
HSNV100 - Thermoplasticity in simple traction

Summary:

This test treats the thermoplasticity of Von Mises with isotropic work hardening on a three-dimensional problem (modeling *A* into axisymmetric) and two-dimensional (modeling *B* in plane constraints). The interest of the test is due to the dependence of the elastic limit with the temperature. It also makes it possible to test the orthotropism in thermoelasticity because it applies to an isotropic material then with an isotropic material declared orthotropic.

One tests there also the calculation of the deformation energy.

Two modelings (*C* with element PIPE, *D* with element TUYAU_6M) are added to test thermoplasticity in these elements.

Modeling *E* allows to test the good taking into account of the variation of the coefficients of the behavior VMIS_CINE_LINE with the temperature (axisymmetric).

Modeling *F* allows to test the calculation of the thermoelastic deformation energy in the beams (modeling POU_D_T).

Modeling *G* allows to test the same features as modelings *A* and *B*, but with a modeling 3D.

Modelings *H* and *I* allow to test, in modeling 3D and in plane constraints, an initial loading in field of unelastic deformations. This one is equivalent to a thermal deformation.

Modeling *J* is resulting from modeling *G*, and allows to validate the features of SIMU_POINT_MAT in thermoplasticity.

Modeling *K* is resulting from modeling *A*, and allows to validate the option AFFE_CHAR_TEMP_R modeling AXIS_INCO_UPG with DEFORMATION=' PETIT'. Even thing for modelings *L* and *M* but for 3D_INCO_UPG with DEFORMATION=' PETIT'.

Modeling *N* is resulting from modeling *A*, and allows to validate the option AFFE_CHAR_TEMP_R modeling AXIS_INCO_UP. Even thing for modelings *O* and *P* but for 3D_INCO_UP.

Modeling Q is resulting from modeling A , and allows to validate the option `AFPE_CHAR_TEMP_R` modeling `AXIS_INCO_UPG` with `DEFORMATION=' SIMO_MIEHE'`. Even thing for modelings R and S but for `3D_INCO_UPG` with `DEFORMATION=' SIMO_MIEHE'`.

Modeling T is resulting from modeling A , and allows to validate the option `AFPE_CHAR_TEMP_R` modeling `AXIS_INCO_UPG` with `DEFORMATION=' GDEF_LOG'` . Even thing for modelings U and V but for `3D_INCO_UPG` with `DEFORMATION=' GDEF_LOG'`.

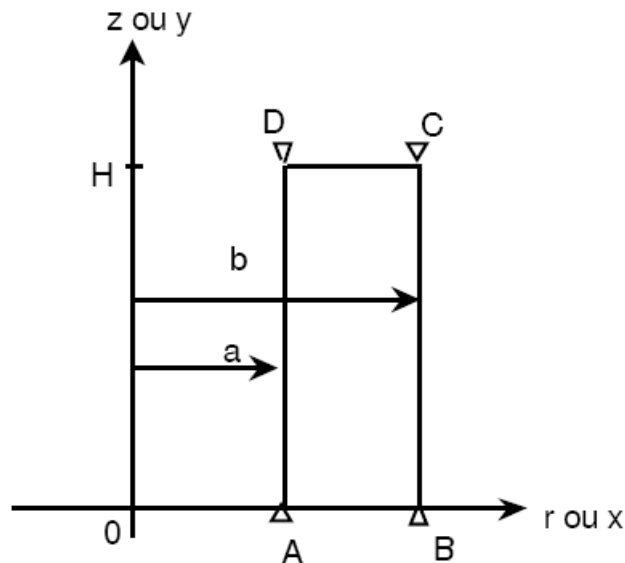
Modeling W is resulting from modeling A , and allows to validate the option `AFPE_CHAR_TEMP_R` modeling `AXIS_INCO_UP`. Even thing for modelings X and Y but for `3D_INCO_UP` .

The solution is analytical.

1 Problem of reference

1.1 Geometry

Axisymmetric cylinder (modelings *A* and *E*) or plates rectangular (modeling *B*) or right pipe (modelings *C* and *D*), or beam (modeling *F*), or parallelepiped 3D (modelings *G*, *H* and *J*).



Geometry of the cylinder (*mm*) :

- $a=1$
- $b=2$
- $H=4$

1.2 Property of materials

For all modelings:

Young modulus: $E=200000 \text{ MPa}$

Tangent module: $E_t=50000 \text{ MPa}$

Poisson's ratio: $\nu=0.3$

$\sigma_0=400 \text{ MPa}$

$s=1.0 \text{ E}^{-2} \text{ }^\circ\text{C}^{-1}$

Thermal dilation coefficient: $\alpha=1.0 \text{ E}^{-5} \text{ }^\circ\text{C}^{-1}$

Voluminal heat: $C^p=0 \text{ J.mm}^{-3} \text{ }^\circ\text{C}^{-1}$

Thermal conductivity: $\lambda=1.0 \text{ E}^{-3} \text{ W.mm}^{-1} \text{ }^\circ\text{C}^{-1}$

For isotropic material declared orthotropic, it comes:

$$E_L = E_T = E_N = E$$

$$\nu_{LT} = \nu_{LN} = \nu_{TN} = \nu$$

$$G_{LT} = G_{LN} = G_{TN} = 75000 \text{ MPa}$$

$$\text{ALPHA_L} = \text{ALPHA_T} = \text{ALPHA_N} = \alpha$$

For modelings of A with G :

$$\sigma_y(T) = \sigma_0 (1 - s \cdot (T - T_0))$$

For modeling H and I :

$$\sigma_y(T) = \sigma_0 \quad (s=0)$$

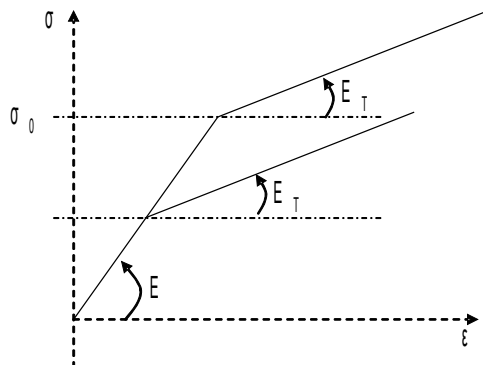


Figure : Traction diagram of material

1.3 Boundary conditions and loadings

- Modelings A , E and K : $uz=0$ on the sides AB and CD (Axis Oz fix)
- Modelings B and I : $uy=0$ on the sides AB and CD , $ux=0$ in A
- Modelings C , D and F : embedding in A , $Uy=0$ in C
- Modelings G , H , L and M : $uy=0$ on the sides AB and CD , $ux=uz=0$ (node $N3$), $uz=0$ (node $N4$)
- $T(t) = \gamma t + T0$ with: $\gamma = 1^\circ C/s$ and $T0 = 0^\circ C$.
- Modelings H and I : Fields of initial deformations: $\epsilon = \alpha(T - T_0) Id$
- Modeling J : Field of imposed deformations: $\epsilon_{yy} = 0$

2 Reference solution

2.1 Method of calculating used for the reference solution

Analytical solution was determined by F. Voldoire (EDF R & D/AMA):

Axisymmetric case (2D)

Fields of displacement $u = U_r(r) e_r$ (blocking in z)

$$\text{Fields of deformation: } \varepsilon(u) = \begin{pmatrix} u_r' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{u_r}{r} \end{pmatrix} \text{ according to } \begin{pmatrix} r \\ z \\ \theta \end{pmatrix}$$

$$\text{Stress fields: } \sigma(u) = \sigma_r \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ (cf. conditions aux limites) according to } \begin{pmatrix} r \\ z \\ \theta \end{pmatrix}$$

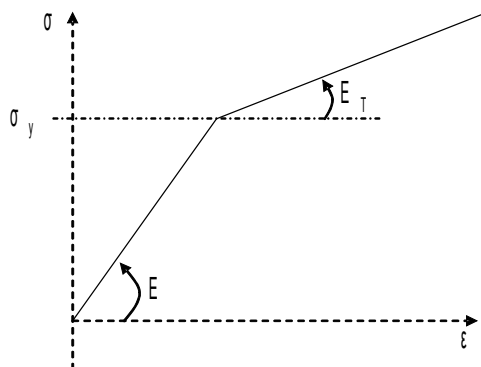
Parallelepipedic case

Fields of displacement $u = U_x(x) e_x + U_y(y) e_y$ (blocking in z)

$$\text{Fields of deformation: } \varepsilon(u) = \begin{pmatrix} u_x' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & u_y' \end{pmatrix} \text{ according to } \begin{pmatrix} x \\ z \\ y \end{pmatrix}$$

$$\text{Stress fields: } \sigma(u) = \sigma \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ (cf. conditions aux limites) according to } \begin{pmatrix} x \\ z \\ y \end{pmatrix}$$

The case could be studied in plane constraints and in 3D .



$$2\nu = \frac{E}{(1+\nu)} \quad K = \frac{E}{(1-2\nu)}$$

The law of behavior (variable scalar intern is written p):

$$\left\{ \begin{array}{l} \varepsilon = \frac{1}{9K} \text{tr} \sigma \text{Id} + \frac{1}{2\mu} \sigma^D + \varepsilon^P + \alpha(T - T^0) \text{Id} \\ \text{with: } \sigma^D = \sigma - \frac{1}{3} \text{tr}(\sigma) \text{Id} \quad (\text{diverter of the constraints}) \\ \dot{\sigma}^P = \frac{3}{2} \dot{p} \frac{\sigma^D}{\|\sigma_{\dot{e}q}\|} \text{ et } \|\sigma_{\dot{e}q}\| = \sqrt{\frac{3}{2} \sigma^D \cdot \sigma^D} \\ \dot{p} = 0 \text{ if } f(\sigma, p) = \|\sigma_{\dot{e}q}\| - R(p) < 0 \\ \dot{p} \geq 0 \text{ if } f(\sigma, p) = 0 \end{array} \right.$$

$R(p)$ indicate the function of work hardening: $R(p) = \sigma_y + \frac{EE_T}{E - E_T} p$

The rate \dot{p} can be expressed, when $f(\sigma, p) = 0$. Indeed, of $\dot{p} f$ identically no one, one draws:
 $\dot{p} \dot{f} + \ddot{p} f = 0$

Thus, when one is on the criterion, necessarily $\dot{f} = 0$. It is - with-to say:

$$\begin{aligned} \frac{3}{2} \frac{\sigma^D \cdot \dot{\sigma}^D}{\|\sigma_{\dot{e}q}\|} - \frac{\partial R}{\partial T} \cdot \dot{T} - \frac{\partial R}{\partial p} \dot{p} &= 0 \\ \frac{3}{2} \frac{\sigma^D \cdot \dot{\sigma}^D}{\|\sigma_{\dot{e}q}\|} + \sigma_y^o \cdot s \cdot \dot{T} - \frac{EE_T}{E - E_T} \dot{p} &= 0 \end{aligned}$$

From where:

$$\dot{p} = \frac{E - E_T}{EE_T} \left(\frac{3}{2} \frac{\sigma^D \cdot \dot{\sigma}^D}{\|\sigma_{\dot{e}q}\|} + \sigma_y^o \cdot s \cdot \dot{T} \right) \text{ if } \dot{p} \geq 0, \text{ for } \|\sigma_{\dot{e}q}\| = R(p)$$

The stress field being uniaxial, one a:

$$\sigma^D = \frac{\sigma_L}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

As follows:

$$\|\sigma_{\dot{e}q}\| = |\sigma_L|$$

and:

$$\dot{\varepsilon}^P = \frac{\dot{p}}{2} \cdot \text{sgn}(\sigma_L) \cdot \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

The relation of behavior leads to:

$$\begin{cases} \dot{\varepsilon}_{rr} = \dot{\varepsilon}_{\theta\theta} = \frac{-\nu}{E} \dot{\sigma}_L - \frac{\dot{p}}{2} \operatorname{sgn}(\sigma_L) + \alpha \dot{T} \quad (= \dot{\varepsilon}_{xx} = \dot{\varepsilon}_{yy} \text{ pour le cas du parallélépipède}) \\ \dot{\varepsilon}_{rr} = \dot{\varepsilon}_{\theta\theta} = \frac{-\nu}{E} \dot{\sigma}_L - \frac{\dot{p}}{2} \operatorname{sgn}(\sigma_L) + \alpha \dot{T} \end{cases}$$

From where:

$$\begin{cases} \dot{\varepsilon}_{rr} = \dot{\varepsilon}_{\theta\theta} = \frac{-3}{2} \alpha \dot{T} + \frac{1-2\nu}{2E} \dot{\sigma}_L \\ \dot{p} = \operatorname{sgn}(\sigma_L) \left(-\alpha \dot{T} - \frac{\dot{\sigma}_L}{E} \right) = 0 \quad \text{si } |\sigma_L| < R(p) \\ \dot{p} = \operatorname{Max} \left[0, \frac{E-E_T}{EE_T} \left(\frac{3}{2} \frac{\sigma^D \dot{\sigma}^D}{\|\sigma_{\dot{\varepsilon}q}\|} + \sigma_y^o . s . \dot{T} \right) \right] \quad \text{sinon} \end{cases}$$

I.e., in the case $|\sigma_L| = R(p)$ (criterion reached):

$$\dot{p} = \max \left[0, \frac{E-E_T}{EE_T} \left(\frac{3}{2} \frac{\sigma^D \dot{\sigma}^D}{\|\sigma_{\dot{\varepsilon}q}\|} + \sigma_y^o . s . \dot{T} \right) \right]$$

2.1.1 Elastic phase

At the beginning of the thermal loading, $|\sigma_L|$ being lower than σ_y , \dot{p} is null.

From where: $\dot{\sigma}_L = -E \alpha \dot{T}$; $\dot{\varepsilon}_{rr} = \dot{\varepsilon}_{\theta\theta} = \alpha \dot{T} (1 + \nu)$

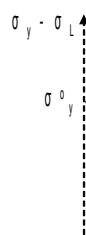
As follows:
$$\begin{cases} \sigma_L = -E \alpha \theta t \\ \varepsilon_{rr} = \varepsilon_{\theta\theta} = \alpha \theta (1 + \nu) t \end{cases} \quad (\text{compression } \sigma_L < 0)$$

This corresponds to the reference solution for the test of orthotropic elasticity

Validity of the elastic solution

The criterion is: $|\sigma_L(t)| - \sigma_y(t) = E \theta t - \sigma_y^o (1 - s \theta t) \leq 0$

The criterion is not crossed for $t \in [0, t_y]$, with: $t_y = \frac{\sigma_y^o}{\theta (E \alpha + \sigma_y^o s)}$



At the moment t_y : $\sigma_L(t_y) = \frac{-E \alpha \sigma_y^o}{E \alpha + \sigma_y^o s}$

The density of deformation energy is worth: $w(t_y) = \frac{-1}{2} E (\alpha \theta)^2$

In the parallelepipedic case one a: $w(t_y) = \frac{-1}{2} E (\alpha \theta)^2 (x_B - x_A) H$

In the axisymmetric case one a: $w(t_y) = \frac{-1}{2} E (\alpha \theta)^2 \cdot \frac{(r_B^2 - r_A^2)}{2} H$ (for 1 radian)

2.1.2 Elastoplastic phase

$t \geq t_y$. One is on the criterion. Then:

$$\dot{p} = \max \left[0, \frac{E - E_T}{E E_T} (\dot{\sigma}_L \operatorname{sgn}(\sigma_L) + \sigma_y^o \cdot s \cdot \dot{T}) \right]$$

By admitting that one is "charges some" $\dot{p} \geq 0$, then one eliminates \dot{p} to have:

$$\dot{\sigma}_L = -E_T \cdot \dot{T} (\alpha + \operatorname{sgn}(\sigma_L)) \cdot \frac{E - E_T}{E E_T} \sigma_y^o \cdot s$$

then:

$$\dot{p} = \frac{E - E_T}{E} \dot{T} \left(-\alpha \operatorname{sgn}(\sigma_L) + \frac{\sigma_y^o \cdot s}{E} \right)$$

$t = t_y$, $\sigma_L = -E \alpha \theta t_y < 0$; one integrates these expressions then for $t \geq t_y$ ($\dot{T} = \theta$):

$$\begin{cases} \sigma_L(t) = -E_T \cdot \theta (t - t_y) \left[\alpha - \frac{E - E_T}{E E_T} s \sigma_y^o \right] - \sigma_L(t - t_y) \\ p(t) = \frac{\sigma_y^o (E - E_T)}{E^2} \left(\frac{t}{t_y} - 1 \right) \end{cases}$$

Maybe, after rearrangement, $t \geq t_y$:

$$\begin{cases} \sigma_L(t) = \sigma_y^o \left(s \theta t - 1 + \frac{E_T}{E} \left(1 - \frac{t}{t_y} \right) \right) \\ p(t) = \frac{\sigma_y^o (E - E_T)}{E^2} \left(\frac{t}{t_y} - 1 \right) \end{cases}$$

Validity of this elastoplastic solution

It should be made sure that $\sigma_L(t)$ remain negative. Knowing that $s \theta t < 1$, and that $t > t_y$, the preceding result confirms that $\sigma_L(t) < 0$. Lastly, it is noticed that:

$$\operatorname{sgn}(\sigma_L) \frac{1 - 2\nu}{2} \dot{p} + \varepsilon_{rr} = \alpha (1 + \nu) \dot{T}$$

from where, since $\sigma_L(t) < 0$:

$$\varepsilon_{rr}(t) = \varepsilon_{\theta\theta}(t) = \alpha \theta (1 + \nu) t + \frac{1-2\nu}{2} p(t), \forall t \in [t_y, t_{fin}]$$

2.1.3 Typical case of modelings H and I

The thermal deformation is replaced by a given unelastic deformation. Like $s=0$,

$$\sigma_L = -E \alpha \theta \quad \varepsilon_{xx} = \varepsilon_{zz} = \alpha \theta (1 + \nu) t$$

The solution remains elastic as long as $t < t_y = \frac{\sigma_0}{\theta E \alpha} = 200 s$

2.2 Results of reference

ε_{rr} Or ε_{xx} , σ_{zz} ET p in t_y and beyond:

Elastic phase : for $t < t_y$:

$$\sigma_L = -E \alpha \theta t \quad \varepsilon_{rr} = \varepsilon_{\theta\theta} = \alpha \theta (1 + \nu) t \quad \text{into axisymmetric}$$

$$\varepsilon_{xx} = \alpha \theta (1 + \nu) t \quad \text{in plane constraints}$$

The yield stress is reached in $t_y = \frac{\sigma_0}{\theta (E \alpha + \sigma_0 s)} = 66.666 s$ from where $\sigma_L(t_y) = \frac{\sigma}{(1 + \sigma_0 \frac{s}{E \alpha})}$

Elastoplastic phase : for $t \geq t_y$

$$\sigma_L(t) = \sigma_0 \left(s \theta t - 1 + \frac{E_t}{E} \left(1 - \frac{t}{t_y} \right) \right)$$

$$p(t) = \frac{\sigma_0 (E - E_T)}{E^2} \left(\frac{t}{t_y} - 1 \right)$$

$$\varepsilon_{rr} = \varepsilon_{\theta\theta} = \alpha \theta (1 + \nu) t + \frac{1-2\nu}{2} p(t) \quad \text{into axisymmetric}$$

$$\text{or } \varepsilon_{xx} = \varepsilon_{\theta\theta} = \alpha \theta (1 + \nu) t + \frac{1-2\nu}{2} p(t) \quad \text{in plane constraints}$$

From where:

elastic phase

$$\left. \begin{array}{l} t_y = 66.666 s \\ \sigma_L(t_y) = -133.333 MPa \\ \varepsilon_{rr}(t_y) = \varepsilon_{\theta\theta}(t_y) = 0.86666 E^{-3} \end{array} \right\} \text{phase élastique}$$

$$W = 4.444 10^{-3} MPa$$

$$W = 0.17778 MPa.mm^2 \quad (\text{PLAN or 3D})$$

$$W = 0.26666 MPa.mm^3 .rad - 1 \quad (\text{axi})$$

Then elastoplastic phase :

$$\begin{aligned} \text{à } t=80 \text{ s} : \quad & \sigma(80) = -100.0 \text{ MPa} \\ & p(80) = 0.300 E^{-3} \\ & \varepsilon_{rr}(80) = \varepsilon_{\theta\theta}(80) = 1.1 E^{-3} \\ \text{with } t=90 \text{ s} : \quad & \sigma(90) = -75.0 \text{ MPa} \\ & p(90) = 0.525 E^{-3} \\ & \varepsilon_{rr}(90) = \varepsilon_{\theta\theta}(90) = 1.275 E^{-3} \end{aligned}$$

2.2.1 Typical case of modelings H and I

$$\begin{aligned} \text{With } t=66.67 \text{ s} \quad & \sigma_L(66.67) = -133.33 \text{ MPa} \\ & \varepsilon_{xx}(66.67) = \varepsilon_{zz}(66.67) = 8.667 E^{-4} \\ \text{With } t=80 \text{ s} \quad & \sigma_L(80) = -160 \text{ MPa} \\ & \varepsilon_{xx}(80) = \varepsilon_{zz}(80) = 8.667 E^{-4} \\ \text{With } t=90 \text{ s} \quad & \sigma_L(90) = -180 \text{ MPa} \\ & \varepsilon_{xx}(90) = \varepsilon_{zz}(90) = 8.667 E^{-4} \end{aligned}$$

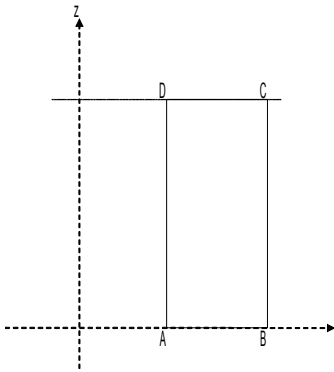
2.3 Uncertainty on the solution

Analytical solution.

3 Modeling A

3.1 Characteristics of modeling

QUAD4 - Axisymmetric



3.2 Characteristics of the grid

Many nodes: 4

Many meshes and types: 1 QUAD4, 4 SEG2

3.3 Sizes tested and results

Variables	Moments (s)	Reference
$\varepsilon_{rr} = \varepsilon_{\theta\theta}$	$t = 66.666$	$8.6666 \cdot 10^{-4}$
	$t = 80$	$1.1000 \cdot 10^{-3}$
	$t = 90$	$1.2750 \cdot 10^{-3}$
p	$t = 66.666$	0
	$t = 80$	$3.0000 \cdot 10^{-4}$
	$t = 90$	$5.2500 \cdot 10^{-4}$
σ_{zz} (MPa)	$t = 66.666$	- 133,333
	$t = 80$	- 100,000
	$t = 90$	- 75,000
ENEL_ELGA (J)	$t = 66.666$	$4.444 \cdot 10^{-2}$
ENER_TOTALE (J)	$t = 66.666$	0.2666
ENER_POT (J)	$t = 66.666$	0.2666

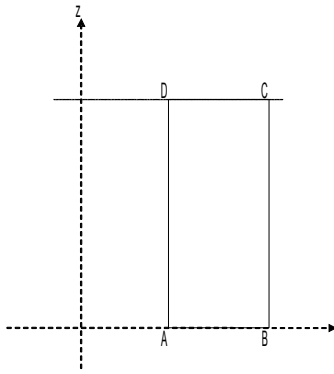
Orthotropism (BEHAVIOR)

Variables	Moments (s)	Reference
$\varepsilon_{rr} = \varepsilon_{\theta\theta}$	$t = 66.666$	$8.6666 \cdot 10^{-4}$
	$t = 80$	$1.04 \cdot 10^{-3}$
	$t = 90$	$1.17 \cdot 10^{-3}$
σ_{zz} (MPa)	$t = 66.666$	- 133,333
	$t = 80$	- 160,000
	$t = 90$	- 180,000

4 Modeling B

4.1 Characteristics of modeling

QUAD4 - Plane constraints



4.2 Characteristics of the grid

Many nodes: 4
Many meshes and types: 1 QUAD4, 4 SEG2

4.3 Sizes tested and results

Variables	Moments (s)	Reference
<i>EPXX</i>	$t=66.666$	$8.6666 \cdot 10^{-4}$
	$t=80$	$1.1000 \cdot 10^{-3}$
	$t=90$	$1.2750 \cdot 10^{-3}$
<i>p</i>	$t=66.666$	0
	$t=80$	$3.0000 \cdot 10^{-4}$
	$t=90$	$5.2500 \cdot 10^{-4}$
<i>SIYY (MPa)</i>	$t=66.666$	- 133,333
	$t=80$	- 100.
	$t=90$	- 75,000
ENEL_ELGA (J)	$t=66.666$	$4.444 \cdot 10^{-2}$
ENER_TOTALE (J)	$t=66.666$	0.17777
ENER_POT (J)	$t=66.666$	0.17777

5 Modeling C

5.1 Characteristics of modeling



1 element PIPE

5.2 Characteristics of the grid

1 element PIPE

5.3 Sizes tested and results

Variables	Moments (s)	Reference
p	$t=66.666$	0
	$t=80$	$3 \cdot 10^{-4}$
	$t=90$	$5.25 \cdot 10^{-4}$
σ_{yy} (MPa)	$t=66.666$	- 1,333
	$t=80$	- 100
	$t=90$	- 75

6 Modeling D

6.1 Characteristics of modeling



1 element PIPE 6M

6.2 Characteristics of the grid

1 element PIPE

6.3 Sizes tested and results

Variables	Moments (s)	Reference
p	$t=66.666$	0
	$t=80$	$3 \cdot 10^{-4}$
	$t=90$	$5.25 \cdot 10^{-4}$
σ_{yy} (MPa)	$t=66.666$	-1,333
	$t=80$	-100
	$t=90$	-75

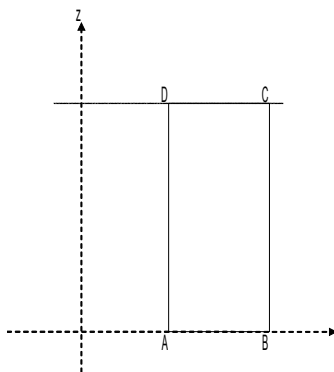
7 Modeling E

7.1 Characteristics of modeling

QUAD4 - Axisymmetric.

Test of the variation of the coefficients of VMIS_CINE_LINE according to the temperature, in this case E_T (given by D_SIGM_EPSI) vary like: $E_T = 10^5 (1 - 10^{-2}(T - T_0))$. The constant of Prager is

worth:
$$C = \frac{2}{3} \frac{E E_T}{E - E_T}$$



7.2 Characteristics of the grid

Many nodes: 4

Many meshes and types: 1 QUAD4, 4 SEG2

7.3 Notice

One tests the variation of E_T (D_SIGM_EPSI) with the temperature by comparison with the behavior VMIS_ECMI_TRAC where C (constant of Prager) varies with the similar temperature of way (not of analytical solution).

7.4 Sizes tested and results

Variables	Moments (s)	Reference (Aster) (VMIS_ECMI_TRAC)
$\varepsilon_{rr} = \varepsilon_{\theta\theta}$	$t = 66.666$	$8.6666 \cdot 10^{-4}$
	$t = 80$	$1,112 \cdot 10^{-3}$
	$t = 90$	$1,303 \cdot 10^{-3}$
σ_{zz} (MPa)	$t = 66.666$	- 133,333
	$t = 80$	- 88
	$t = 90$	- 47

7.5 Notice

One gets well the same results with the behavior VMIS_CINE_LINE that with the behavior VMIS_ECMI_TRAC what validates the taking into account of the temperature in this model.

8 Modeling F

8.1 Characteristics of modeling



1 element POU_D_T

8.2 Characteristics of the grid

1 mesh SEG2

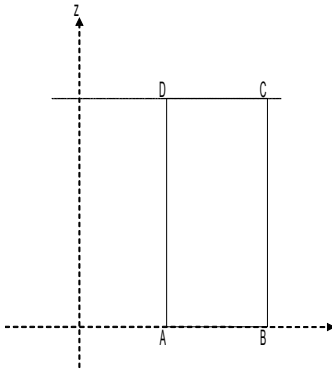
8.3 Sizes tested and results

Variables	Moments (s)	Reference
σ_{yy}	$t=66.666$	- 1,333
ENER_POT (J)	$t=66.666$	0.3555

9 Modeling G

9.1 Characteristics of modeling

3D, $H=1$



9.2 Characteristics of the grid

Many nodes: 8
Many meshes and types: 1 HEXA8

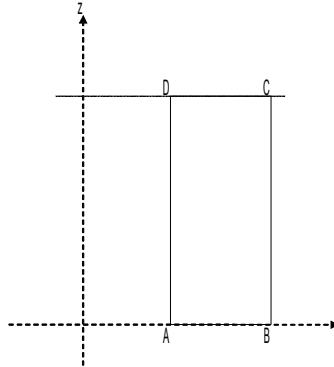
9.3 Sizes tested and results

Variables	Moments (s)	Reference
ε_{xx}	$t=66.666$	$8.6666 \cdot 10^{-4}$
	$t=80$	$1.1000 \cdot 10^{-3}$
	$t=90$	$1.2750 \cdot 10^{-3}$
p	$t=66.666$	0
	$t=80$	$3.0000 \cdot 10^{-4}$
	$t=90$	$5.2500 \cdot 10^{-4}$
σ_{yy} (MPa)	$t=66.666$	- 133,333
	$t=80$	- 100.
	$t=90$	- 75,000
ENEL_ELGA	$t=66.666$	$4.444 \cdot 10^{-2}$
ENER_TOTALE	$t=66.666$	$4.444 \cdot 10^{-2}$
ENER_POT	$t=66.666$	$4.444 \cdot 10^{-2}$

10 Modeling H

10.1 Characteristics of modeling

3D, $H=1$



10.2 Characteristics of the grid

Many nodes: 8
Many meshes and types: 1 HEXA8

10.3 Sizes tested and results

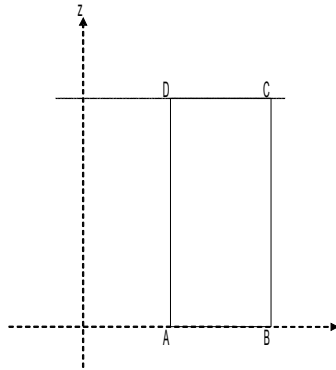
Variables	Moments (s)	Reference
ε_{xx}	$t=66.666$	$8.9930 \cdot 10^{-4}$
	$t=80$	$1.0980 \cdot 10^{-3}$
	$t=90$	$1.2480 \cdot 10^{-3}$
p	$t=66.666$	0
	$t=80$	$2.9400 \cdot 10^{-4}$
	$t=90$	$3.9200 \cdot 10^{-4}$
σ_{yy} (MPa)	$t=66.666$	- 100,666
	$t=80$	- 101.2
	$t=90$	- 101.6
ENEL_ELGA	$t=66.666$	$2.53344 \cdot 10^{-2}$
EPSP_ELGA		
ε_{xx}	$t=90$	$1,960 \cdot 10^{-4}$
ξ_{yy}	$t=90$	$-3,920 \cdot 10^{-4}$
EPSP_ELNO		
ε_{xx}	$t=90$	$1,960 \cdot 10^{-4}$
ξ_{yy}	$t=90$	$-3,920 \cdot 10^{-4}$
EPME_ELGA		
ε_{xx}	$t=90$	$3,484 \cdot 10^{-4}$
ξ_{yy}	$t=90$	$-9,000 \cdot 10^{-4}$
EPME_ELNO		
ε_{xx}	$t=90$	$3,484 \cdot 10^{-4}$
ξ_{yy}	$t=90$	$-9,000 \cdot 10^{-4}$

EPMG_ELGA		
ε_{xx}	$t=90$	$3,491 \cdot 10^{-4}$
ξ_{yy}	$t=90$	$-9,000 \cdot 10^{-4}$
EPMG_ELNO		
ε_{xx}	$t=90$	$3,491 \cdot 10^{-4}$
ξ_{yy}	$t=90$	$-9,000 \cdot 10^{-4}$
ENER_TOTALE	$t=66.666$	$4.17215 \cdot 10^{-2}$

11 Modeling I

11.1 Characteristics of modeling

QUAD4 - Plane constraints



Many nodes: 4

Many meshes and types: 1 QUAD4, 4 SEG2

11.2 Sizes tested and results

Variables	Moments (s)	Reference
p	$t=66.666$	0
	$t=80$	$2,940 \cdot 10^{-4}$
	$t=90$	$3.9200 \cdot 10^{-4}$
σ_{yy} (MPa)	$t=66.666$	- 100,666
	$t=80$	- 101.2
	$t=90$	- 101.6
ENEL_ELGA	$t=66.666$	$2.53344 \cdot 10^{-2}$
EPSP_ELGA		
ϵ_{xx}	$t=90$	$1,960 \cdot 10^{-4}$
ξ_{yy}	$t=90$	$-3,920 \cdot 10^{-4}$
EPSP_ELNO		
ϵ_{xx}	$t=90$	$1,959 \cdot 10^{-4}$
ξ_{yy}	$t=90$	$-3.9199 \cdot 10^{-4}$
INDIC_ENER	$t=90$	0.00
INDIC_SEUIL	$t=90$	0.00

12 Modeling J

12.1 Characteristics of modeling

Pas de grid: not material (SIMU_POINT_MAT)

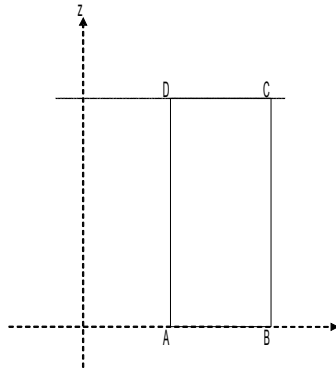
12.2 Sizes tested and results

Variables	Moments (s)	Reference
ε_{xx}	$t = 66.666$	$8.6666 \cdot 10^{-4}$
	$t = 80$	$1.1000 \cdot 10^{-3}$
	$t = 90$	$1.2750 \cdot 10^{-3}$
p	$t = 66.666$	0
	$t = 80$	$3.0000 \cdot 10^{-4}$
	$t = 90$	$5.2500 \cdot 10^{-4}$
σ_{yy} (MPa)	$t = 66.666$	- 133,333
	$t = 80$	- 100.
	$t = 90$	- 75,000

13 Modeling K

13.1 Characteristics of modeling

QUAD8 - Axisymmetric



13.2 Characteristics of the grid

Many nodes: 8
Many meshes and types: 1 QUAD8, 4 SEG3

13.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t = 66,666 \text{ s})$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.1%
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t = 80\text{s})$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.1%
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t = 90\text{s})$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.1%
$p(t = 66,666 \text{ s})$	'ANALYTICAL'	0	0.1%
$p(t = 80\text{s})$	'ANALYTICAL'	$3 \cdot 10^{-4}$	0.1%
$p(t = 90\text{s})$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	0.1%
$\sigma_{zz}(t = 66,666 \text{ s})$	'ANALYTICAL'	-133,333 MPa	0.1%
$\sigma_{zz}(t = 80\text{s})$	'ANALYTICAL'	-100 MPa	0.3%
$\sigma_{zz}(t = 90\text{s})$	'ANALYTICAL'	-75 MPa	0.1%

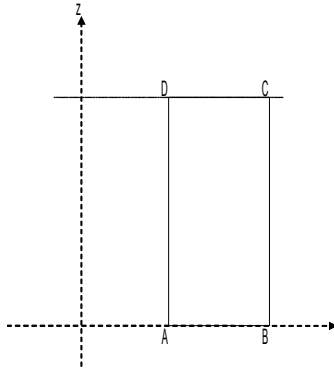
13.4 Notice

One gets well the same results with the formulation AXIS_INCO_UPG that with the formulation AXIS classic.

14 Modeling L

14.1 Characteristics of modeling

3D, $H=1$



14.2 Characteristics of the grid

Many nodes: 20
Many meshes and types: 1 HEXA20

14.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66,666\text{ s})$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.1%
$\varepsilon_{xx}(t=80\text{ s})$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.1%
$\varepsilon_{xx}(t=90\text{ s})$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.1%
$p(t=66,666\text{ s})$	'ANALYTICAL'	0	0.1%
$p(t=80\text{ s})$	'ANALYTICAL'	$3 \cdot 10^{-4}$	0.1%
$p(t=90\text{ s})$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	0.1%
$\sigma_{yy}(t=66,666\text{ s})$	'ANALYTICAL'	-133,333 MPa	0.1%
$\sigma_{yy}(t=80\text{ s})$	'ANALYTICAL'	-100 MPa	0.1%
$\sigma_{yy}(t=90\text{ s})$	'ANALYTICAL'	-75 MPa	0.1%

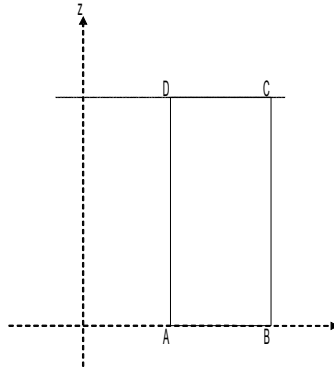
14.4 Notice

One gets well the same results with the formulation 3D_INCO_UPG that with the formulation 3D classic.

15 Modeling M

15.1 Characteristics of modeling

3D, $H=1$



15.2 Characteristics of the grid

Many nodes: 35
Many meshes and types: 12 TETRA10

15.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66,666\text{ s})$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.1%
$\varepsilon_{xx}(t=80\text{ s})$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.1%
$\varepsilon_{xx}(t=90\text{ s})$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.1%
$p(t=66,666\text{ s})$	'ANALYTICAL'	0	0.1%
$p(t=80\text{ s})$	'ANALYTICAL'	$3 \cdot 10^{-4}$	0.1%
$p(t=90\text{ s})$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	0.1%
$\sigma_{yy}(t=66,666\text{ s})$	'ANALYTICAL'	-133,333 MPa	0.1%
$\sigma_{yy}(t=80\text{ s})$	'ANALYTICAL'	-100 MPa	0.1%
$\sigma_{yy}(t=90\text{ s})$	'ANALYTICAL'	-75 MPa	0.1%

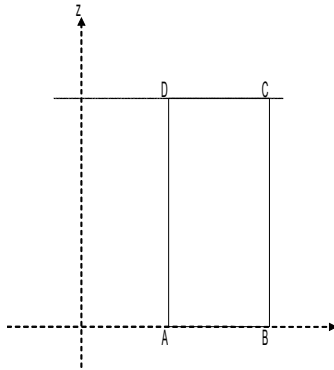
15.4 Notice

One gets well the same results with the formulation 3D_INCO_UPG that with the formulation 3D classic.

16 Modeling NR

16.1 Characteristics of modeling

QUAD8 - Axisymmetric



16.2 Characteristics of the grid

Many nodes: 8

Many meshes and types: 1 QUAD8, 4 SEG3

16.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t=66,666 s)$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.1%
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t=80s)$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.1%
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t=90s)$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.1%
$p(t=66,666 s)$	'ANALYTICAL'	0	0.1%
$p(t=80s)$	'ANALYTICAL'	$3 \cdot 10^{-4}$	0.1%
$p(t=90s)$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	0.3%
$\sigma_{zz}(t=66,666 s)$	'ANALYTICAL'	-133,333 MPa	0.1%
$\sigma_{zz}(t=80s)$	'ANALYTICAL'	-100 MPa	0.1%
$\sigma_{zz}(t=90s)$	'ANALYTICAL'	-75 MPa	0.1%

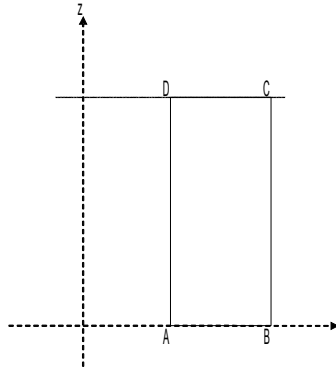
16.4 Notice

One gets well the same results with the formulation AXIS_INCO_UP that with the formulation AXIS classic.

17 Modeling O

17.1 Characteristics of modeling

3D, $H=1$



17.2 Characteristics of the grid

Many nodes: 20
Many meshes and types: 1 HEXA20

17.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66,666\text{ s})$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.1%
$\varepsilon_{xx}(t=80\text{ s})$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.1%
$\varepsilon_{xx}(t=90\text{ s})$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.1%
$p(t=66,666\text{ s})$	'ANALYTICAL'	0	0.1%
$p(t=80\text{ s})$	'ANALYTICAL'	$3 \cdot 10^{-4}$	0.1%
$p(t=90\text{ s})$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	0.1%
$\sigma_{yy}(t=66,666\text{ s})$	'ANALYTICAL'	-133,333 MPa	0.1%
$\sigma_{yy}(t=80\text{ s})$	'ANALYTICAL'	-100 MPa	0.1%
$\sigma_{yy}(t=90\text{ s})$	'ANALYTICAL'	-75 MPa	0.1%

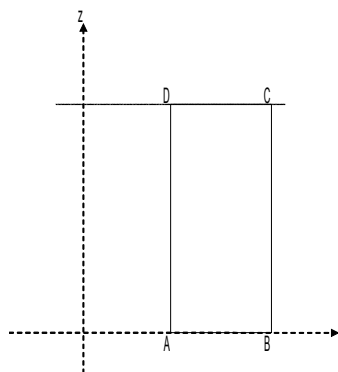
17.4 Notice

One gets well the same results with the formulation 3D_INCO_UP that with the formulation 3D classic.

18 Modeling P

18.1 Characteristics of modeling

3D, $H=1$



18.2 Characteristics of the grid

Many nodes: 35
Many meshes and types: 12 TETRA10

18.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66,666\text{ s})$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.1%
$\varepsilon_{xx}(t=80\text{ s})$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.1%
$\varepsilon_{xx}(t=90\text{ s})$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.1%
$p(t=66,666\text{ s})$	'ANALYTICAL'	0	0.1%
$p(t=80\text{ s})$	'ANALYTICAL'	$3 \cdot 10^{-4}$	0.1%
$p(t=90\text{ s})$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	0.1%
$\sigma_{yy}(t=66,666\text{ s})$	'ANALYTICAL'	-133,333 MPa	0.1%
$\sigma_{yy}(t=80\text{ s})$	'ANALYTICAL'	-100 MPa	0.1%
$\sigma_{yy}(t=90\text{ s})$	'ANALYTICAL'	-75 MPa	0.1%

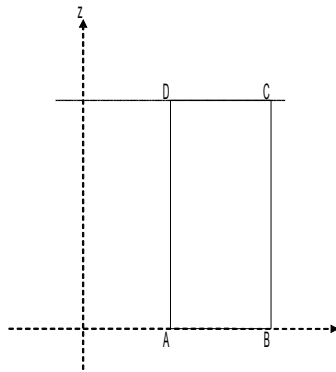
18.4 Notice

One gets well the same results with the formulation 3D_INCO_UP that with the formulation 3D classic.

19 Modeling Q

19.1 Characteristics of modeling

QUAD8 - Axisymmetric



19.2 Characteristics of the grid

Many nodes: 8

Many meshes and types: 1 QUAD8, 4 SEG3

19.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t = 66,666 \text{ s})$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.1%
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t = 80\text{s})$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.15%
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t = 90\text{s})$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.4%
$p(t = 66,666 \text{ s})$	'ANALYTICAL'	0	0.2%
$p(t = 80\text{s})$	'ANALYTICAL'	$3 \cdot 10^{-4}$	0.5%
$p(t = 90\text{s})$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	0.4%
$\sigma_{zz}(t = 66,666 \text{ s})$	'ANALYTICAL'	-133,333 MPa	0.2%
$\sigma_{zz}(t = 80\text{s})$	'ANALYTICAL'	-100 MPa	0,525%
$\sigma_{zz}(t = 90\text{s})$	'ANALYTICAL'	-75 MPa	0.5%

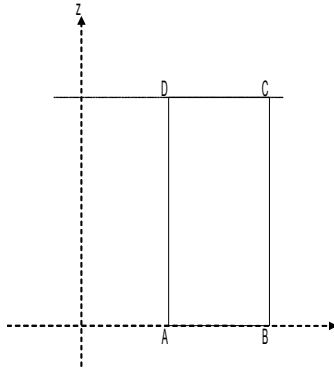
19.4 Notice

One gets well the same results with the formulation AXIS_INCO_UPG that with the formulation AXIS classic.

20 Modeling R

20.1 Characteristics of modeling

3D, $H=1$



20.2 Characteristics of the grid

Many nodes: 20
Many meshes and types: 1 HEXA20

20.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66,666\text{ s})$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.1%
$\varepsilon_{xx}(t=80\text{ s})$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.15%
$\varepsilon_{xx}(t=90\text{ s})$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.4%
$p(t=66,666\text{ s})$	'ANALYTICAL'	0	0.11%
$p(t=80\text{ s})$	'ANALYTICAL'	$3 \cdot 10^{-4}$	0.3%
$p(t=90\text{ s})$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	0.4%
$\sigma_{yy}(t=66,666\text{ s})$	'ANALYTICAL'	-133,333 MPa	0.1275%
$\sigma_{yy}(t=80\text{ s})$	'ANALYTICAL'	-100 MPa	0,525%
$\sigma_{yy}(t=90\text{ s})$	'ANALYTICAL'	-75 MPa	0.5%

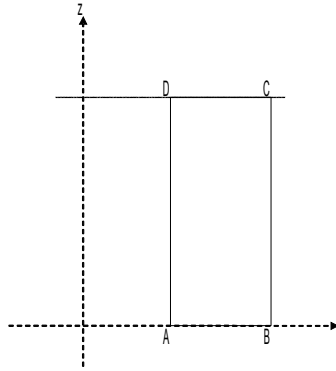
20.4 Notice

One gets well the same results with the formulation 3D_INCO_UPG that with the formulation 3D classic.

21 Modeling S

21.1 Characteristics of modeling

3D, $H=1$



21.2 Characteristics of the grid

Many nodes: 35
Many meshes and types: 12 TETRA10

21.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66,666 s)$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.1%
$\varepsilon_{xx}(t=80s)$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.15%
$\varepsilon_{xx}(t=90s)$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.4%
$p(t=66,666 s)$	'ANALYTICAL'	0	0.2%
$p(t=80s)$	'ANALYTICAL'	$3 \cdot 10^{-4}$	0.5%
$p(t=90s)$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	0.35%
$\sigma_{yy}(t=66,666 s)$	'ANALYTICAL'	-133,333 MPa	0.2%
$\sigma_{yy}(t=80s)$	'ANALYTICAL'	-100 MPa	0,525%
$\sigma_{yy}(t=90s)$	'ANALYTICAL'	-75 MPa	0.5%

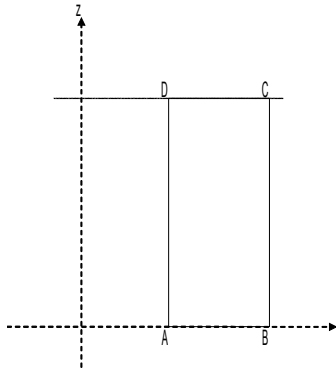
21.4 Notice

One gets well the same results with the formulation 3D_INCO_UPG that with the formulation 3D classic.

22 Modeling T

22.1 Characteristics of modeling

QUAD8 - Axisymmetric



22.2 Characteristics of the grid

Many nodes: 8

Many meshes and types: 1 QUAD8, 4 SEG3

22.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t=66,666 s)$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.05%
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t=80s)$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.1%
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t=90s)$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.1%
$p(t=66,666 s)$	'ANALYTICAL'	0	0.2%
$p(t=80s)$	'ANALYTICAL'	$3 \cdot 10^{-4}$	$1 \cdot 10^{-4}\%$
$p(t=90s)$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	$1 \cdot 10^{-4}\%$
$\sigma_{zz}(t=66,666 s)$	'ANALYTICAL'	-133,333 MPa	0.2%
$\sigma_{zz}(t=80s)$	'ANALYTICAL'	-100 MPa	0.25%
$\sigma_{zz}(t=90s)$	'ANALYTICAL'	-75 MPa	0.3%

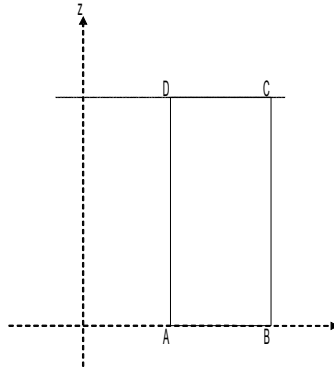
22.4 Notice

One gets well the same results with the formulation AXIS_INCO_UPG that with the formulation AXIS classic.

23 Modeling U

23.1 Characteristics of modeling

3D, $H=1$



23.2 Characteristics of the grid

Many nodes: 20
Many meshes and types: 1 HEXA20

23.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66,666\text{ s})$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.05%
$\varepsilon_{xx}(t=80\text{ s})$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.06%
$\varepsilon_{xx}(t=90\text{ s})$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.1%
$p(t=66,666\text{ s})$	'ANALYTICAL'	0	0.11%
$p(t=80\text{ s})$	'ANALYTICAL'	$3 \cdot 10^{-4}$	0.05%
$p(t=90\text{ s})$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	0.01%
$\sigma_{yy}(t=66,666\text{ s})$	'ANALYTICAL'	-133,333 MPa	0.2%
$\sigma_{yy}(t=80\text{ s})$	'ANALYTICAL'	-100 MPa	0.25%
$\sigma_{yy}(t=90\text{ s})$	'ANALYTICAL'	-75 MPa	0.3%

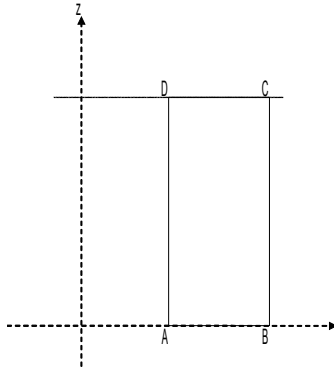
23.4 Notice

One gets well the same results with the formulation 3D_INCO_UPG that with the formulation 3D classic.

24 Modeling V

24.1 Characteristics of modeling

3D, $H=1$



24.2 Characteristics of the grid

Many nodes: 35
Many meshes and types: 12 TETRA10

24.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66,666\text{ s})$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.05%
$\varepsilon_{xx}(t=80\text{ s})$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.06%
$\varepsilon_{xx}(t=90\text{ s})$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.1%
$p(t=66,666\text{ s})$	'ANALYTICAL'	0	0.2%
$p(t=80\text{ s})$	'ANALYTICAL'	$3 \cdot 10^{-4}$	0,005%
$p(t=90\text{ s})$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	0,001%
$\sigma_{yy}(t=66,666\text{ s})$	'ANALYTICAL'	-133,333 MPa	0.2%
$\sigma_{yy}(t=80\text{ s})$	'ANALYTICAL'	-100 MPa	0.25%
$\sigma_{yy}(t=90\text{ s})$	'ANALYTICAL'	-75 MPa	0.3%

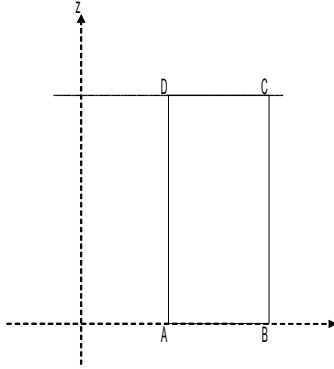
24.4 Notice

One gets well the same results with the formulation 3D_INCO_UPG that with the formulation 3D classic.

25 Modeling W

25.1 Characteristics of modeling

QUAD8 - Axisymmetric



25.2 Characteristics of the grid

Many nodes: 8

Many meshes and types: 1 QUAD8, 4 SEG3

25.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t=66,666\text{ s})$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.05%
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t=80\text{ s})$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.1%
$\varepsilon_{rr} = \varepsilon_{\theta\theta}(t=90\text{ s})$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.1%
$p(t=66,666\text{ s})$	'ANALYTICAL'	0	0.2%
$p(t=80\text{ s})$	'ANALYTICAL'	$3 \cdot 10^{-4}$	$1 \cdot 10^{-4}\%$
$p(t=90\text{ s})$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	$1 \cdot 10^{-4}\%$
$\sigma_{zz}(t=66,666\text{ s})$	'ANALYTICAL'	-133,333 MPa	0.2%
$\sigma_{zz}(t=80\text{ s})$	'ANALYTICAL'	-100 MPa	0.25%
$\sigma_{zz}(t=90\text{ s})$	'ANALYTICAL'	-75 MPa	0.3%

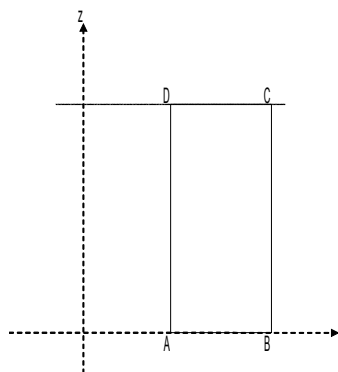
25.4 Notice

One gets well the same results with the formulation AXIS_INCO_UP that with the formulation AXIS classic.

26 Modeling X

26.1 Characteristics of modeling

3D, $H=1$



26.2 Characteristics of the grid

Many nodes: 20
Many meshes and types: 1 HEXA20

26.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66,666\text{ s})$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.05%
$\varepsilon_{xx}(t=80\text{ s})$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.06%
$\varepsilon_{xx}(t=90\text{ s})$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.1%
$p(t=66,666\text{ s})$	'ANALYTICAL'	0	0.11%
$p(t=80\text{ s})$	'ANALYTICAL'	$3 \cdot 10^{-4}$	0.05%
$p(t=90\text{ s})$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	0.01%
$\sigma_{yy}(t=66,666\text{ s})$	'ANALYTICAL'	-133,333 MPa	0.2%
$\sigma_{yy}(t=80\text{ s})$	'ANALYTICAL'	-100 MPa	0.25%
$\sigma_{yy}(t=90\text{ s})$	'ANALYTICAL'	-75 MPa	0.3%

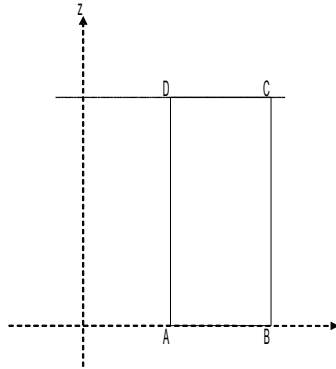
26.4 Notice

One gets well the same results with the formulation 3D_INCO_UP that with the formulation 3D classic.

27 Modeling Y

27.1 Characteristics of modeling

3D, $H=1$



27.2 Characteristics of the grid

Many nodes: 35
Many meshes and types: 12 TETRA10

27.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
$\varepsilon_{xx}(t=66,666\text{ s})$	'ANALYTICAL'	$8.6666 \cdot 10^{-4}$	0.05%
$\varepsilon_{xx}(t=80\text{ s})$	'ANALYTICAL'	$1.1 \cdot 10^{-3}$	0.06%
$\varepsilon_{xx}(t=90\text{ s})$	'ANALYTICAL'	$1,275 \cdot 10^{-3}$	0.1%
$p(t=66,666\text{ s})$	'ANALYTICAL'	0	0.2%
$p(t=80\text{ s})$	'ANALYTICAL'	$3 \cdot 10^{-4}$	0,005%
$p(t=90\text{ s})$	'ANALYTICAL'	$5.25 \cdot 10^{-4}$	0,001%
$\sigma_{yy}(t=66,666\text{ s})$	'ANALYTICAL'	-133,333 MPa	0.2%
$\sigma_{yy}(t=80\text{ s})$	'ANALYTICAL'	-100 MPa	0.25%
$\sigma_{yy}(t=90\text{ s})$	'ANALYTICAL'	-75 MPa	0.3%

27.4 Notice

One gets well the same results with the formulation 3D_INCO_UP that with the formulation 3D classic.

28 Summary of the results

The results are satisfactory and validate the behaviors thermoelastic and thermoplastic of Von Mises with isotropic and kinematic work hardening linear. The finite elements used are the elements 2D (quadrilaterals in constraints plane or axisymmetric), 3D, incompressible elements (3D_INCO_UPG, 3D_INCO_UP, AXIS_INCO_UPG and AXIS_INCO_UP) and elements PIPE.

One notes in particular a good modeling of the variation of the elastic limit and constant of Prager with the temperature.

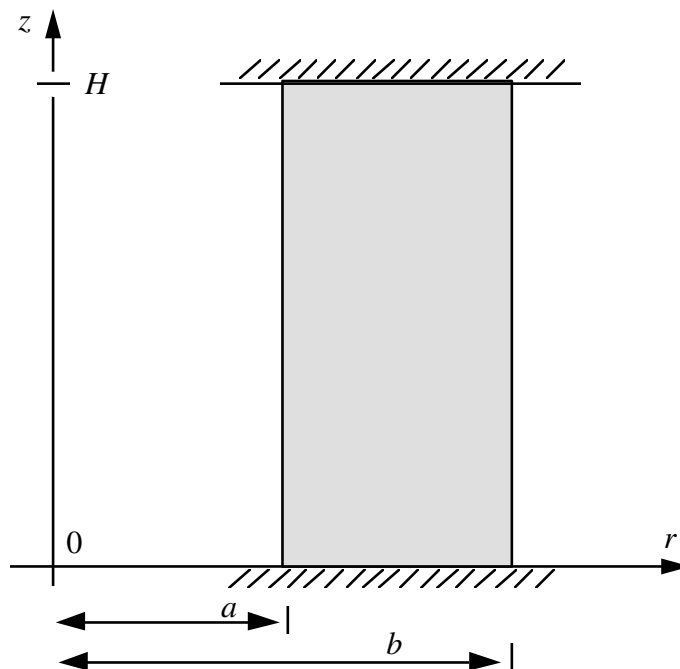
29 Appendix

29.1 Presentation

The studied problem is such as the solution is uniform in space, without any external effort given, so as to test only the treatment of the relation of behavior.

The following solid thus is considered:

- height H ,
- axisymmetric (of rays a and b),
- or parallelepipedic (thickness $b-a$).



It is placed between two lubricated rigid plates.

The material is thermoelastoplastic homogeneous (see hereafter) with isotropic work hardening and criterion of Von Mises.

One supposes the uniform temperature spaces some, and increasing.

29.2 Kinematics, balance

29.2.1 Axisymmetric case (2D)

Fields of displacement: $u = u_r(r) e_r$ (blocking in z)

$$\text{fields of deformation: } \varepsilon(u) = \begin{pmatrix} u_r' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{u_r}{r} \end{pmatrix} \quad \left(\text{according to } \begin{pmatrix} r \\ z \\ \theta \end{pmatrix} \right)$$

$$\text{fields of deformation: } \sigma(u) = \sigma_L \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \left(\text{cf boundary conditions} \right) \left(\text{according to } \begin{pmatrix} r \\ z \\ \theta \end{pmatrix} \right)$$

29.2.2 Parallelepipedic case

Fields of displacement: $u = u_x(x) e_x + u_y(y) e_y$ (blocking in z)

$$\text{fields of deformation: } \varepsilon(u) = \begin{pmatrix} u_x' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & u_y' \end{pmatrix} \quad \left(\text{according to } \begin{pmatrix} x \\ z \\ y \end{pmatrix} \right)$$

$$\text{fields of deformation: } \sigma(u) = \sigma_L \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \left(\text{cf boundary conditions} \right) \left(\text{according to } \begin{pmatrix} x \\ z \\ y \end{pmatrix} \right)$$

The case could be studied in D_PLAN and in 3D.

29.3 Relation of behavior

Isotropic, linear work hardening (tangent module E_T constant).

Criterion of Von Mises.

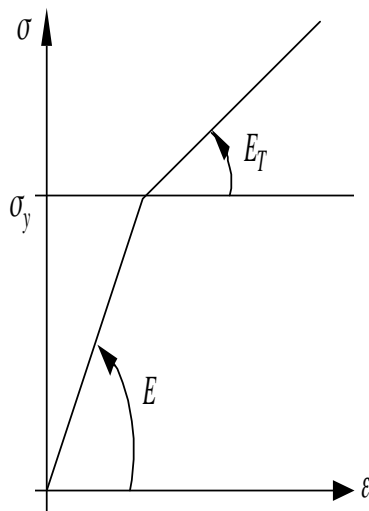
Elastic coefficients, E and ν , as well as the tangent module E_T are invariants according to the temperature.

Elastic limit σ_y vary according to the temperature T :

$$\sigma_y(T) = \sigma_y^o (1 - s(T - T^o))$$

(for the temperature range studied, σ_y is positive!).

The thermal dilation coefficient α is constant.



$$2\mu = \frac{E}{1 + \nu}$$

$$3K = \frac{E}{1 - 2\nu}$$

$$2\mu = \frac{E}{1 + \nu} \quad 3K = \frac{E}{1 - 2\nu}$$

The law of behavior (variable scalar intern is written P):

$$\varepsilon = \frac{1}{9K} \text{tr} \sigma Id + \frac{1}{2\mu} \sigma^D + \varepsilon^p = \alpha(T - T^o) Id$$

with: $\sigma^D = \sigma - \frac{1}{3} \text{tr} \sigma Id$ (diverter of the constraints)

$$\dot{\sigma}^p = \frac{3}{2} \dot{p} \frac{\sigma^D}{\|\sigma_{\dot{e}q}\|}, \text{ with } \|\sigma_{\dot{e}q}\| = \sqrt{\frac{3}{2}} \sqrt{\sigma^D \sigma^D}$$

$$\dot{p} = 0 \text{ if } f(\sigma, p) = \|\sigma_{\dot{e}q}\| - R(p) < 0$$

$$\dot{p} \geq 0 \text{ if } f(\sigma, p) = 0$$

$R(p)$ indicate the function of work hardening:

$$R(p) = \sigma_y + \frac{EE_T}{E - E_T} p$$

The rate \dot{p} can be expressed, when $f(\sigma, p) = 0$. Indeed, of $\dot{p} f$ identically no one, one draws: $\dot{p} \dot{f} + \ddot{p} f = 0$. Thus, when one is on the criterion $f = 0$, necessarily $\dot{f} = 0$. It is - with-to say:

$$\begin{aligned} \frac{3}{2} \frac{\sigma^D \dot{\sigma}^D}{\|\sigma_{\dot{\epsilon}q}\|} - R_{,T} \dot{T} - R_{,p} \dot{p} &= 0 \\ \Leftrightarrow \frac{3}{2} \frac{\sigma^D \dot{\sigma}^D}{\|\sigma_{\dot{\epsilon}q}\|} - \sigma_y^o s \dot{T} - \frac{EE_T}{E - E_T} \dot{p} &= 0 \end{aligned}$$

From where:

$$\dot{p} = \frac{E - E_T}{EE_T} \left(\frac{3}{2} \frac{\sigma^D \dot{\sigma}^D}{\|\sigma_{\dot{\epsilon}q}\|} + \sigma_y^o s \dot{T} \right) \text{ if } \dot{p} \geq 0, \text{ for } \|\sigma_{\dot{\epsilon}q}\| = R(p)$$

(criterion reached, in "load")

29.4 Thermal loading

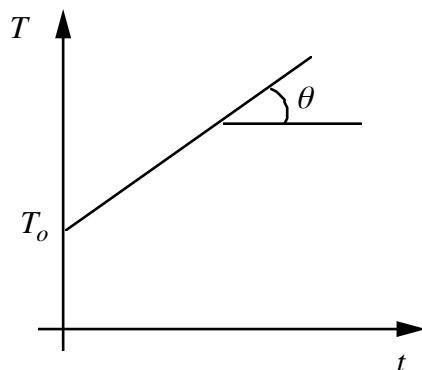
Uniform temperature in space

$$T(t) = \theta t + T_o, \quad \theta > 0$$

$$t \in \left[0, t_{fin} \right]; \quad \text{avec } t_{fin} < \frac{1}{s\theta}$$

$$T(t) = \theta t + T_o, \quad \theta > 0$$

$$t \in \left[0, t_{fin} \right] \quad \text{with } t_{fin} < \frac{1}{s\theta}$$



Virgin initial state: $\sigma_L = 0$
 $p = 0$

29.5 Solution

The stress field being uniaxial, one a:

$$\sigma^D = \frac{\sigma_L}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

As follows:

$$\|\sigma_{\dot{\epsilon}q}\| = |\sigma_L|$$

and:

$$\dot{\epsilon}^p = \frac{\dot{p}}{2} \operatorname{sgn}(\sigma_L) \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

The relation of behavior leads to

$$\begin{aligned} \dot{\epsilon}_{rr} = \dot{\epsilon}_{\theta\theta} &= \frac{-\nu}{E} \dot{\sigma}_L - \frac{\dot{p}}{2} \operatorname{sgn}(\sigma_L) + \alpha \dot{T} \quad (\dot{\epsilon}_{xx} = \dot{\epsilon}_{yy} \text{ for the case of the parallelepiped}) \\ \dot{\epsilon}_{zz} = 0 &= \frac{1}{E} \dot{\sigma}_L + \dot{p} \operatorname{sgn}(\sigma_L) + \alpha \dot{T} \end{aligned}$$

From where:

$$\begin{aligned} \dot{\epsilon}_{rr} = \dot{\epsilon}_{\theta\theta} &= \frac{3}{2} \alpha \dot{T} + \frac{1-2\nu}{2E} \dot{\sigma}_L \\ \dot{p} = \operatorname{sgn}(\sigma_L) \left(-\alpha \dot{T} - \frac{\dot{\sigma}_L}{E} \right) &= 0 \quad \text{if } |\sigma_L| < R(p) \\ \dot{p} = \operatorname{Max} \left[0; \frac{E-E_T}{EE_T} \left(\frac{3}{2} \frac{\sigma^D \sigma^D}{\|\sigma_{\dot{\epsilon}q}\|} + \sigma_y^o s \dot{T} \right) \right] & \quad \text{if not} \end{aligned}$$

I.e., in the case $|\sigma_L| = R(p)$ (criterion reached)

$$\dot{p} = \operatorname{Max} \left[0; \frac{E-E_T}{EE_T} (\operatorname{sgn}(\sigma_L) \dot{\sigma}_L + \sigma_y^o s \dot{T}) \right]$$

29.5.1 Elastic phase

At the beginning of the thermal loading, $|\sigma_L|$ being lower than σ_y , \dot{p} is null.

From where:

$$\begin{aligned}\dot{\sigma}_L &= -E \alpha \dot{T} \\ \dot{\varepsilon}_{rr} = \dot{\varepsilon}_{\theta\theta} &= \alpha \dot{T} (1 + \nu)\end{aligned}$$

As follows:

$$\begin{aligned}\sigma_L &= -E \alpha \theta t \quad (\text{compression } \sigma_L < 0) \\ \varepsilon_{rr} = \varepsilon_{\theta\theta} &= \alpha \theta (1 + \nu) t\end{aligned}$$

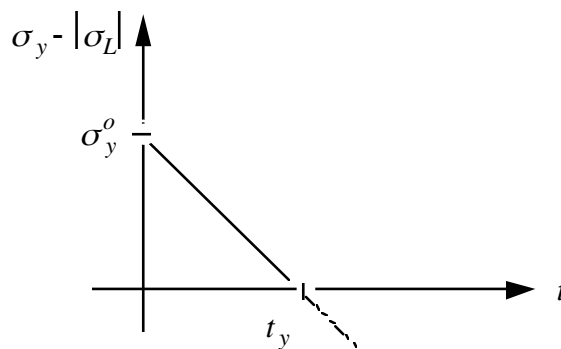
Validity of the elastic solution

The criterion is:

$$|\sigma_L(t)| - \sigma_y(t) = E \alpha \theta t - \sigma_y^o (1 - s \theta t) \leq 0$$

The criterion is not crossed for $t \in [0, t_y]$, with:

$$t_y = \frac{\sigma_y^o}{\theta (E \alpha + \sigma_y^o s)}$$



At the moment t_y :

$$\sigma_L(t_y) = \frac{-E \alpha \sigma_y^o}{E \alpha + \sigma_y^o s}$$

29.5.2 Elastoplastic phase

$t \geq t_y$. One is on the criterion. Then:

$$\dot{p} = \text{Max}\left[0; \frac{E - E_T}{EE_T} (\dot{\sigma}_L \text{sgn}(\sigma_L) + \sigma_y^o s \dot{T})\right]$$

By admitting that one is "charges some" ($\dot{p} > 0$), then one eliminates \dot{p} to have

$$\dot{\sigma}_L = -E_T \dot{T} \left(\alpha + \text{sgn}(\sigma_L) \frac{E - E_T}{EE_T} s \sigma_y^o \right)$$

then:

$$\dot{p} = \frac{E - E_T}{E} \dot{T} \left(-\alpha \text{sgn}(\sigma_L) + \frac{s \sigma_y^o}{E} \right)$$

with $t = t_y$, $\sigma_L = -E \alpha \theta t_y < 0$; one integrates these expressions then for $t \geq t_y$ ($\dot{T} = \theta$):

$$\sigma_L(t) = -E_T \theta (t - t_y) \left[\alpha - \frac{E - E_T}{EE_T} s \sigma_y^o \right] - \sigma_L(t_y)$$

$$p(t) = \frac{E - E_T}{E^2} \theta [\alpha E + s \sigma_y^o] (t - t_y)$$

Maybe, after rearrangement, ($t > 0 t_y$):

$$\sigma_L(t) = \sigma_y^o \left(s \theta t - 1 + \frac{E_T}{E} \left(1 - \frac{t}{t_y} \right) \right)$$

$$p(t) = \frac{\sigma_y^o (E - E_T)}{E^2} \left(\frac{t}{t_y} - 1 \right)$$

Validity of this elastoplastic solution

It should be made sure that $\sigma_L(t)$ remain negative. Knowing that $s \theta t < 1$, and that $t > t_y$, the result precedent confirms that $\sigma_L(t) < 0$.

Lastly, it is noticed that:

$$\text{sgn}(\sigma_L) \frac{1 - 2\nu}{2} \dot{p} + \varepsilon_{rr} = \alpha (1 + \nu) \dot{T}$$

from where:

$$\varepsilon_{rr}(t) = \varepsilon_{\theta\theta}(t) = \alpha \theta (1 + \nu) t + \frac{1 - 2\nu}{2} p(t), \quad \forall t \in [t_y, t_{fin}]$$

(since $\sigma_L(t) < 0$).

29.6 Digital application

$$\begin{aligned} E &= 200000 \text{ MPa}; & \nu &= 0.3; & \alpha &= 10.0 E-5 \text{ } ^\circ\text{C}^{-1}; & \theta &= 1.0 \text{ s}^{-1} \\ \sigma_y^o &= 400 \text{ MPa}; & T^o &= 0 \text{ } ^\circ\text{C}; & s &= 10.0 E-2 \text{ } ^\circ\text{C}^{-1}; & t_{fin} &< 100 \text{ s} \\ E_T &= 50000 \text{ MPa}; \end{aligned}$$

From where one obtains in elastic phase:

$$\begin{aligned} t_y &= 66.6666 \text{ s} \\ \sigma_L(t_y) &= -133.333 \text{ MPa} \\ \varepsilon_{rr}(t_y) = \varepsilon_{\theta\theta}(t_y) &= 0.866666 E-3 \end{aligned}$$

Then, elastoplastic phase:

$$\begin{aligned} \text{with } t=80\text{s} : & \quad \sigma_L(80) = -100 \text{ MPa} \\ & \quad p(80) = 0.30 E-3 \\ & \quad \varepsilon_{rr}(80) = \varepsilon_{\theta\theta}(80) = 1.100 E-3 \\ \\ \text{with } t=90\text{s} : & \quad \sigma_L(90) = -75 \text{ MPa} \\ & \quad p(90) = 0.525 E-3 \\ & \quad \varepsilon_{rr}(90) = \varepsilon_{\theta\theta}(90) = 1.275 E-3 \end{aligned}$$