 1 - 0.6366\eta - 0.4053\eta^2 + 2.0163\eta^3 - 0.6773\eta^4 - 3.8523\eta^5 + 4.1687\eta^6 + 3.2741\eta^7 .$$

Note:

For $\eta = 0$ *(infinite medium), the solution is exact. For a medium finished, uncertainty on the solution is unknown. In this test,* $\eta = 1/30$.

2.2 Result of reference

The result of reference is thus: $K_I = 157.73\,10^3\,Pa\,m^{1/2}$ and $G = 1.132\,10^{-1}\,J/m^2$

2.3 Bibliographical references

1) Stress intensity factors Handbook (Y. MURAKAMI), box 11.39, pp. 1089 - 1090, the Society of Material Science, Japan, Pergamon Near, 1987.

3 Modeling A

3.1 Characteristics of modeling

It is about a three-dimensional modeling. The grid was carried out using procedure GIBI of fissured block $3D$ [bib1]. One represented only the eighth of the structure (and thus a quarter of the face of the crack), the quarter of this face being discretized in 16 sectors.

3.2 Characteristics of the grid

The grid is composed of quadratic elements

Many meshs and types: 624 `PENTA15`, 5600 `HEXA20`

3.3 Sizes tested and results

The values tested are those of the rate of refund of energy G total and of the rate of refund of energy room at the points A and B starting from the various crowns of integration and the two methods of definition of the fields θ :

Identification	Reference	*Aster*	$\%$ difference
G **total**			
Crown 1 G	8.8910^{-8}	$8.66\ 10^{-8}$	2.53
Crown 2 G	8.8910^{-8}	$8.68\ 10^{-8}$	2.31
Crown 3 G	8.8910^{-8}	$8.69\ 10^{-8}$	2.17
Crown 4 G	8.8910^{-8}	$8.68\ 10^{-8}$	2.31
G **room Lagrange – Legendre (degree 7)**			
G room 1 in A	$5.66\ 10^{-8}$	$6.13\ 10^{-8}$	8.31
G room 2 in A	$5.66\ 10^{-8}$	$6.19\ 10^{-8}$	9.37
G room 3 in A	$5.66\ 10^{-8}$	$6.42\ 10^{-8}$	13.46
G **room Lagrange – Legendre (degree 7)**			
G room 1 in B	$5.66\ 10^{-8}$	$5.50\ 10^{-8}$	2.75
G room 2 in B	$5.66\ 10^{-8}$	$5.51\ 10^{-8}$	2.62
G room 3 in B	$5.66\ 10^{-8}$	$5.50\ 10^{-8}$	2.84
G **room Legendre – Legendre (degree 7)**			
G room 1 in A	$5.66\ 10^{-8}$	$5.54\ 10^{-8}$	2.07
G room 2 in A	$5.66\ 10^{-8}$	$5.58\ 10^{-8}$	1.39
G room 3 in A	$5.66\ 10^{-8}$	$5.71\ 10^{-8}$	1.01
G **room Legendre – Legendre (degree 7)**			
G room 1 in B	$5.66\ 10^{-8}$	$5.51\ 10^{-8}$	2.62
G room 2 in B	$5.66\ 10^{-8}$	$5.52\ 10^{-8}$	2.46
G room 3 in B	$5.66\ 10^{-8}$	$5.52\ 10^{-8}$	2.52

Crown 1: $Rinf=0.07$ $\mathrm{Rsup}=0.2$
Crown 2: $Rinf=0.2$ $\mathrm{Rsup}=0.4$
Crown 3: $Rinf=0.4.$ $\mathrm{Rsup}=0.6.$
Crown 4: $Rinf=0.07$ $\mathrm{Rsup}=0.6$

Supports of the field θ room correspond to the first three crowns of the total field.

3.4 Remarks

- The value of reference is the value of the rate of refund of energy room: $G_{réf}=5.66\,10^{-8}\,J/m^2$. The total rate of refund of energy provided by *Code_Aster* is:

  $G_{Aster}=G_{réf}\times\frac{2\Pi a}{8}$, since by reason of symmetry one models only one quarter of the plan of the crack and only one lip.

- Results of G room are given only for the points A and B respectively located on a symmetry plane and face of crack. Results concerning the point B (medium of the face) reveal a variation of approximately 3% compared to the result of reference. Results concerning the point A are less good (the variation is located between 3% and 13.5%), which is a usual report for the estimate of G room for the points located on a symmetry plane.

4 Summary of the results

- The passage of a quadratic grid to a linear grid for mechanical calculation decreases the precision of the result: G total have a variation of 4.8% on average with the reference for the linear grid against 2.2% for the quadratic grid.
- Smoothing LEGENDRE-LEGENDRE conduit, on this case test, with the most precise results for the local values of G. For the calculation of K buildings, smoothing is advised LAGRANGE-LAGRANGE.
- Precision on the calculation of K_I room is satisfactory, the average deviation being limited to 2.3%.