

HSNV121 - Traction in great deformations plastics of a bar under loading thermics

Summary:

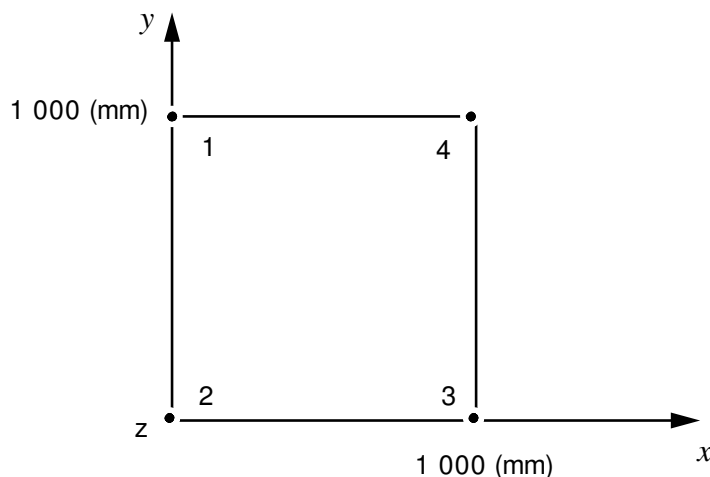
This quasi-static thermomechanical test consists in heating a bar of rectangular section uniformly (3D) or cylindrical (2D axisymmetric) then to subject it to a traction. One thus validates the kinematics of the great deformations in plasticity (order STAT_NON_LINE, keyword DEFORMATION: 'SIMO_MIEHE' or 'PETIT_REAC') for a relation of behavior in great deformations with linear isotropic work hardening (order STAT_NON_LINE, keyword RELATION: 'VMIS_ISOT_LINE' and 'VMIS_ISOT_TRAC') with thermomechanical loading. With modelings hull or plate, the great deformations in plasticity are accessible thanks to the keyword DEFORMATION: 'PETIT_REAC' provided that rotations remain weak.

The bar is modelled by a voluminal element (HEXA20, modeling A) or quadrangular (QUAD4, for an axisymmetric modeling, modeling B) or by elements of plate or hull (DKT for modeling C and COQUE_3D for modeling D).

The solution is analytical.

1 Problem of reference

1.1 Geometry

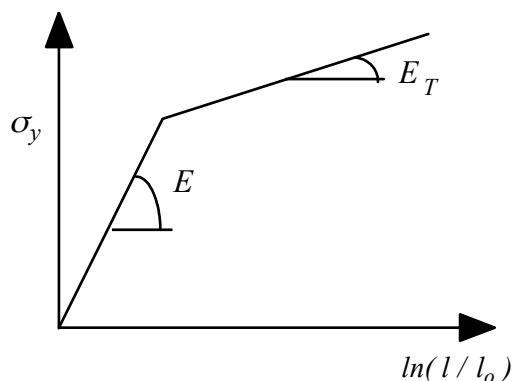


1.2 Properties of material

The material obeys a law of behavior in great deformations figure with linear isotropic work hardening, whose characteristics depend on the temperature.

The traction diagram is given in the plan deformation logarithmic curve - rational constraint.

$$\sigma = \frac{F}{S} = \frac{F}{S_0} \cdot \frac{l}{l_0}$$



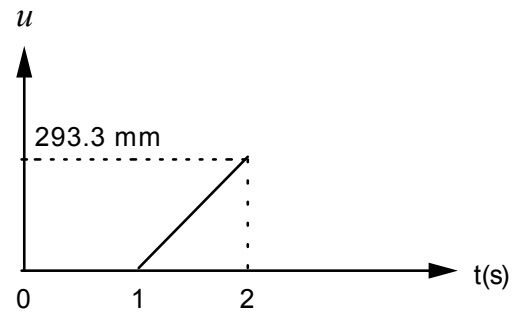
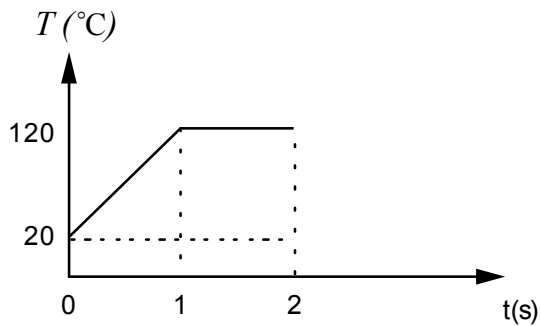
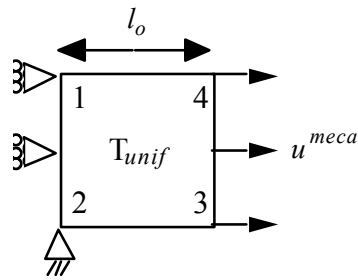
ν	$= 0.3$
α	$= 10^{-4} K^{-1}$
σ_y	$= 1000 \text{ MPa}$
à T	$= 20^\circ \text{ C}$
E	$= 250000 \text{ MPa}$
E_T	$= 2500 \text{ MPa}$
à T	$= 120^\circ \text{ C}$
E	$= 200000 \text{ MPa}$
E_T	$= 2000 \text{ MPa}$

l_0 and l are, respectively, the initial length and the current length of the useful part of the test-tube.

S_0 and S are, respectively, initial and current surface. Between the temperatures 20° C and 120° C , the characteristics are interpolated linearly.

1.3 Boundary conditions and loadings

The bar, initial length l_0 , blocked in the direction Ox on the face [1,2] is subjected to a uniform temperature T and with a mechanical displacement of traction u^{meca} on the face [3,4]. The sequences of loading are the following ones:



Temperature of reference: $T_{réf} = 20^\circ\text{C}$.

Note:

Mechanical displacement is measured starting from the configuration deformed by the thermal loading ($t=1$ s). To have total displacement, it is thus necessary to add the thermal displacement obtained at time $t=1$ s.

2 Reference solution

2.1 Result of the reference solution

For a tensile test according to the direction x , the tensor of Kirchhoff τ is form:

$$\tau = \begin{pmatrix} \tau & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Tensor gradients of the transformation \mathbf{F} and $\bar{\mathbf{F}}$ and the isochoric tensor of plastic deformations \mathbf{G}^p are form:

$$\mathbf{F} = \begin{pmatrix} F & 0 & 0 \\ 0 & F_{yy} & 0 \\ 0 & 0 & F_{yy} \end{pmatrix} \quad \text{et } J = \det \mathbf{F} = FF_{yy}^2 \Rightarrow F_{yy} = \sqrt{J/F}$$

$$\bar{\mathbf{F}} = J^{-1/3} \mathbf{F} = \begin{pmatrix} \bar{F} & 0 & 0 \\ 0 & \bar{F}_{yy} & 0 \\ 0 & 0 & \bar{F}_{yy} \end{pmatrix} \quad \text{et } \det \bar{\mathbf{F}} = 1 \Rightarrow \begin{cases} \bar{F} = J^{-1/3} F \\ \bar{F}_{yy} = \bar{F}^{-1/2} \end{cases}$$

$$\mathbf{G}^p = \begin{pmatrix} G^p & 0 & 0 \\ 0 & G_{yy}^p & 0 \\ 0 & 0 & G_{yy}^p \end{pmatrix} \quad \text{et } \det \mathbf{G}^p = 1 \Rightarrow G_{yy}^p = (G^p)^{-1/2}$$

By the law of behavior, one obtains the following relation:

$$\tau = \frac{3K}{2}(J^2 - 1) - \frac{9K\alpha(T - T_{ref})}{2}(J + \frac{1}{J})$$

that is to say

$$J^3 - 3\alpha(T - T_{ref})J^2 - J(1 + \frac{2\tau}{3K}) - 3\alpha(T - T_{ref}) = 0$$

The constraint of Cauchy is written:

$$J\sigma = \tau$$

In plastic load for an isotropic work hardening R linear, such as:

$$R(p) = \frac{EE_T}{E - E_T} p$$

one a:

$$p = \frac{E - E_T}{E E_T} (\tau - \sigma_y)$$

The integration of the law of flow of the plastic deformation G^P give (knowing that $G^P(p=0)=1$):

$$G^P = e^{-2p}$$

The component \bar{F} gradient of the transformation is given by the resolution of:

$$\bar{F}^3 - \frac{\tau}{\mu G^P} \bar{F} - \frac{1}{(G^P)^{3/2}} = 0$$

The field of displacement \mathbf{u} (in the initial configuration) is form $\mathbf{u} = u_x \mathbf{X} + u_y \mathbf{Y} + u_z \mathbf{Z}$. The components are given by:

$$\begin{aligned} u_x &= \frac{\tilde{u}}{l_o} X & \text{avec } \tilde{u} &= (F - 1) \cdot l_o \\ u_y &= \frac{\tilde{v}}{l_o} Y & \text{avec } \tilde{v} &= \left[\sqrt{\frac{J}{F} - 1} \right] l_o \\ u_z &= \frac{\tilde{v}}{l_o} Z \end{aligned}$$

2.2 Results of reference

One will adopt like results of reference displacements, the constraint of Cauchy σ and cumulated plastic deformation p .

At time $t = 2 \text{ s}$ ($\Delta T = 100 \text{ }^\circ\text{C}$, traction u)

One seeks total displacement (thermal + mechanical) such as the constraint τ that is to say equalizes with:

$\tau = 1500 \text{ MPa}$ (with $T = 120 \text{ }^\circ\text{C}$)

- $3K = 500\,000 \text{ MPa}$ $\mu = 76923 \text{ MPa}$
- $J = 1.03$
- $\sigma = 1453 \text{ MPa}$
- $p = 0,2475$
- $G^P = 0,609$
- $\bar{F} = 1,289$
- $F = 1,303$
- $\tilde{u} = 303 \text{ mm}$
- $\tilde{v} = -110 \text{ mm}$

With these sizes, it is possible to determine the elastic energy of the bar. Attention, the presence of thermics generates a high jacobien, requiring a specific correction as described in R5.03.21. With final, one obtains at the material point: $\Psi_{elas} = 5,6 \text{ MPa}$

2.3 Uncertainty on the solution

The solution is analytical. With the rounding errors near, one can consider it exact.

2.4 Bibliographical references

One will be able to refer to:

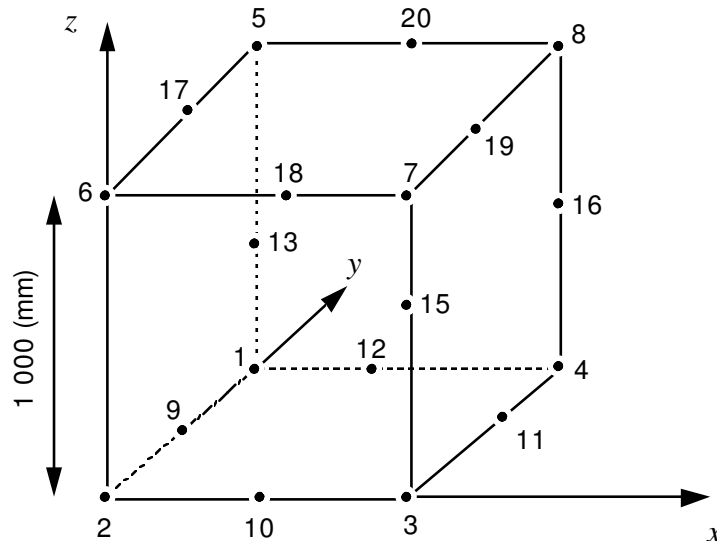
- 1) V. CANO, E. LORENTZ: Introduction into *Code_Aster* of a model of behavior in great deformations elastoplastic with isotropic work hardening - Note interns EDF DER HI - 74/98/006/0

3 Modeling A

3.1 Characteristics of modeling

Voluminal modeling:

1 mesh HEXA20
1 mesh QUAD8



Boundary conditions:

$$\begin{aligned}
 N2 : \quad & U_x = U_y = U_z = 0 & N9, N13, N14, N5, N17 : U_x = 0 \\
 N1 : \quad & U_x = U_z = 0 \\
 N6 : \quad & U_x = U_y = 0
 \end{aligned}$$

Load: Traction on the face [3 4 8 7 1 1 1 6 1 9 1 5] + assignment of the same temperature on all the nodes.

The full number of increments is of 21 (1 increment enters $t=0s$ and $1s$, 20 increments enters $t=1s$ and $2s$)

Convergence is carried out if the residue RESI_GLOB_RELA is lower or equal to 10^{-6} .

3.2 Characteristics of the grid

Many nodes: 20
Many meshes: 2

1 HEXA20
1 QUAD8

3.3 Sizes tested and results

Identification	Reference	Tolerance
$t=2$ Displacement DX ($N8$)	303	1,00%
$t=2$ Displacement DY ($N8$)	- 110	1,00%
$t=2$ Displacement DZ ($N8$)	- 110	1,00%
$t=2$ Constraints $SIGXX$ ($PG1$)	1453	1,00%
$t=2$ Variable P $VARI$ ($PG1$)	0.2475	1.50%
$t=2$ Variable $INDIPLAS$ ($PG1$)	1.	0.10%
$t=2$ ENER_ELAS, TOTAL	5,60E+009	5,00%

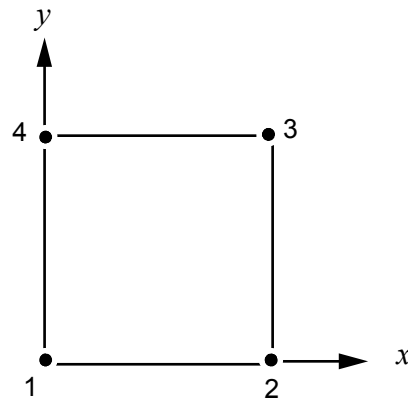
With *INDIPLAS* the indicator of plasticity.

4 Modeling B

4.1 Characteristics of modeling

Modeling 2D axisymmetric:

1 mesh QUAD4
1 mesh SEG2



Boundary conditions:

$$N1 : U_y = 0$$

$$N2 : U_y = 0$$

Loading:

Traction on the face [3 4] (mesh SEG2) + assignment of the same temperature on all the nodes
The full number of increments is of 21 (1 increment enters $t=0s$ and $1s$, 20 increments enters $t=1s$ and $2s$)

Convergence is carried out if the residue RESI_GLOB_REL is lower or equal to 10^{-6} .

4.2 Characteristics of the grid

Many nodes: 4
Many meshes: 2

1 QUAD4
1 SEG2

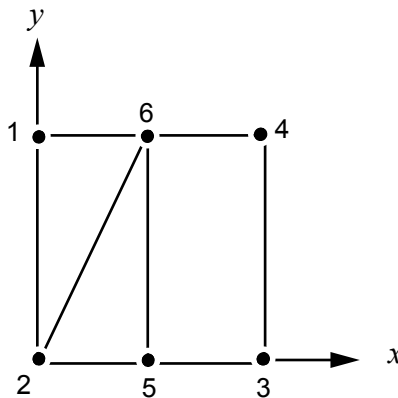
4.3 Sizes tested and results

Identification	Reference	Tolerance
$t=2$ Displacement DX (N3)	- 110	1,00%
$t=2$ Displacement DY (N3)	303	1,00%
$t=2$ Constraints $SIGYY$ (PGI)	1453	1,00%
$t=2$ Variable P $VARI$ (PGI)	0.2475	1,00%
$t=2$ ENER_ELAS, TOTAL	5,60E+009	5,00%

5 Modeling C

5.1 Characteristics of modeling

Modeling plates DKT of thickness 1000 mm : 1 mesh QUAD4, 2 meshes TRIA3
1 mesh SEG2



Boundary conditions:

$$N2 : \quad U_x = 0 \quad U_y = 0 \quad U_z = 0 \quad \theta_x = 0 \quad \theta_y = 0 \quad \theta_z = 0$$

$$N1 : \quad U_x = 0 \quad U_z = 0$$

Loading:

Traction on the face [3 4] (mesh SEG2) + assignment of the same temperature on all the nodes
The full number of increments is of 21 (1 increment enters $t=0s$ and $1s$, 20 increments enters $t=1s$ and $2s$)

Convergence is carried out if the residue RESI_GLOB_REL is lower or equal to 10^{-6} .

5.2 Characteristics of the grid

Many nodes: 8
Many meshes: 4

1 QUAD4
2 TRIA3
1 SEG2

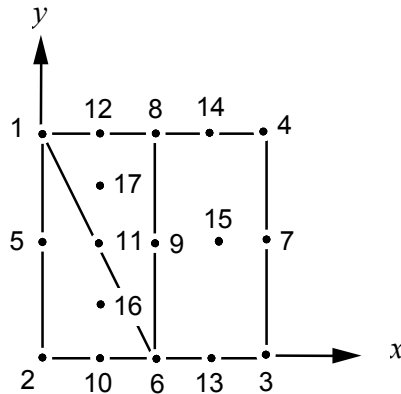
5.3 Sizes tested and results

Identification	Reference	Tolerance
$t=2$ Displacement DX (N3)	- 110	1,50%
$t=2$ Displacement DY (N3)	303	1,00%
$t=2$ Effort NXX (PGI)	1497	1,00%
$t=2$ Effort NYX (PGI)	0,79	1,00%
$t=2$ Effort NXY (PGI)	0,61	1,00%
$t=2$ Variable P $VARI$ (PGI)	0.2475	1,50%

6 Modeling D

6.1 Characteristics of modeling

Modeling COQUE_3D of thickness 1000 mm : 1 mesh QUAD9, 2 meshes TRIA7
1 mesh SEG3



Boundary conditions:

$$N2 : \quad U_x = 0 \quad U_y = 0 \quad U_z = 0 \quad \theta_x = 0 \quad \theta_y = 0 \quad \theta_z = 0$$

$$N5 : \quad U_x = 0 \quad U_z = 0$$

$$N1 : \quad U_x = 0 \quad U_z = 0$$

Loading:

Traction on the face [3 4] (mesh SEG3) + assignment of the same temperature on all the nodes
The full number of increments is of 21 (1 increment enters $t=0s$ and $1s$, 20 increments enters $t=1s$ and $2s$)

Convergence is carried out if the residue RESI_GLOB_REL is lower or equal to 10^{-6} .

6.2 Characteristics of the grid

Many nodes: 17

Many meshes: 4

1 QUAD9

2 TRIA7

1 SEG3

6.3 Sizes tested and results

Identification	Reference	Tolerance
$t=2$ Displacement DX ($N3$)	- 110	1,50%
$t=2$ Displacement DY ($N3$)	303	1,00%
$t=2$ Constraint $SIXX$ (PGI)	1453	3,10%
$t=2$ Variable P $VARI$ (PGI)	0.2475	1,50%

7 Summary of the results

Results found with *Code_Aster* and DEFORMATION: 'SIMO_MIEHE' are very satisfactory with percentages of error lower than 0.4% on the constraint and with 1.2% on the variable of work hardening. For elements of plate and hull the use of DEFORMATION: 'PETIT_REAC' give satisfactory results with percentages of error of 3% on the effort or the constraint and lower than 0.7% on the variable of work hardening.