# HSNV124 - Element of volume in traction and temperature variables

**Summary:**

This test, suggested by the IPSI for the Phi2As day of March 30th, 2000 on the nonlinear behaviors makes it possible to validate the good taking into account of the variation of the coefficients with the temperature for four models of behavior (nonlinear isotropic work hardening, linear kinematic work hardening, and two types of nonlinear kinematic work hardening) in $3D$ .

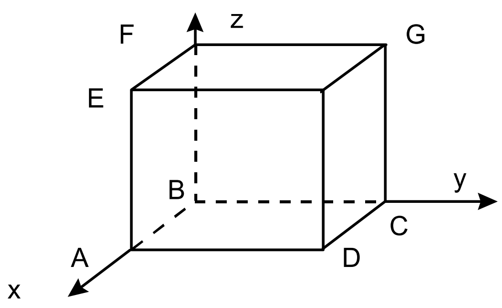
Four modelings make it possible to validate each one of these behaviors.

The reference solution is analytical for the first three behaviors, and the results will be compared with those of the other participants in the Phi2as day for the fourth.

# 1 Problem of reference

## 1.1 Geometry

Element of volume materialized by a unit cube on side:




## 1.2 Properties of materials

$E=2.10^5\,MPa$ , $\nu=0.3$ , $\alpha=10^{-5}\,°C^{-1}$

The material is elastoplastic with various types of behaviors:

$C1$ : Isotropic work hardening: the traction diagram is form:

$\sigma(\varepsilon^P,T)=\sigma_y(T)+Q(T)(1-e^{-b(T)\varepsilon^P})$

$SIGY=200.-1.7.\mathrm{T}$ (in $MPa$ )
$Q(T)=100.+1.7.\mathrm{T}$ (in $MPa$ )
$b(T)=50.+2.\mathrm{T}$

$C2$ : Linear kinematic work hardening:

$\sigma(\varepsilon^P,T)=\pm\sigma_y(T)+C(T)\varepsilon^P$

$SIGY=200.-1.7.\mathrm{T}$ (in $MPa$ )
$C(T)=1000+2990.\mathrm{T}$ (in $MPa$ )

$C3$ : Nonlinear kinematic work hardening ( $I$ ):

$$\sigma(\varepsilon^P,\dot{\varepsilon}^P,T)=\pm\sigma_y(T)+Q(T)\alpha$$

$$\dot{\alpha}=\dot{\varepsilon}^P-D(T)\alpha\left|\dot{\varepsilon}^P\right|$$

$SIGY=200.-1.7.\mathrm{T}$ (in $MPa$ )
$C(T)=(100+1.7.\mathrm{T})(50+$ (in $MPa$ )
$D(T)=50$

$C4$ : Nonlinear kinematic work hardening ( $II$ ):

$$\sigma(\varepsilon^P,\dot{\varepsilon}^P,T)=\pm\sigma_y(T)+Q(T)\alpha$$

$$\dot{\alpha}=\dot{\varepsilon}^P-D(T)\alpha\left|\dot{\varepsilon}^P\right|$$

same characteristics as for the behavior $C3$ , except $D(T)=50+2\mathrm{T}$

## 1.3 Boundary conditions and loadings

Such that the stress and strain state are uniform in the element of volume:
Not $B$ blocked in $x$, $y$ and $z$. Not $A$ blocked in $z$, $DY=0$ on the face $ABFE$
Force distributed on the face $CDHG$: $Fy$
Uniform temperature $T(t)$ on the cube. The temperature of reference is worth $0°C$.
$Fy$ and $T$ vary according to time in the following way:

| moment $t$ | 0 | 1 | 2 |
|---|---|---|---|
| $Fy(t)$ | 0 | $210\,MPa$ | $210\,MPa$ |
| $T(t)$ | 0 | 0 | $100°C$ |

# 2 Reference solution

## 2.1 Method of calculating used for the reference solution [bib1]

### 2.1.1 Isotropic work hardening: analytical

$t=1\text{s}$ **(** $T=0°C$ **)**: $\varepsilon^P = \frac{1}{b}\ln\left(\frac{Q+\sigma_y-\sigma}{Q}\right)$ with $\sigma=210MPa=cste$ that is to say

$\varepsilon^P = 0.21072\%$

**Heating:** Maximum plastic deformation with $t=1.45222\text{s}$ ( $T=45.222°C$ ):

$$\varepsilon^P = \frac{1}{b(T)}\ln\left(\frac{Q(T)+\sigma_y(T)-\sigma}{Q(T)}\right)$$ that is to say

$$\varepsilon^P = 0.48108\%$$

Then: the plastic deformation does not evolve any more.

### 2.1.2 Linear kinematic work hardening: analytical

$t=1\text{s}$ **(** $T=0°C$ **)**: $\varepsilon^P = 1\%$

**Heating:** Constant plastic deformation until $t=356/316=1.12658\text{s}$ ( $T=12.658°C$ ):

Then, the plastic deformation decreases to reach with $t=2\text{s}$ : $\varepsilon^P = 0.08\%$

### 2.1.3 Kinematic work hardening nonlinear I: analytical

$t=1\text{s}$ **(** $T=0°C$ **)**: $\varepsilon^P = \frac{1}{D}\ln\left(\frac{A+\sigma_y-\sigma}{A}\right)$ with $A=\frac{C}{D}=100$ that is to say

$\varepsilon^P = 0.21072\%$

**Heating:** Maximum plastic deformation with $t=1.26011\text{s}$ ( $T=26.011°C$ ):

$$\varepsilon^P = \frac{1}{D}\ln\left(\frac{A(T)+\sigma_y(T)-\sigma}{A(T)}\right)$$ that is to say

$$\varepsilon^P = 0.40729\% = \varepsilon_0^p$$

The plastic deformation does not evolve any more until $t_1=1.98332\text{s}$ ( $T_1=98.332°C$ ) where one meets the other end of the field of elasticity.

Then: the plastic deformation decreases to reach with $t=2\text{s}$ :

$$\varepsilon^P = \varepsilon_0^p + \frac{1}{D}\ln\left(\frac{A(T)+\sigma_y(T)-\sigma}{X_0(T_1)+A(T)}\right)$$ with $X_0(T_1)=A(T_1)\left(1-e^{-D\varepsilon_0^p}\right)$ that is to say

$\varepsilon^P = 0.4037229\%$

### 2.1.4 Nonlinear kinematic work hardening II

Comparison with the reference solution suggested to the day $\Phi^2 As$ . (digital result got with 10 pas de time), and comparison with the results got with *Code_Aster* with a very fine discretization in time

| Plastic deformation $YY$ | Calculation fine Aster: 100 pas until $1.26\text{s}$ , 100 pas between 1.98 and 2s | Reference $\Phi^2 As$ : Result for 10 pas |
|---|---|---|
| $t=1\text{s}$ | 2.1072 $10^{-03}$ | 2.1072 $10^{-03}$ |
| $1.26\text{s}<t<1.98\text{s}$ | 4.18947 $10^{-03}$ | 4.38 $10^{-03}$ |
| $t=2\text{s}$ | 4.12131 $10^{-03}$ | 4.32 $10^{-03}$ |

## 2.2 Precision on the results of reference

There is an analytical solution for the first three behaviors, uncertainty is thus worthless. It is estimated at $4\%$ for the fourth (variation enters the result for 10 pas and that for 200 pas, the solution strongly depending on the temporal discretization).

## 2.3 References bibliographical

1) IPSI: day of Phi2AS study of March 30th, 2000 on the nonlinear behaviors of materials.

# 3 Modeling A

## 3.1 Characteristics of modeling

Behavior $C1$ : isotropic work hardening, in $3\mathrm{D}$ . It is modelled in two ways:

- maybe using the behavior VMIS_ISOT_TRAC, with traction diagrams given all them $10°C$, and interpolated for each temperature, which can be vague,
- maybe using the behavior VMIS_CIN1_CHAB, by cancelling nonlinear kinematic work hardening, and by keeping only the isotropic work hardening which is precisely expressed in the form:

$$\sigma(\varepsilon^P, T) = \sigma_y(T) + Q(T)\left(1 - e^{-b(T)\varepsilon^P}\right)$$

It is enough to take then:

$$R_0 = SIGY = 200. \text{-1.7.T}$$
$$R_1 = SIGY + Q(T) = 200. - 1.7.\mathrm{T} + 100. + 1.7.\mathrm{T} = 300$$
$$C_1 = G_0 = 0$$

1 pas de time enters $t = 0\mathrm{s}$ and $t = 1\mathrm{s}$ and 20 pas de time enters $t = 1\mathrm{s}$ and $t = 2\mathrm{s}$ .

## 3.2 Characteristics of the grid

The grid comprises a mesh HEXA8.

## 3.3 Sizes tested and results

Behavior VMIS_ISOT_TRAC :

| Moment ( $s$ ) | Plastic deformation according to $Y$ | Reference | Aster | $\%$ difference |
|---|---|---|---|---|
| $t = 1.$ | $EPYY$ | 2.1072 $10^{-03}$ | 2.1096 $10^{-03}$ | 0.1 |
| $t1 = 1.45$ | $EPYY$ | 4.8108 $10^{-03}$ | 4.8135E-03 | 0,056 |
| 2 | $EPYY$ | 4.8108 $10^{-03}$ | 4.8135E-03 | 0,056 |

Behavior VMIS_CIN1_CHAB :

| Moment ( $s$ ) | Plastic deformation according to $Y$ | Reference | Aster | $\%$ Difference |
|---|---|---|---|---|
| $t1 = 1.$ | $EPYY$ | 2.1072 $10^{-03}$ | 2.1096 $10^{-03}$ | 0.1 |
| $t1 = 1.45$ | $EPYY$ | 4.8108 $10^{-03}$ | 4.81079 $10^{-03}$ | 0.0001 |
| 2 | $EPYY$ | 4.8108 $10^{-03}$ | 4.81079 $10^{-03}$ | 0.0001 |

# 4 Modeling B

## 4.1 Characteristics of modeling

Behavior $C2$ : linear kinematic work hardening, in $3D$ . It is modelled in three ways:

- maybe using the behavior VMIS_CINE_LINE, while taking: $D\_SIGM\_EPSI = E.C(T)/(E+C(T))$ with $C(T)=(1000+2990.T)$
- maybe using the behavior VMIS_ECMI_LINE, while taking: $D\_SIGM\_EPSI = E.C(T)/(E+C(T))$ and the constant of Prager $PRAG=2/3\,C(T)$
- maybe using the behavior VMIS_CIN1_CHAB, by keeping only linear kinematic work hardening: It is enough to take then: $R_0=R_I=SIGY$ , $b=0$ , $C_I=C(T)$ , $G_0=0$

Temporal discretization: 1 pas de time enters $t=0\text{s}$ and $t=1\text{s}$ and 20 pas de time enters $t=1\text{s}$ and $t=2\text{s}$ .

## 4.2 Characteristics of the grid

The grid comprises a mesh HEXA8.

## 4.3 Sizes tested and results

Behavior VMIS_CINE_LINE :

| **Moment ( $s$ )** | **Plastic deformation according to $Y$** | **Reference** | ***Aster*** | **% difference** |
|---|---|---|---|---|
| $t1=1.1$ | $EPYY$ | 0.01 | 0.01 | 0. |
| 2 | $EPYY$ | 8.E-4 | 8.E-4 | 0 |

Behavior VMIS_ECMI_LINE :

| **Moment ( $s$ )** | **Plastic deformation according to $Y$** | **Reference** | ***Aster*** | **% difference** |
|---|---|---|---|---|
| $t1=1.1$ | $EPYY$ | 0.01 | 0.01 | 0. |
| 2 | $EPYY$ | 8.E-4 | 8.E-4 | 0 |

Behavior VMIS_CIN1_CHAB :

| **Moment ( $s$ )** | **Plastic deformation according to $Y$** | **Reference** | ***Aster*** | **% difference** |
|---|---|---|---|---|
| $t1=1.1$ | $EPYY$ | 0.01 | 0.01 | 0. |
| 2 | $EPYY$ | 8.E-4 | 8.E-4 | 0 |

# 5 Modeling C

## 5.1 Characteristics of modeling

Behavior $C3$ : nonlinear kinematic work hardening ( $I$ ) in $3\mathrm{D}$ . It is modelled in two ways:

- maybe using the behavior VMIS_CIN1_CHAB. It is enough to take then:
  $R_0 = R_I = SIGY$ , $b=0$ , $C_I = C(T) = (100+1.7.\mathrm{T})(50+2.\mathrm{T})$ , $G_0 = 50$
- maybe using the behavior VMIS_CIN2_CHAB, by choosing the parameters in such way that two variable kinematics are identical: It is enough to take then:
  $R_0 = R_I = SIGY$ , $b=0$ , $C1_I = C2_I = C(T)/2$ , $G1_0 = G2_0 = 50$

Temporal discretization: 20 pas de time enters $t=0\mathrm{s}$ and $t=1\mathrm{s}$ and 60 pas de time enters $t=1\mathrm{s}$ and $t=2\mathrm{s}$ .

## 5.2 Characteristics of the grid

The grid comprises a mesh HEXA8.

## 5.3 Sizes tested and results

Behavior VMIS_CIN1_CHAB :

| **Moment (S)** | **Plastic deformation according to** $Y$ | **Reference** | ***Aster*** | **% difference** |
|---|---|---|---|---|
| 1 | $EPYY$ | 2.1072 $10^{-03}$ | 2,113 $10^{-03}$ | 0.27 |
| 1.26 | $EPYY$ | 4.0729 $10^{-03}$ | 4.0875 $10^{-03}$ | 0.36 |
| 1.98 | $EPYY$ | 4.0729 $10^{-03}$ | 4.0875 $10^{-03}$ | 0.36 |
| 2 | $EPYY$ | 4.0372 $10^{-03}$ | 3,978 $10^{-03}$ | 1.46 |

Behavior VMIS_CIN2_CHAB :

| **Moment (S)** | **Plastic deformation according to** $Y$ | **Reference** | ***Aster*** | **% difference** |
|---|---|---|---|---|
| 1 | $EPYY$ | 2.1072 $10^{-03}$ | 2,113 $10^{-03}$ | 0.27 |
| 1.26 | $EPYY$ | 4.0729 $10^{-03}$ | 4.0875 $10^{-03}$ | 0.36 |
| 1.98 | $EPYY$ | 4.0729 $10^{-03}$ | 4.0875 $10^{-03}$ | 0.36 |
| 2 | $EPYY$ | 4.0372 $10^{-03}$ | 3,978 $10^{-03}$ | 1.46 |

## 5.4 Notice

The variation with the reference solution comes from the temporal discretization. While refining more, the solution approaches the analytical solution. One chose a compromise between a reasonable temporal discretization in time calculation and nevertheless rather precise.

# 6 Modeling D

## 6.1 Characteristics of modeling

Behavior $C4$ : nonlinear kinematic work hardening ( $II$ ) in $3\mathrm{D}$ . It is modelled in two ways:

- maybe using the behavior VMIS_CIN1_CHAB. It is enough to take then:
  $R_0 = R_I = SIGY$ , $b=0$ , $C_I = C(T) = (100+1.7.\mathrm{T})(50+2.\mathrm{T})$ , $G_0 = D(T)$
- maybe using the behavior VMIS_CIN2_CHAB, by choosing the parameters in such way that two variable kinematics are identical: It is enough to take then:
  $R_0 = R_I = SIGY$ , $b=0$ , $C1_I = C2_I = C(T)/2$ , $G1_0 = G2_0 = D(T)$

Temporal discretization: 40 pas de time enters $t=0\mathrm{s}$ and $t=1\mathrm{s}$ and 30 pas de time enters $t=1\mathrm{s}$ and $t=2\mathrm{s}$ .

## 6.2 Characteristics of the grid

The grid comprises a mesh HEXA8.

## 6.3 Sizes tested and results

Behavior VMIS_CIN1_CHAB :

| Moment (S) | Plastic deformation according to $Y$ | Reference | *Aster* | % difference |
|---|---|---|---|---|
| 1 | $EPYY$ | $2.1072\ 10^{-03}$ | $2.11\ 10^{-03}$ | 0.14 |
| 1.26 | $EPYY$ | $4.18947\ 10^{-03}$ | $4{,}231\ 10^{-03}$ | 0.99 |
| 1.98 | $EPYY$ | $4.18947\ 10^{-03}$ | $4{,}231\ 10^{-03}$ | 0.99 |
| 2 | $EPYY$ | $4.12131\ 10^{-03}$ | $4{,}163\ 10^{-03}$ | 1.00 |

Behavior VMIS_CIN2_CHAB :

| Moment (S) | Plastic deformation according to $Y$ | Reference | *Aster* | % difference |
|---|---|---|---|---|
| 1 | $EPYY$ | $2.1072\ 10^{-03}$ | $2.11\ 10^{-03}$ | 0.14 |
| 1.26 | $EPYY$ | $4.18947\ 10^{-03}$ | $4{,}231\ 10^{-03}$ | 0.99 |
| 1.98 | $EPYY$ | $4.18947\ 10^{-03}$ | $4{,}231\ 10^{-03}$ | 0.99 |
| 2 | $EPYY$ | $4.12131\ 10^{-03}$ | $4{,}163\ 10^{-03}$ | 1.00 |

## 6.4 Notice

The variation with the reference solution (obtained for a very fine temporal discretization) comes from the temporal discretization. One chose here a compromise between a reasonable temporal discretization in time calculation and nevertheless rather precise.

# 7 Summary of the results

This test makes it possible to highlight the effects of variation of the coefficients of the elastoplastic behaviors with the temperature.

The results are identical to the analytical solution for the linear kinematic work hardening (where the solution does not depend on the temporal discretization). For the other behaviors, the precision is less good (lower deviation than $1.5\%$ ) because the solution strongly depends on the selected temporal discretization.

This test thus makes it possible to validate the integration of the behaviors VMIS_ISOT_TRAC, VMIS_CINE_LINE, VMIS_ECMI_LINE, VMIS_CIN1_CHAB, and VMIS_CIN2_CHAB compared to the variation of the coefficients with the temperature.