HSNV132 - Crack X-FEM in thermoelasticity

Summary

The purpose of this test is to validate the taking into account of a thermal loading for a calculation of cracking by method X-FEM [bib1] on an academic case $2D/3D$.

This test brings into play a square plate with a right crack leading, embedded to the lower edge, and subjected to a horizontal variation in temperature. This loading causes to open the crack. One compares displacement for the node end in top with right, for FEM and X-FEM. This test comprises a call to `THER_LINEAIRE`, then `MECA_STATIQUE`.

Four modelings are considered:

- modeling A : $\mathrm{FEM\,2D}$ (taken as reference),
- modeling B : $\mathrm{X\text{-}FEM\,2D}$, crack in the middle of the elements,
- modeling C : $\mathrm{X\text{-}FEM\,3D}$ (one blocks displacements according to z), crack in the middle of the elements,
- modeling D : $\mathrm{X\text{-}FEM\,3D}$, modeling in truth $3\mathrm{d}$, without putting $DZ=0$, to validate the calculation of G in $3\mathrm{D}$.
- modeling E : $\mathrm{X\text{-}FEM\,3D}$, modeling in truth $3\mathrm{d}$, without putting $DZ=0$, to validate the calculation of G in $3\mathrm{D}$. Similar to modeling D except the fact that the surfaces upper and lower were embedded to induce important constraints.

One finds a lower deviation than 1% for displacement, and a variation of $3,5\%$ on K_I and $1,4\ \%$ on K_{II}.

1 Problem of reference

1.1 Geometry

The structure 2d is a unit square ($LX=1\,m$, $LY=1\,m$), comprising an on the right emerging right crack, located at middle height. [Figure 1.1-1]. One calls the line of left the line in $x=0$, the line of right-hand side the line in $x=LX$ and the line the lower line in $y=0$.

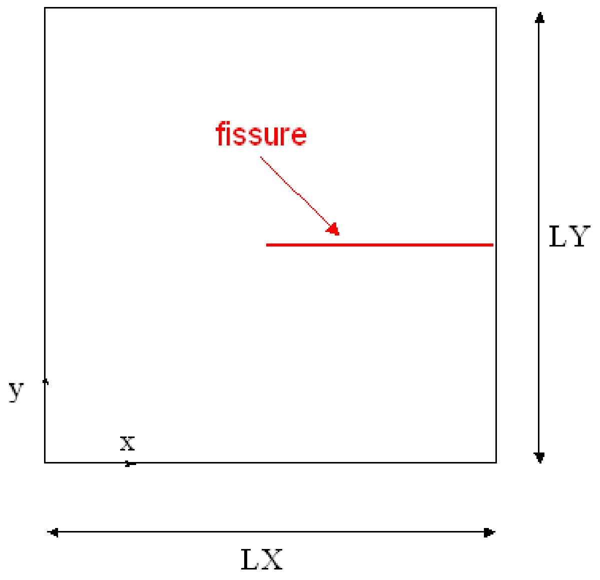

Figure 1.1-1 : geometry of the fissured square plate

1.2 Properties of material

Young modulus: $E=205000\,MPa$
Poisson's ratio: $\nu=0.3$
Thermal dilation coefficient: $\alpha=1{,}282\,10^{-5}$
Thermal conductivity: $\lambda=1\,W.m^{-1}.K^{-1}$
Voluminal heat-storage capacity: $\rho C_p=0\,J.m-3.\mathrm{K}^{-1}$

1.3 Boundary conditions and loadings

1.3.1 For the thermal part

The thermal loading consists in applying an imposed temperature $T=20\,°C$ on the nodes of the line of left and an imposed temperature $T=220\,°C$ on the nodes of the line of right-hand side. The variation in temperature is uniform and opens the crack in mixed mode.

1.3.2 For the mechanical part

The nodes of the lower line are embedded.

1.4 Bibliography

1. GENIAUT S., MASSIN P.: eXtended Finite Method Element, Handbook of reference of *Code_Code_Aster*, [R7.02.12]
2. CRYSTAL E., PROIX J.M.: Calculation of the stress intensity factors, [R7.02.08].

2 Modeling a: fissures with a grid in dimension 2

In this modeling, one considers the structure in $2D$. The classical finite element method is used. This modeling is used as reference for the continuation.

2.1 Characteristics of the grid

The structure is modelled by a regular grid composed of 100×100 QUAD8, respectively along the axes x, y. The crack is with a grid.

2.2 Sizes tested and results

One tests the values of following displacement X and Y point end 'PTEXTR' coordinates $(1;1)$.

One tests also the value of the stress intensity factor K_I data by CALC_G, option CALC_K_G like that given by K_I of POST_K1_K2_K3 [bib2].

Finally, it is tested G given by CALC_G, option CALC_G.

All these tests are tests of not-regression and will be used as reference for following modelings.

Identification	*Code_Aster*
DX (PTEXTR)	- 8.7404263 10^{-4}
DY (PTEXTR)	3.826096 10^{-3}
K1 (CALC_G/CALC_K_G)	9.0328413 10^{6}
K1 (POST_K1_K2_K3)	8.4543655 10^{6}
G (CALC_G/CALC_G)	492,82

3 Modeling b: fissures not-with a grid in dimension 2

In this modeling, one considers the structure in $2D$. The wide finite element method (X-FEM) is used.

This modeling utilizes two command files (hsnv132b.comm and hsnv132b.com1). In each file, one models the same problem strictly, but by a different strategy (only at ends of validation). Sizes tested, as well as the values of nonregression are identical of one file to the other.

file hsnv132b.comm:

Elements X-FEM play a part only on the level it calculation mechanical to represent the discontinuity of displacement through the crack. For the thermal part, the temperature is calculated on a healthy thermal model, the temperature is in fact continuous through the lips of the crack.

file hsnv132b.com1:

Elements X-FEM play a part on the level of thermal calculation and the level of mechanical calculation. For the thermal part, one imposes a continuous temperature through the interface (via AFFE_CHAR_THER / ECHANGE_PAROI / TEMP_CONTINUE = 'YES'). One thus models the same problem as in the preceding command file, but different discretization.

3.1 Characteristics of the grid

The structure is modelled by a regular grid composed of 101×101 QUAD4, respectively along the axes x , y . The crack is not with a grid.

3.2 Sizes tested and results

One tests the values of following displacement X and Y point end 'PTEXTR' coordinates $(1,1)$.

One tests also the value of the stress intensity factor K_I data by CALC_G, option CALC_K_G like that given by K_I of POST_K1_K2_K3.

Finally, it is tested G given by CALC_G, option CALC_G.

The values of reference are those obtained by modeling A.

Identification	*Type of reference*	Reference	% Tolerance
DX (PTEXTR)	AUTRE_ASTER	$-8.740426\ 10^{-4}$	2.0
DY (PTEXTR)	AUTRE_ASTER	$3.826096\ 10^{-3}$	1.0
K1 (CALC_G/CALC_K_G)	AUTRE_ASTER	$9.0328413\ 10^{6}$	2.0
K1 (POST_K1_K2_K3)	AUTRE_ASTER	$8.4543655\ 10^{6}$	4.0
G (CALC_G/CALC_G)	AUTRE_ASTER	492,82	2.0

4 Modeling C: crack not-with a grid in false 3D

In this modeling, one considers the structure in $3\mathrm{d}$, but all degrees of freedom according to z (not only displacements) are put at zero to be reduced to the case $2\mathrm{d}$. Wide finite element method ($\mathrm{X\text{-}FEM}$) is used.

This modeling utilizes two command files (hsnv132c.comm and hsnv132c.com1). In each file, one models the same problem strictly, but by a different strategy (only at ends of validation). Sizes tested, as well as the values of nonregression are identical of one file to the other.

file hsnv132c.comm:

Elements X-FEM play a part only on the level it calculation mechanical to represent the discontinuity of displacement through the crack. For the thermal part, the temperature is calculated on a healthy thermal model, the temperature is in fact continuous through the lips of the crack.

file hsnv132c.com1:

Elements X-FEM play a part on the level of thermal calculation and the level of mechanical calculation. For the thermal part, one imposes a continuous temperature through the interface (via AFFE_CHAR_THER / ECHANGE_PAROI / TEMP_CONTINUE = 'YES'). One thus models the same problem as in the preceding command file, but different discretization.

4.1 Characteristics of the grid

The structure is modelled by a regular grid composed of $11 \times 11 \times 1$ HEXA8, respectively along the axes x , y and z . The crack is not with a grid.

4.2 Boundary conditions and loadings

To be reduced to the case $2\mathrm{D}$, it is necessary to block all the following degrees of freedom z .
To block displacements according to z is not enough, the degrees of freedom nouveau riches have a strong importance. It is thus necessary to impose $DZ=0$ on all the nodes, and to impose too $H1Z=0$ on the nodes nouveau riches by Heaviside and $E1Z=E2Z=E3Z=E4Z=0$ on all the nodes nouveau riches by the asymptotic functions.

4.3 Sizes tested and results

One tests the values of following displacement X and Y point end 'PTEXTR' coordinates $(1,1)$.

One does not test the value of the rate of refund of energy G data by CALC_G nor that of the stress intensity factor K_I data by K_I of POST_K1_K2_K3 because the fact of forcing displacements according to Z is not in accordance with $2\mathrm{D}$ from the energy point of view.

The values of reference are those obtained by modeling A.

Identification	*Type of reference*	Reference	% tolerance
DX (PTEXTR)	AUTRE_ASTER	$-8.740426\ 10^{-4}$	2
DY (PTEXTR)	AUTRE_ASTER	$3.826095\ 10^{-3}$	1

5 Modeling D: crack not-with a grid in true 3D

In this modeling, one considers the structure in $3D$. Wide finite element method ($X\text{-}FEM$) is used.

This modeling utilizes two command files (hsnv132d.comm and hsnv132d.com1). In each file, one models the same problem strictly, but by a different strategy (only at ends of validation). Sizes tested, as well as the values of nonregression are identical of one file to the other.

file hsnv132d.comm:

Elements X-FEM play a part only on the level it calculation mechanical to represent the discontinuity of displacement through the crack. For the thermal part, the temperature is calculated on a healthy thermal model, the temperature is in fact continuous through the lips of the crack.

file hsnv132d.com1:

Elements X-FEM play a part on the level of thermal calculation and the level of mechanical calculation. For the thermal part, one imposes a continuous temperature through the interface (via AFFE_CHAR_THER / ECHANGE_PAROI / TEMP_CONTINUE = 'YES'). One thus models the same problem as in the preceding command file, but different discretization.

5.1 Characteristics of the grid

The structure is modelled by a regular grid composed of $31\times31\times10$ HEXA8, respectively along the axes x , y and z . The crack is not with a grid.

5.2 Sizes tested and results

One tests the value of the stress intensity factors G given by CALC_G and CALC_K_G on item 5 of the bottom of the crack. These values are identical.

Identification	*Type of reference*	Values of référenece
G	'NON_REGRESSION'	231.412800803

Note:
1. It is noted that the bar is quasi-free of thermal dilation.
2. It is noted that G_IRWIN is very different from G calculated by CALC_G. It is a purely thermal problem. The fact that one uses the singular solutions of a thermomechanical problem involved values of K bad. However, the values of G is right. See the documents R for more information.
3. It is noted that the values of G obtained with option CALC_G and CALC_K_G are identical.

6 Modeling E: crack not-with a grid in 3D with 2 embedded surfaces

In this modeling, one considers the structure in $3D$ identical to modeling D suuf the fact that the 2 surfaces upper and lower were embedded to induce important constraints. Wide finite element method ($X\text{-}FEM$) is used.

This modeling utilizes two command files (hsnv132e.comm and hsnv132e.com1). In each file, one models the same problem strictly, but by a different strategy (only at ends of validation). Sizes tested, as well as the values of nonregression are identical of one file to the other.

file hsnv132e.comm:

Elements X-FEM play a part only on the level it calculation mechanical to represent the discontinuity of displacement through the crack. For the thermal part, the temperature is calculated on a healthy thermal model, the temperature is in fact continuous through the lips of the crack.

file hsnv132e.com1:

Elements X-FEM play a part on the level of thermal calculation and the level of mechanical calculation. For the thermal part, one imposes a continuous temperature through the interface (via AFFE_CHAR_THER / ECHANGE_PAROI / TEMP_CONTINUE = 'YES'). One thus models the same problem as in the preceding command file, but different discretization.

6.1 Characteristics of the grid

The structure is modelled by a regular grid composed of $31\times31\times10$ HEXA8, respectively along the axes x , y and z . The crack is not with a grid.

6.2 Sizes tested and results

One tests the value of the stress intensity factors G given by CALC_G and CALC_K_G on item 5 of the bottom of the crack. These values are identical. One thus tests the values of the factors of intensity of the constraints.

Identification	*Type of reference*	Values of référenece
G obtained with CALC_G/CALC_G	'NON_REGRESSION'	393366.055819
G obtained with CALC_G/CALC_G/	'NON_REGRESSION'	393366.055819
K_I obtained with CALC_G/CALC_G/	'NON_REGRESSION'	-310148521.794
K_II obtained with CALC_G/CALC_G/	'NON_REGRESSION'	-16044.9748463
K_III obtained with CALC_G/CALC_G/	'NON_REGRESSION'	884808.766742

Note:

1. It is noted that G_IRWIN is not very different from G calculated by CALC_G. The fact that one uses the singular solutions of a thermomechanical problem involved approximate values of K. However, the values of G is right. See the documents R for more information.
2. It is noted that the values of G obtained with option CALC_G and CALC_K_G are identical.

7 Summaries of the results

The goals of this test are achieved:

- to validate on a simple case the taking into account of the temperature on mechanical calculation with X-FEM ,
- to validate the calculation of the stress intensity factors for the elements X-FEM , in particular terms related to thermics.