HSNV104 - Thermoplasticity and metallurgy in plane deformations with restoration of work hardening

Summary:

One treats the determination of the mechanical evolution of a right-angled parallelepiped in plane deformations subjected to evolutions thermics $T_{(t)}$ and metallurgical $Z_{(t)}$ known and uniform (the metallurgical transformation is of bainitic type).

The elements used are two-dimensional elements in plane deformations and the relation of behavior is the plasticity of von Mises with linear isotropic work hardening. One takes account of the restoration of work hardening, but not of the plasticity of transformation.

The dilation coefficient α depends on the metallurgical composition.

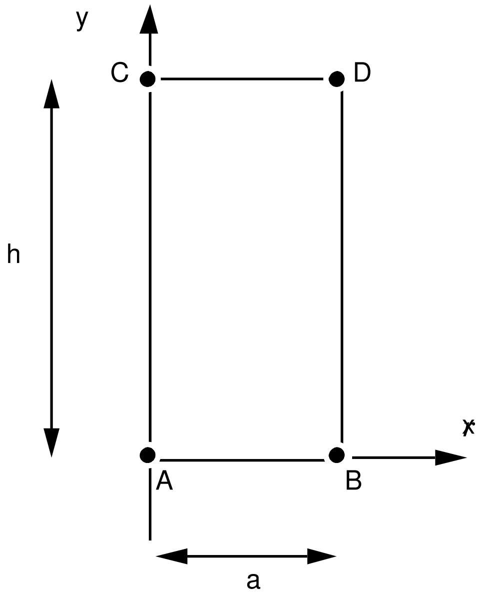
The reference solution is obtained by the analytical resolution of the problem.

Results provided by *Code_Aster* are very satisfactory with errors lower than $0{,}8\%$.

1 Problem of reference

1.1 Geometry

Largeur : a = 0.05 m.
Hauteur : h = 0.2 m.



1.2 Properties of materials

$E=2.0\text{E}+11\text{Pa}$ $\alpha^{fbm}=20.0\text{E}-06\,°C^{-1}$ $notons\ H_{(T)}=\dfrac{\alpha_{(T)}E_{(T)}}{E_{(T)}-\alpha_{(T)}}$

$\alpha_o^{aust}=20.0\text{E}-06\,°C^{-1}$ $H^{aust}=2000.0\text{E}+06\text{Pa}$

$\nu=0.3$ $\epsilon_{ref_{fbm}}=2.52\text{E}-03$ $H^{fbm}=2000.0\text{E}+06\text{Pa}$

$T^{ref}=900\,°C$ $cp=2.0\text{E}+06\,J.m^{-3}.°C^{-1}$

$\lambda=9999.9\,W.m^{-1}.°C^{-1}$

$*^{aust}$	=	characteristics relating to the austenitic phase,
$*^{fbm}$	=	characteristics relating to the phases ferritic, bainitic and martensitic,
α^{fbm}	=	thermal dilation coefficient of the phases ferritic, bainitic and martensitic,
α^{aust}	=	dilation coefficient of the austenitic phase
$\epsilon_{ref_{fbm}}$	=	deformation of the phases ferritic, bainitic and martensitic at the temperature of reference, austenite being regarded as not deformed at this temperature. That translated the difference in compactness between the cubic crystallographic structures with centered faces (austenite) and cubic centered (ferrite).

TRC to model a metallurgical evolution of bainitic type, on all the structure, of the form:

$$Z_{fbm}=\begin{cases} 0.0 & si\,t\le\tau_1 & \tau_1=60\text{sec} \\ \dfrac{t-\tau_1}{\tau_2-\tau_1} & si\,\tau_1\le t\le\tau_2 & \tau_2=112\text{sec} \\ 1.0 & si\,\tau_2\le t & \end{cases}$$

Law of plasticity of transformation: $\dot{\epsilon}^{pt}=K^{fbm}F(Z_{fbm})\langle\dot{Z}_{fbm}\rangle$

with $F(Z_{fbm})=Z_{fbm}(Z-Z_{fbm})$

one thus does not take account of the plasticity of transformation one takes $K^{fbm}=0$

Notations: $T_{(\tau_1)}=T_1$
$T_{(\tau_2)}=T_2$

1.3 Boundary conditions and loadings

- $u_Y=0$ on the side AB ; $u_X=0$ in A .
- $T=T^0+\mu\, t$, $\mu=-5°C.s^{-1}$ on all the structure.
- The loading on the structure is due with the phenomena of thermal and metallurgical dilation constrained in the direction z by the condition of plane deformations.

1.4 Initial conditions

$T^0=900°C=T^{ref}$

2 Reference solution

2.1 Method of calculating used for the reference solution

Before transformation, thermoelastic solution until in t_1 such as in t_1 :

$$\sigma_{zz} = -E\varepsilon^{th} = \sigma_y \quad \Leftrightarrow T - T^o = \frac{-\sigma_y^{aust}}{E\alpha + a} = -100^\circ C$$
$$\Leftrightarrow t_1 = 20s$$

$$\text{donc pour } t \leq t_1 \qquad \sigma_{zz} = -E\alpha_\gamma (T - T^o)$$

Before transformation, and for $t \geq t_1$, thermoelastoplastic solution such as:

$$\begin{cases} \varepsilon_{zz} = \varepsilon_{zz}^{th} + \varepsilon_{zz}^{p} + \frac{\sigma}{E} = 0 \\ \sigma_{zz} = R_0 \varepsilon_{zz}^{p} + \sigma_y \end{cases}$$

$$\text{d'où} \quad \sigma\left(\frac{1}{R_0^{aust}} + \frac{1}{E}\right) = \frac{\sigma_y^{aust}}{R_0^{aust}} - \alpha_\gamma (T - T^0) \quad et \quad \varepsilon_{zz}^{p} = p = \frac{\sigma - \sigma}{R_0^{aust}}$$

During the transformation, one is in elastic mode, one thus has an elastic solution thermo - with phase shift.

$$\sigma = -E\left[\alpha(T - T^0) + Z\varepsilon_{réf_{fbm}} + \varepsilon_{zz}^{p}(\tau_1)\right]$$

After the transformation, there is always a thermoelastic solution until in t_2.

In t_2 : $\sigma_{zz} = R(T, Z, \varepsilon^{eff}) + \sigma_y(T, Z)$

Because of the restoration of work hardening and owing to the fact that one was in elastic mode during all the transformation: $R=0$ before replastification.

One thus has in t_2 :

$$\sigma_{zz} = -E\left[\alpha(T - T^0) + \varepsilon_{réf_{fbm}} + \varepsilon_{zz}^{p}(\tau_1)\right] = \sigma_y^{fbm} \Leftrightarrow (T - T^0) = -\frac{\left[\sigma_y^{fbm} + E(\varepsilon_{réf_{fbm}} + \varepsilon_{zz}^{p}(\tau_1))\right]}{E\alpha}$$
$$\Leftrightarrow (T - T^0) = -624^\circ C \qquad t_2 \approx 125s$$

For $T < 276\,°C$ there is a thermoelastoplastic solution such as:

$$\begin{cases} \varepsilon_{zz} = \varepsilon^{th} + \frac{\sigma}{E} + \varepsilon_{zz}^{p}(t) \\ \sigma_{zz} = R_0\left[\varepsilon_{zz}^{p}(t) - \varepsilon_{zz}^{p}(\tau_1)\right] + \sigma_y \end{cases}$$

$$\text{d'où} \quad \sigma\left(\frac{1}{R_0^{fbm}} + \frac{1}{E}\right) = \frac{\sigma_y^{fbm}}{R_0^{fbm}} - \alpha(T - T^0) - \varepsilon_{réf_{fbm}} - \varepsilon_{zz}^{p}(\tau_1)$$

2.2 Results of reference

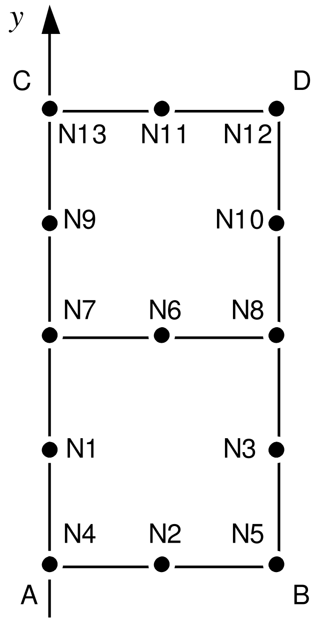
σ_{zz}, $\varepsilon_{\gamma}^{eff}$ et $\varepsilon_{\alpha}^{eff}$ à $t=60s$
σ_{zz}, $\varepsilon_{\gamma}^{eff}$ et $\varepsilon_{\alpha}^{eff}$ à $t=89s$
σ_{zz}, $\varepsilon_{\gamma}^{eff}$ et $\varepsilon_{\alpha}^{eff}$ à $t=112s$
σ_{zz}, $\varepsilon_{\gamma}^{eff}$ et $\varepsilon_{\alpha}^{eff}$ à $t=176s$

2.3 Bibliographical references

1. DONORE A.M. - WAECKEL F. - Influence of structure transformations in the elastoplastic laws of behavior Notes HI-74/93/024.
2. DONORE.A.M. - WAECKEL.F. - RAZAKANAIVO.A. - Doc. Aster [R4.04.02].

3 Modeling A

3.1 Characteristics of modeling



$A=N4$, $B=N5$, $C=N13$, $D=N12$.

3.2 Characteristics of the grid

Many nodes: 13.
Many meshs and types: 2 meshs `QUAD8`, 6 meshs `SEG3`.

3.3 Sizes tested and results

Identification	Type of Reference	Reference	Tolerance (%)
σ_{zz} $t=60s$	ANALYTICAL	4.0792E8	0.10
$\varepsilon_{\gamma}^{eff}$ $t=60s$	ANALYTICAL	3.9604E-3	0.02
$\varepsilon_{\alpha}^{eff}$ $t=60s$	ANALYTICAL	0.	0.00
σ_{zz} $t=89s$	ANALYTICAL	7.0684E8	0.80
$\varepsilon_{\gamma}^{eff}$ $t=89s$	ANALYTICAL	3.9604E-3	0.02
$\varepsilon_{\alpha}^{eff}$ $t=89s$	ANALYTICAL	0.	0.00
σ_{zz} $t=112s$	ANALYTICAL	9.4392E8	0.05
$\varepsilon_{\gamma}^{eff}$ $t=112s$	ANALYTICAL	0.	0.00
$\varepsilon_{\alpha}^{eff}$ $t=112s$	ANALYTICAL	0.	0.00
σ_{zz} $t=176s$	ANALYTICAL	12.101E8	0.10
$\varepsilon_{\gamma}^{eff}$ $t=176s$	ANALYTICAL	0.	0.00
$\varepsilon_{\alpha}^{eff}$ $t=176s$	ANALYTICAL	5.068921E-3	0.04

3.4 Remarks

In this modeling:

$$\varepsilon_{zz}^{pt}(T,Z) = 0$$

The error on σ_{ZZ} at 89 seconds comes by way of the mistake made on the digital description of the metallurgical transformation which is, at this moment, of approximately 56%.

4 Summary of the results

Results found with *Code_Aster* are very satisfactory, with percentages of error lower than 0.8%.