
HSNV101 - Thermoplasticity and metallurgy uncoupled in simple traction

Summary:

One treats the determination of the mechanical evolution of a cylindrical bar subjected to thermal evolutions $T(t)$ and metallurgical $Z(t)$ known and uniform (the metallurgical transformation is of bainitic type).

The elements used are axisymmetric elements and the relation of behavior is the plasticity of von Mises with linear isotropic work hardening (for modeling B, one also takes account of the plasticity of transformation).

The yield stress and the slope of the traction diagram depend on the temperature and the metallurgical composition.

The dilation coefficient α depends on the metallurgical composition.

The metallurgical transformations take place with $\dot{\epsilon}^p = 0$ (it is in the sense that the test **uncouple** the plasticity of transformation of classical plasticity).

Results provided by *Code_Aster* are very satisfactory with errors lower than 2 % .

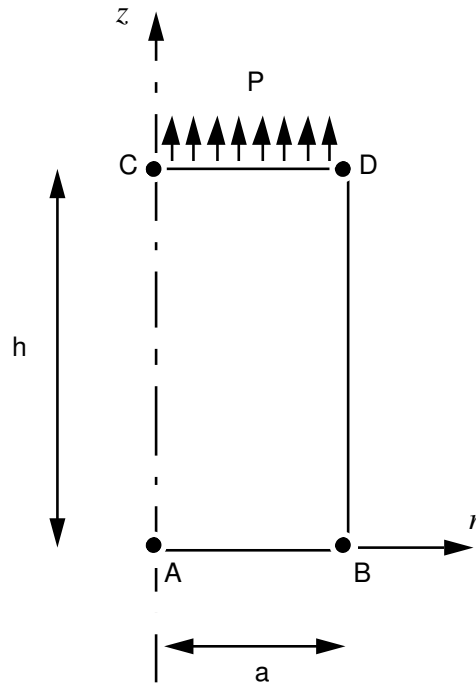
Modeling A, B, D use grids made up of two meshes QUAD8, modeling C uses meshes TRIA6, and modeling E validates the metallurgy on material point.

Modelings A, C and E use behavior META_P_IL, modeling B validates behavior META_P_IL_PT, and modeling D uses META_P_CL.

1 Problem of reference

1.1 Geometry

Rayon : $a = 0.05$ m.
Hauteur : $h = 0.2$ m.



1.2 Properties of materials

$E = 200000 \cdot 10^6 \text{ Pa}$	$\sigma_y^{aust} = \sigma_o^{aust} + s^{aust}(T - T^o)$	notons $H(t) = \frac{\alpha(t) \cdot E(t)}{E(t) - \alpha(t)}$
$\nu = 0.3$	$\sigma_o^{aust} = 400 \cdot 10^6 \text{ Pa}$	$H^{aust} = H_o^{aust} + \lambda^{aust}(T - T^o)$
$\alpha_{fbm} = 15 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1}$	$s^{aust} = 0.5 \cdot 10^6 \text{ Pa} \cdot ^\circ\text{C}^{-1}$	$H_o^{aust} = 1250 \cdot 10^6 \text{ Pa}$
$\alpha_{aust} = 23.5 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1}$	$\sigma_y^{fbm} = \sigma_o^{fbm} + s^{fbm}(T - T^o)$	$\lambda^{aust} = -5 \cdot 10^6 \text{ Pa} \cdot ^\circ\text{C}^{-1}$
$\varepsilon_{ref\,fbm} = 2.52 \cdot 10^{-3}$	$\sigma_o^{fbm} = 530 \cdot 10^6 \text{ Pa}$	$H^{fbm} = H_o^{fbm} + \lambda^{fbm}(T - T^o)$
$T^{ref} = 900^\circ \text{ C}$	$s^{fbm} = 0.5 \cdot 10^6 \text{ Pa} \cdot ^\circ\text{C}^{-1}$	$H_o^{fbm} = -50 \cdot 10^6 \text{ Pa}$
$cp = 2\,000\,000 \text{ J} \cdot \text{m}^{-3} \cdot ^\circ\text{C}^{-1}$	$\lambda = 9999.9 \text{ W} \cdot \text{m}^{-1} \cdot ^\circ\text{C}^{-1}$	$\lambda^{fbm} = -5 \cdot 10^6 \text{ Pa} \cdot ^\circ\text{C}^{-1}$
		$k^m = 1 \cdot 10^{-10} \text{ Pa}^{-1}$

- **aust* = characteristics relating to the austenitic phase
- **fbm* = characteristics relating to the phases ferritic, bainitic and martensitic
- α_{fbm} = thermal dilation coefficient of the phases ferritic, bainitic and martensitic
- α_{aust} = dilation coefficient of the austenitic phase
- $\varepsilon_{ref\,fbm}$ = deformation of the phases ferritic, bainitic and martensitic at the temperature of reference, austenite being regarded as not deformed at this temperature: translated the difference in compactness between the cubic crystallographic structures with centered faces (austenite) and cubic centered (ferrite).

TRC to model a metallurgical evolution of bainitic type, on all the structure, of the form:

$$Z_{fbm} = \begin{cases} 0, & \text{si } t \leq \tau_1 & \tau_1 = 60 \text{ s} \\ \frac{t - \tau_1}{\tau_2 - \tau_1} & \text{si } \tau_1 \leq t < \tau_2 & \tau_2 = 112 \text{ s} \\ 1, & \text{si } t \geq \tau_2 \end{cases}$$

Law of plasticity of transformation: $\dot{\varepsilon}^{pt} = K^{fbm} F(Z_{fbm}) \dot{Z}_{fbm}$

$$\text{with } F(Z_{fbm}) = Z_{fbm} (2 - Z_{fbm})$$

1.3 Boundary conditions and loadings

- $u_z = 0$ on the side AB (condition of symmetry).
- traction imposed on the side CD

$$p(t) = \begin{cases} p_o t & \text{pour } t \leq \tau_1 & p_o = 6 \cdot 10^6 \text{ Pa} \\ 360 \cdot 10^6 \text{ Pa} & \text{pour } t \geq \tau_1 & \tau_1 = 60 \text{ s} \end{cases}$$

- $T = T^o + \mu t$, $\mu = -5 \text{ } ^\circ\text{C} \cdot \text{s}^{-1}$ on all the structure.

1.4 Initial conditions

$$T^o = 900 \text{ } ^\circ\text{C} = T^{ref}$$

2 Reference solution

2.1 Method of calculating used for the reference solution

Before transformation, elastic solution for $t < \tau_1$.

$$\sigma(t) = p_o t \quad \varepsilon_{zz}(t) = \varepsilon_{zz}^e(t) + \varepsilon_{zz}^{th}(t) = \frac{\sigma(t)}{E} + \alpha_{aust}(T - T^o)$$

The yield stress is reached for $\tau_1 = \frac{\sigma_o^{aust}}{p_o - s^{aust} \times \mu} = 47.06 \text{ s}$.

Before transformation, thermoelastoplastic solution, $\tau_1 \leq t \leq \tau_1$, $\tau_1 = 60 \text{ s}$.

$$\begin{aligned} \sigma(t) &= p_o t & \varepsilon_{zz}(t) &= \varepsilon_{zz}^e(t) + \varepsilon_{zz}^{th}(t) + \varepsilon_{zz}^p(t) \\ \varepsilon_{zz}^e(t) &= \frac{\sigma(t)}{E} & \varepsilon_{zz}^{th}(t) &= Z_{aust} \times \alpha_{aust}(T - T^o) \\ \varepsilon_{zz}^p(t) &= \frac{\sigma(t) - (\sigma_y^{aust} + s^{aust} \mu t)}{H_o^{aust} + \lambda^{aust} \mu t} \end{aligned}$$

During the transformation, thermo-élasto-metallurgical solution, $\tau_1 < t < \tau_2$, $\tau_2 = 112 \text{ s}$.

$$\begin{aligned} \sigma(t) &= 360 \cdot 10^6 \text{ Pa} & \varepsilon_{zz}(t) &= \varepsilon_{zz}^e(t) + \varepsilon_{zz}^{th}(t) + \varepsilon_{zz}^{pt}(t) + \varepsilon_{zz}^p(60) \\ \varepsilon_{zz}^{th}(t) &= Z_{aust} \times \alpha_{aust}(T - T^o) + Z_{fbm} \times \alpha_{fbm}(T - T^o) + Z_{fbm} \times \varepsilon_{ref_{fbm}} \\ \varepsilon_{zz}^p(t) &= k^{fbm} F(Z_{fbm}) p_o \tau_1 \end{aligned}$$

After the transformation, thermoelastoplastic solution, $\tau_2 < t < \tau_3$, $\tau_3 = 176 \text{ s}$.

$$\begin{aligned} \sigma(t) &= 360 \cdot 10^6 \text{ Pa} & \varepsilon_{zz}(t) &= \varepsilon_{zz}^e(t) + \varepsilon_{zz}^{th}(t) + \varepsilon_{zz}^p(t) + \varepsilon_{zz}^{pt}(112) \\ \varepsilon_{zz}^p(t) &= \frac{\sigma(t) - (\sigma_o^{fbm} + s^{fbm} \mu t)}{H_o^{fbm} + \lambda^{fbm} \mu t} \end{aligned}$$

2.2 Results of reference

ε_{zz}^p , χ , σ and ε_{zz} for $t=47, 48, 64$ and 114 seconds.

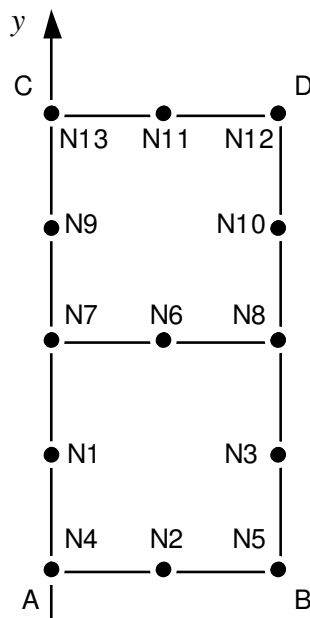
ε_{zz}^p for $t=60$ and 176 seconds.

2.3 Bibliographical references

- 1) DONORE A.M. - WAECKEL F.: Influence of structure transformations in the elastoplastic laws of behavior Notes HI-74/93/024.

3 Modeling A

3.1 Characteristics of modeling



$$A=N4, B=N5, C=N13, D=N12.$$

3.2 Characteristics of the grid

Many nodes: 13

Many meshes and types: 2 meshes QUAD8, 6 meshes SEG3

3.3 Sizes tested and results

One tests the structural parameters of data results:

Identification	Reference	Test	Tolerance
INST for NUME_ORDRE= 7 0	176	ANALYTICAL	0,10%
ITER_GLOB for NUME_ORDRE=70	5	NON_REGRESSION	0,00%

Identification	Reference	Test	Tolerance
ε_{zz}^P $t=47 s$	0	NON_DEFINI	1,0E-12 (absolute)
χ $t=47 s$	0	NON_DEFINI	1,0E-12 (absolute)
σ $t=47 s$	282. 10 ⁶	NON_DEFINI	0,1%
ε_{zz} $t=47 s$	- 4.1125 10 ⁻³	NON_DEFINI	0,1%
ε_{zz}^P $t=48 s$	3.2653 10 ⁻³	NON_DEFINI	0,15%
χ $t=48 s$	1	NON_DEFINI	0,1%
σ $t=48 s$	288. 10 ⁶	NON_DEFINI	0,1%
ε_{zz} $t=48 s$	- 9.3469 10 ⁻⁴	NON_DEFINI	0,007%
ε_{zz}^P $t=60 s$	0.04	NON_DEFINI	0,1%
ε_{zz}^P $t=64 s$	0,040	NON_DEFINI	0,022%
χ $t=64 s$	0	NON_DEFINI	1,0E-12 (absolute)
σ $t=64 s$	360. 10 ⁶	NON_DEFINI	0,01%
ε_{zz} $t=64 s$	3.4683 10 ⁻²	NON_DEFINI	0,025%
ε_{zz}^P $t=114 s$	0.04107	NON_DEFINI	0,01%
χ $t=114 s$	1	NON_DEFINI	0,1%
σ $t=114 s$	360. 10 ⁶	NON_DEFINI	0,020%
ε_{zz} $t=114 s$	0.03684	NON_DEFINI	0,026%
ε_{zz}^P $t=176 s$	0.06206	NON_DEFINI	0,20%

3.4 Remarks

In this modeling: $\varepsilon_{zz}^{Pt}(T, Z) = 0$

4 Modeling B

4.1 Characteristics of modeling

The grid and the data are identical to modeling A; the only difference comes from behavior META_P_IL_PT (taken into account of the plasticity of transformation)

4.2 Sizes tested and results

Identification	Reference	Aster	% difference
ε_{zz}^p $t=47 s$	0	0	0
χ $t=47 s$	0	0	0
σ $t=47 s$	$282. 10^6$	$282. 10^6$	0
ε_{zz} $t=47 s$	$- 4.1125 10^{-3}$	$- 4.1125 10^{-3}$	0
ε_{zz}^p $t=48 s$	$3.2653 10^{-3}$	$3.26535 10^{-3}$	0,011
χ $t=48 s$	1	1	0
σ $t=48 s$	$288. 10^6$	$288. 10^6$	0
ε_{zz} $t=48 s$	$- 9.3469 10^{-4}$	$- 9.34644 10^{-4}$	- 0,005
ε_{zz}^p $t=60 s$	0.04	0.04	0
ε_{zz}^p $t=64 s$	0.04	$4.0 10^{-2}$	0
χ $t=64 s$	0	0	0
σ $t=64 s$	$360. 10^6$	$359.99 10^6$	- 0,004
ε_{zz} $t=64 s$	$4.00085 10^{-2}$	$4.000268 10^{-2}$	- 0,015
ε_{zz}^p $t=114 s$	0.041071	$4.10751 10^{-2}$	+0,004
χ $t=114 s$	1	1	0
σ $t=114 s$	$360. 10^6$	$360.01 10^6$	0,000
ε_{zz} $t=114 s$	0.072841	$7.144112 10^{-2}$	- 1,915
ε_{zz}^p $t=176 s$	0.06206	$6.2066 10^{-2}$	0,000

4.3 Remarks

In this modeling, one takes into account the term due to the plasticity of transformation:

$$\varepsilon^{pl}(T, Z) \neq 0 \text{ when } \dot{Z} \neq 0$$

5 Modeling C

5.1 Characteristics of modeling

Identical to modeling A, only the grid is different (triangular meshes).

5.2 Characteristics of the grid

Many nodes: 13

Many meshes and types: 4 meshes TRIA6

5.3 Sizes tested and results

One tests the structural parameters of data results:

Identification	Reference	Test	Tolerance
INST for NUME_ORDRE= 7 0	176	ANALYTICAL	0,10%
ITER_GLOB for NUME_ORDRE=70	5	NON_REGRESSION	0,00%

Identification	Reference	Test	Tolerance
ε_{zz}^P $t=47 s$	0	NON_DEFINI	1,0E-12 (absolute)
χ $t=47 s$	0	NON_DEFINI	1,0E-12 (absolute)
σ $t=47 s$	$282 \cdot 10^6$	NON_DEFINI	0,1%
ε_{zz} $t=47 s$	$-4.1125 \cdot 10^{-3}$	NON_DEFINI	0,1%
ε_{zz}^P $t=48 s$	$3.2653 \cdot 10^{-3}$	NON_DEFINI	0,15%
χ $t=48 s$	1	NON_DEFINI	0,1%
σ $t=48 s$	$288 \cdot 10^6$	NON_DEFINI	0,1%
ε_{zz} $t=48 s$	$-9.3469 \cdot 10^{-4}$	NON_DEFINI	0,007%
ε_{zz}^P $t=60 s$	0.04	NON_DEFINI	0,1%
ε_{zz}^P $t=64 s$	0,040	NON_DEFINI	0,022%
χ $t=64 s$	0	NON_DEFINI	1,0E-12 (absolute)
σ $t=64 s$	$360 \cdot 10^6$	NON_DEFINI	0,01%
ε_{zz} $t=64 s$	$3.4683 \cdot 10^{-2}$	NON_DEFINI	0,025%
ε_{zz}^P $t=114 s$	0.04107	NON_DEFINI	0,01%
χ $t=114 s$	1	NON_DEFINI	0,1%
σ $t=114 s$	$360 \cdot 10^6$	NON_DEFINI	0,020%
ε_{zz} $t=114 s$	0.03684	NON_DEFINI	0,026%
ε_{zz}^P $t=176 s$	0.06206	NON_DEFINI	0,20%

6 Modeling D

6.1 Characteristics of modeling

Grid identical to that of modeling A.
Linear kinematic work hardening: META_P_CL

6.2 Sizes tested and results

Test of nonregression

Identification	Moment	Reference
EPYY	114	0.0368416
EPYY	206	0.0522886
EPYY	251	-0.0111300
EPYY	296	-0.0745489
EPYY	47	-0.0041125
EPYY	48	-0.0009346
EPYY	64	0.0346908
V14	114	0.0410716
V14	206	0.0620686
V26	48	0.0032653
V26	60	0.0400000
V26	64	0.0400000

7 Modeling E

7.1 Characteristics of modeling

Not material (use of SIMU_POINT_MAT)

7.2 Sizes tested and results

One tests the structural parameters of data results (same values of reference as modeling A)

Identification	Reference	Tolerance
ε_{zz}^p $t = 114 s$	0.04107	0,0001%
ε_{zz} $t = 114 s$	0.03684	0,0001%
ε_{zz}^p $t = 176 s$	0.06206	0,0001%

8 Summary of the results

Results found with *Code_Aster* are very satisfactory, with percentages of error lower than 0.025% except for the deformation at the moment 114 s where the error reached 2% for modeling B.