WTNL100 - Consolidation of a column of ground saturated poro-rubber band (Terzaghi)

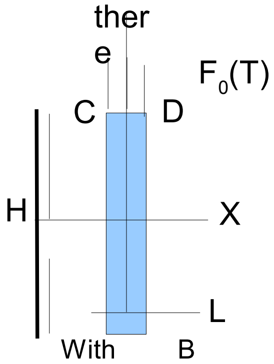
Summary:

This CAS-test relates to the consolidation of a column of ground saturated poro-rubber band, seals laterally and at the base, subjected to a level of force at the head. The objective is to test displacements (compressing of the ground), the pressures and to compare the whole of the results with an analytical solution, whose broad outlines are presented.

1 Problem of reference

1.1 Geometry

This unidimensional CAS-test is resulting from the literature and has an analytical solution [1]. It is inspired by the problem of consolidation of a porous ground saturated with water to room temperature (problem of Terzaghi). Water is supposed to be incompressible ($1/K_{lq}=0{,}0$). The temperature is uniform. The action of gravity in this case is neglected. The problem coupled poro-mechanics saturated is described by the variables of displacements u_y (compressing of the ground) and of pressure of fluid P_{lq} (the hydraulic head being P_{lq}/ρ_{eau}). In order to obtain a purely unidimensional solution, the Poisson's ratio is selected equal to $\nu_0=0{,}0$. Dimensions are the following ones: $L=1{,}00\,m$, $H=10{,}00\,m$.



Drained Young modulus: $E_0=10\,MPa$	Intrinsic permeability: $K_{intr}=10\text{x}10^{-8}$
Poisson's ratio: $\nu_0=0{,}0$	Density of the fluid: $\rho_{lq}=1000\,kg/m^3$
Density: $r_0=2800\,kg/m^3$	Porosity: $\phi^0=0{,}5$
Coefficient of Biot: $b=1{,}0$	Dynamic viscosity of water: $\mu_{lq}(T)=1$
Saturation $S_{lq}(p_c)=1{,}0$	Permeability relating to the fluid: $k_{lq}^{rel}(S_{lq})=1$

The characteristics of behavior and thermal coupling are not significant.

The hydraulic permeability of the medium to water is then: $\lambda_{lq}^{H}=\dfrac{K_{intr}(\varphi).\,k_{lq}^{rel}(S_{lq})}{\mu_{lq}(T)}$ in ($m^3\,s/kg$).

In soil mechanics, it is often noted the permeability by $k=\lambda_{lq}^{H}\,\rho_{lq}\,g$, that is to say here: $k\approx 10^{-14}\,m/s$.

The coefficient of consolidation $c_\nu=\lambda_{lq}^{H}E_0/b^2$ is worth here: $c_\nu=0{,}1\,m^2/s$.

1.2 Boundary conditions and loadings

1.2.1 Boundary conditions

The pressure of interstitial fluid remains worthless on all the higher face CD : $P_{lq}=0$. The side faces have the displacements blocked in x . The lower face AB has the displacements blocked in x and in y , and it is tight: $P_{lq,y}=0$

1.2.2 Initial conditions

The column is initially at rest in a virgin state: $P_{lq}=0$, $\sigma_{yy}=0$.

1.2.3 Loading

He is exerted a level of pressure $F_0=-1,0\,Pa$ on the face higher CD than $t=0\mathrm{s}$.
Gravity is neglected here.

2 Reference solution

The total constraints poro-rubber bands are: $\sigma=E_0\varepsilon(u)-bP_{lq}$. Only the vertical component is present: $\sigma_{yy}(y,t)=E_0 u_{y,y}(y,t)-bP_{lq}(y,t)$.

Hydro-mechanical balance poro-rubber band $1\mathrm{D}$ is thus written, in the absence of force of gravity, for $t\geq 0$:

$$\begin{cases} E_0 u_{y,yy}(y,t)-bP_{lq,y}(y,t)=0\text{, équilibre mécanique} \\ \lambda_{lq}^H P_{lq,yy}(y,t)-b\dot{u}_{y,y}(y,t)=0\text{, équilibre hydraulique} \end{cases}$$

with the initial conditions: $u_y(y,0)=0$, $P_{lq}(y,0)=0$ and boundary conditions for $t>0$: $u_y(0,t)=0$, $P_{lq}(0,t)=0$, $\sigma_{yy}(H,t)=F_0\eta(t)=E_0 u_{y,y}(H,t)-bP_{lq}(H,t)$ where $\eta(t)$ is the function level in $t=0$ (Heaviside).

Mechanical balance directly gives the uniformity of the total constraints for $t>0$ on $|0,H|$, from where: $\sigma_{yy}(y,t)=F_0=E_0 u_{y,y}(y,t)-bP_{lq}(y,t)$, that is to say $u_{y,y}(y,t)=\frac{1}{E_0}(F_0+bP_{lq}(y,t))$, for $t>0$ on $|0,H|$.

Hydraulic balance leads then to:
$\frac{b^2}{E_0}\dot{P}_{lq}(y,t)-\lambda_{lq}^H P_{lq,yy}(y,t)=0$, for $t>0$ on $|0,H|$

with like initial conditions $P_{lq}(y,0)=-\frac{F_0}{b}$ on $|0,H|$, and two boundary conditions $P_{lq}(H,t)=0$ and $P_{lq,y}(0,t)=0$ for $t>0$, i.e. a problem of the standard thermal shock on $|0,H|$.
The coefficient of consolidation $c_\nu=\lambda_{lq}^H E_0/b^2$ is worth here: $c_\nu=0,1\,m^2/s$. It controls the duration of the process of consolidation.

It results from this thus a characteristic time $\tau_c=H^2/c_\nu$ ($=1000\mathrm{s}$ here) being used to identify the step of the temporal discretization for the digital method of integration. With this value τ_c , it was reached a little more than 90% of the consolidation.

The solution is, cf [2]:

$$P_{lq}(y,t)=\frac{-4F_0}{\pi b}\sum_{m=1,2,3..}^{+\infty}\frac{(-1)^{m-1}}{2\mathrm{m}-1}e^{-\lambda_{lq}^H E_0\pi^2(2\mathrm{m}-1)^2\frac{t}{(4\mathrm{b}^2 H^2)}}.\cos(\frac{\pi y(2\mathrm{m}-1)}{2\mathrm{H}})$$

and: $u_y(y,t)=\frac{F_0 y}{E_0}+\frac{b}{E_0}\int_0^y P_{lq}(\xi,t)d\xi$

that is to say: $u_y(y,t)=\frac{F_0 y}{E_0}-\frac{8HF_0}{\pi^2 E_0}\sum_{m=1,2,3..}^{+\infty}\frac{(-1)^{m-1}}{(2\mathrm{m}-1)^2}e^{-\lambda_{lq}^H E_0\pi^2(2\mathrm{m}-1)^2\frac{t}{(4\mathrm{b}^2 H^2)}}.\sin(\frac{\pi y(2\mathrm{m}-1)}{2\mathrm{H}})$

The effective constraints (acting on the skeleton) are: $\sigma_{yy}^{eff}(y,t)=E_0 u_{y,y}(y,t)$. For moments $t\to\infty$, we obtain: $P_{lq}(0,\infty)=0$ and $u_y(H,\infty)=\frac{F_0 H}{E_0}$ (either here $-10^{-6} m$).

2.1 Uncertainties on the solution

The reference solution is analytical.

2.2 Influence of the choice of modeling

In this paragraph, one wishes to draw attention to a difficulty of digital modeling related to this kind of test (column of Terzaghi). In what follows one thus discusses the influence of modeling selected and one is based in particular on the calculations carried out with the code Lagamine (University of Liege) which has elements different from those from Code_Aster. For this reason, the test presented in this paragraph comes from an external study and is not quantitatively exactly the same one as that studied in the rest of the page. It thus does not give the same reference solutions. This paragraph thus constitutes a complement given on a purely informative basis.

At the moment $t=0$, solution is discontinuous at free surface $y=H$:

$$\begin{cases} \frac{P_{lq}(y,0)}{F_0}=1 & \forall y< H \\ P_{lq}(H,0)=0 \end{cases}$$

Because of this discontinuity, this test is an academic case highlighting the digital phenomena of oscillations of the water pressure related to the use of a mixed formulation by the finite element method. These oscillations generally occur in the vicinity of a draining wall, and are due to:

- the violation of the condition inf-sup;
- choice of a step of too small time, violating the principle of the maximum [**R3.06.07**];

The choice of the type of finite elements has a considerable influence on the solution obtained in the vicinity of free surface, around $t=0$. In Code_Aster, selective modeling (HMS) allows to eliminate these oscillations (**Figure 2.2-a**), contrary to classical modeling (HM).

However, obtaining exact solution for the water pressure $P_{lq}(y,0)$ by modeling HMS results in a larger approximation (compared to modeling HM, to see the case of the fine grid of **Figure 2.2-b**) for solution in vertical displacement $u_y(y,0)$. This approximation is all the more large as the grid is coarser around free surface.

Lastly, it is noticed that the displacement calculated by the code Lagamine (University of Liege) is perfectly exact and independent of the smoothness of the grid, but with on the other hand a solution in very oscillating water pressure [**HT66-05-012-A**]. That is of with the type of finite elements used by Lagamine which are different as of ours since it is about a P2P2 with a squaring 9PG/9PG. This element does not check the condition inf-sup and cannot thus give a correct solution in water pressure.

2.3 Bibliographical references

1. L.MEIROVITCH: *Analytical methods in vibrations*. McMillan ED., 1967.

2. J.J.MARIGO, E.PLANCHAIS: *Introduction to the asymptotic methods. Application to linear thermal problems*. Note EDF/DER/IMA/MMN HI-70/7563, 8/31/1992.

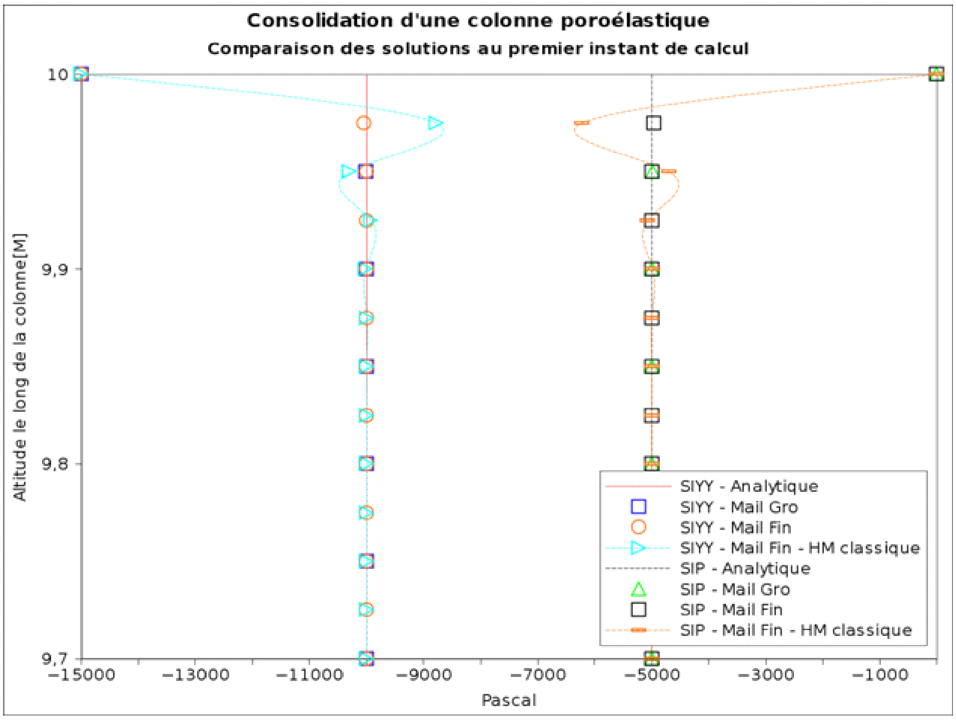


Figure 2.2-a: Comparison of the solutions in vertical effective constraints (SIYY) and in water pressure (SIP) obtained with the first step of calculation in the vicinity of free surface (according to a vertical cut) for various smoothnesses of grid and modelings selective (HMS) and classic (HM) in the case of "fine" grid. Comparison with the analytical solutions with $t=0$.

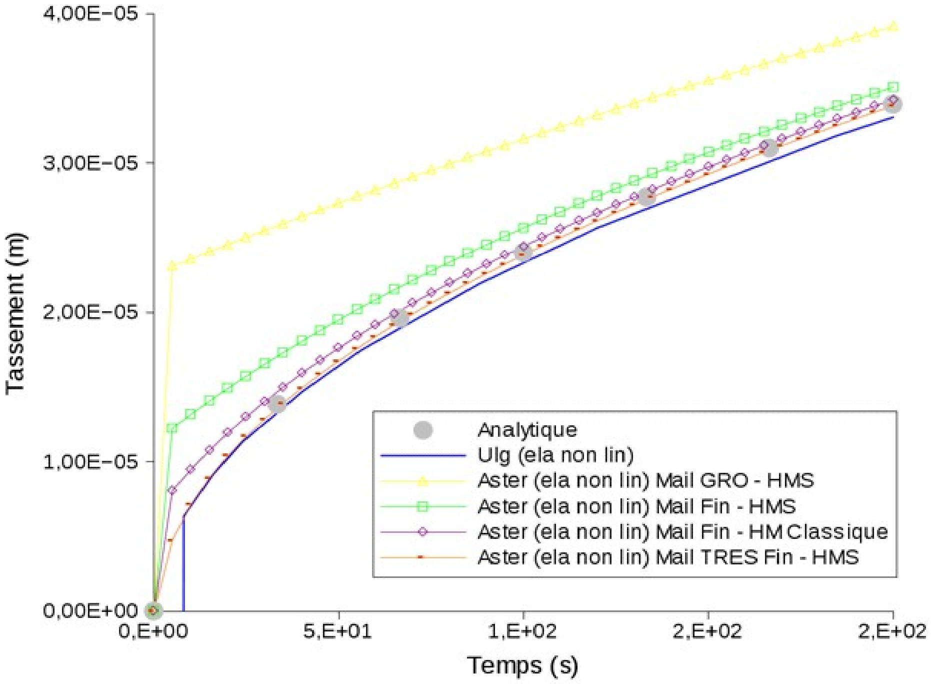


Figure 2.2-b: Comparison of the vertical displacement of free surface according to the time obtained for 3 different grids and Lagamine for the "fine" grid. Comparison of modelings classical (HM) and selective (HMS) of Code_Aster for the "fine" grid. The analytical solution of the problem is also represented.

3 Modeling A

3.1 Characteristics of modeling

The characteristics are identical to the reference solution.

3.2 Sizes tested and results

Value tested	Node	Moment (S)	Type	Reference
Displacement `PRE1`	No29	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No31	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No1	250.	`ANALYTICAL`	0.68544576689
Displacement `PRE1`	NO3	250.	`ANALYTICAL`	0.682208147164
Displacement `PRE1`	NO5	250.	`ANALYTICAL`	0.67252104433
Displacement `PRE1`	No7	250.	`ANALYTICAL`	0.656461946263
Displacement `PRE1`	No9	250.	`ANALYTICAL`	0.634160686593
Displacement `PRE1`	No11	250.	`ANALYTICAL`	0.605800331394
Displacement `PRE1`	No13	250.	`ANALYTICAL`	0.571618145927
Displacement `PRE1`	No15	250.	`ANALYTICAL`	0.531906397249
Displacement `PRE1`	No17	250.	`ANALYTICAL`	0.487012719208
Displacement `PRE1`	No19	250.	`ANALYTICAL`	0.437339762565
Displacement `PRE1`	No21	250.	`ANALYTICAL`	0.38334387542
Displacement `PRE1`	No23	250.	`ANALYTICAL`	0.32553260623
Displacement `PRE1`	No25	250.	`ANALYTICAL`	0.264460889851
Displacement `PRE1`	No27	250.	`ANALYTICAL`	0.200725860656
Displacement `PRE1`	No29	250.	`ANALYTICAL`	0.134960328921
Displacement `PRE1`	No31	250.	`ANALYTICAL`	0.0678250497631
Displacement `PRE1`	No33	250.	`ANALYTICAL`	0.00
Constraint `SIYY`	No48	0.00	`ANALYTICAL`	0.00
Constraint `SIYY`	No34	250.	`ANALYTICAL`	-0.31455423311
Constraint `SIYY`	No35	250.	`ANALYTICAL`	-0.317791852836
Constraint `SIYY`	No36	250.	`ANALYTICAL`	-0.32747895567
Constraint `SIYY`	No37	250.	`ANALYTICAL`	-0.343538053737
Constraint `SIYY`	No38	250.	`ANALYTICAL`	-0.365839313407
Constraint `SIYY`	No39	250.	`ANALYTICAL`	-0.394199668606
Constraint `SIYY`	No40	250.	`ANALYTICAL`	-0.428381854073
Constraint `SIYY`	No41	250.	`ANALYTICAL`	-0.468093602751
Constraint `SIYY`	No42	250.	`ANALYTICAL`	-0.512987280792
Constraint `SIYY`	No43	250.	`ANALYTICAL`	-0.562660237435
Constraint `SIYY`	No44	250.	`ANALYTICAL`	-0.61665612458
Constraint `SIYY`	No45	250.	`ANALYTICAL`	-0.67446739377
Constraint `SIYY`	No46	250.	`ANALYTICAL`	-0.735539110149
Constraint `SIYY`	No47	250.	`ANALYTICAL`	-0.799274139344
Constraint `SIYY`	No48	250.	`ANALYTICAL`	-0.865039671079
Constraint `SIYY`	No49	250.	`ANALYTICAL`	-0.932174950237
Constraint `SIYY`	No50	250.	`ANALYTICAL`	-1.0
Constraint `VMIS`	No50	250.	`NON-REGRESSION`	1.0
Constraint `VMIS_SG`	No50	250.	`NON-REGRESSION`	-1.0
Constraint `PRIN_1`	No50	250.	`NON-REGRESSION`	-1.0
Constraint `PRIN_2`	No50	250.	`NON-REGRESSION`	0.00
Constraint `PRIN_3`	No50	250.	`NON-REGRESSION`	0.00
Constraint `TRESCA`	No50	250.	`NON-REGRESSION`	1.0

Values of reference in `NON_REGRESSION` are obtained for version 10.01.21.

4 Modeling B

4.1 Characteristics of modeling

Modeling is generalized in the case of `3D`.

4.2 Sizes tested and results

Value tested	Node	Moment (S)	Type	Reference
Displacement `PRE1`	No168	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No170	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No172	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No174	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No176	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No178	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No180	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No182	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No184	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No186	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No188	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No190	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No192	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No194	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No196	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No198	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No200	0.	`ANALYTICAL`	0.0
Constraint `VMIS`	No83	0.	`NON-REGRESSION`	1.64519502
Constraint `VMIS_SG`	No83	0.	`NON-REGRESSION`	1.8098495947
Constraint `PRIN_1`	No83	0.	`NON-REGRESSION`	-1.4049
Constraint `PRIN_2`	No83	0.	`NON-REGRESSION`	0.0
Constraint `PRIN_3`	No83	0.	`NON-REGRESSION`	0.4049
Constraint `TRESCA`	No83	0.	`NON-REGRESSION`	-0.1645

Values of reference in `NON_REGRESSION` are obtained for version 10.01.21.

5 Modeling C

5.1 Characteristics of modeling

Modeling is identical to modeling A.

5.2 Sizes tested and results

Value tested	Node	Moment (S)	Type	Reference
Displacement `PRE1`	No29	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No31	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No1	250.	`ANALYTICAL`	0.68544576689
Displacement `PRE1`	NO3	250.	`ANALYTICAL`	0.682208147164
Displacement `PRE1`	NO5	250.	`ANALYTICAL`	0.67252104433
Displacement `PRE1`	No7	250.	`ANALYTICAL`	0.656461946263
Displacement `PRE1`	No9	250.	`ANALYTICAL`	0.634160686593
Displacement `PRE1`	No11	250.	`ANALYTICAL`	0.605800331394
Displacement `PRE1`	No13	250.	`ANALYTICAL`	0.571618145927
Displacement `PRE1`	No15	250.	`ANALYTICAL`	0.531906397249
Displacement `PRE1`	No17	250.	`ANALYTICAL`	0.487012719208
Displacement `PRE1`	No19	250.	`ANALYTICAL`	0.437339762565
Displacement `PRE1`	No21	250.	`ANALYTICAL`	0.38334387542
Displacement `PRE1`	No23	250.	`ANALYTICAL`	0.32553260623
Displacement `PRE1`	No25	250.	`ANALYTICAL`	0.264460889851
Displacement `PRE1`	No27	250.	`ANALYTICAL`	0.200725860656
Displacement `PRE1`	No29	250.	`ANALYTICAL`	0.134960328921
Displacement `PRE1`	No31	250.	`ANALYTICAL`	0.0678250497631
Displacement `PRE1`	No33	250.	`ANALYTICAL`	0.00
Constraint `SIYY`	No48	0.00	`ANALYTICAL`	0.00
Constraint `SIYY`	No34	250.	`ANALYTICAL`	-0.31455423311
Constraint `SIYY`	No35	250.	`ANALYTICAL`	-0.317791852836
Constraint `SIYY`	No36	250.	`ANALYTICAL`	-0.32747895567
Constraint `SIYY`	No37	250.	`ANALYTICAL`	-0.343538053737
Constraint `SIYY`	No38	250.	`ANALYTICAL`	-0.365839313407
Constraint `SIYY`	No39	250.	`ANALYTICAL`	-0.394199668606
Constraint `SIYY`	No40	250.	`ANALYTICAL`	-0.428381854073
Constraint `SIYY`	No41	250.	`ANALYTICAL`	-0.468093602751
Constraint `SIYY`	No42	250.	`ANALYTICAL`	-0.512987280792
Constraint `SIYY`	No43	250.	`ANALYTICAL`	-0.562660237435
Constraint `SIYY`	No44	250.	`ANALYTICAL`	-0.61665612458
Constraint `SIYY`	No45	250.	`ANALYTICAL`	-0.67446739377
Constraint `SIYY`	No46	250.	`ANALYTICAL`	-0.735539110149
Constraint `SIYY`	No47	250.	`ANALYTICAL`	-0.799274139344
Constraint `SIYY`	No48	250.	`ANALYTICAL`	-0.865039671079
Constraint `SIYY`	No49	250.	`ANALYTICAL`	-0.932174950237
Constraint `SIYY`	No50	250.	`ANALYTICAL`	-1.0
Constraint `VMIS`	No50	250.	`NON-REGRESSION`	0.4915089111
Constraint `VMIS_SG`	No50	250.	`NON-REGRESSION`	-0.4915089111
Constraint `PRIN_1`	No50	250.	`NON-REGRESSION`	-0.4915089111
Constraint `PRIN_2`	No50	250.	`NON-REGRESSION`	-1.826682526E-17
Constraint `PRIN_3`	No50	250.	`NON-REGRESSION`	1.60461944E-17
Constraint `TRESCA`	No50	250.	`NON-REGRESSION`	0.4915089111

Values of reference in `NON_REGRESSION` are obtained for version 10.01.21.

6 Modeling D

6.1 Characteristics of modeling

Modeling is identical to modeling B.

6.2 Sizes tested and results

Value tested	Node	Moment (S)	Type	Reference
Displacement `PRE1`	No168	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No170	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No172	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No174	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No176	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No178	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No180	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No182	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No184	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No186	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No188	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No190	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No192	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No194	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No196	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No198	0.	`ANALYTICAL`	1.0
Displacement `PRE1`	No200	0.	`ANALYTICAL`	0.0
Constraint `VMIS`	No83	0.	`NON-REGRESSION`	0.375001823900
Constraint `VMIS_SG`	No83	0.	`NON-REGRESSION`	-0.375001823900
Constraint `PRIN_1`	No83	0.	`NON-REGRESSION`	-0.375001823900
Constraint `PRIN_2`	No83	0.	`NON-REGRESSION`	0.O
Constraint `PRIN_3`	No83	0.	`NON-REGRESSION`	0.O
Constraint `TRESCA`	No83	0.	`NON-REGRESSION`	-0.375001823900

Values of reference in `NON_REGRESSION` are obtained for version 10.01.21.

7 Modeling E

7.1 Characteristics of modeling

Modeling is identical to modeling C but with a successive adaptation of the grid via `MACR_ADAP_MAIL`.

7.2 Sizes tested and results

Value tested	Sequence number	Type	Reference 9.02.13
`ESTERG1` component of the field `ERRE_NOEU_ELEM`	25	`NON-REGRESSION`	5.1838946544447E-03
`ERRE_TPS_GLOB`	25	`NON-REGRESSION`	0.0902526795091860

8 Modeling F

8.1 Characteristics of modeling

Modeling is identical to modeling E but without the indicator of error in time.

8.2 Sizes tested and results

Value tested	Sequence number	Type	Reference 9.02.13
`ESTERG1` component of the field `ERRE_NOEU_ELEM`	25	`NON-REGRESSION`	5.1838946544447E-03

9 Modeling G

9.1 Characteristics of modeling

The characteristics are identical to modeling A. the grid is composed of 83 nodes and of 16 `QUAD8`.

9.2 Sizes tested and results

Value tested	Node	Moment (S)	Type	Reference
Displacement `PRE1`	No29	1.E-4	`AUTRE_ASTER`	1.0
Displacement `PRE1`	No31	1.E-4	`AUTRE_ASTER`	1.0
Displacement `PRE1`	No1	250.	`AUTRE_ASTER`	0.68544576689
Displacement `PRE1`	NO3	250.	`AUTRE_ASTER`	0.682208147164
Displacement `PRE1`	NO5	250.	`AUTRE_ASTER`	0.67252104433
Displacement `PRE1`	No7	250.	`AUTRE_ASTER`	0.656461946263
Displacement `PRE1`	No9	250.	`AUTRE_ASTER`	0.634160686593
Displacement `PRE1`	No11	250.	`AUTRE_ASTER`	0.605800331394
Displacement `PRE1`	No13	250.	`AUTRE_ASTER`	0.571618145927
Displacement `PRE1`	No15	250.	`AUTRE_ASTER`	0.531906397249
Displacement `PRE1`	No17	250.	`AUTRE_ASTER`	0.487012719208
Displacement `PRE1`	No19	250.	`AUTRE_ASTER`	0.437339762565
Displacement `PRE1`	No21	250.	`AUTRE_ASTER`	0.38334387542
Displacement `PRE1`	No23	250.	`AUTRE_ASTER`	0.32553260623
Displacement `PRE1`	No25	250.	`AUTRE_ASTER`	0.264460889851
Displacement `PRE1`	No27	250.	`AUTRE_ASTER`	0.200725860656
Displacement `PRE1`	No29	250.	`AUTRE_ASTER`	0.134960328921
Displacement `PRE1`	No31	250.	`AUTRE_ASTER`	0.0678250497631
Displacement `PRE1`	No33	250.	`AUTRE_ASTER`	0.00

Values of pressure tested Scompared with the values of pressure obtained with modeling A. the differences observed between two modelings are lower than 6.E-3%.

10 Modeling H

10.1 Characteristics of modeling

Modeling is identical to modeling E but with under-integrated modeling `D_PLAN_HM_SI`.

10.2 Sizes tested and results

Value tested	Sequence number	Type	Reference 11.3.7
`ESTERG1` component of the field `ERRE_NOEU_ELEM`	25	`NON-REGRESSION`	5.2034169568591E-03
`ERRE_TPS_GLOB`	25	`NON-REGRESSION`	0.095460688688661

11 Summary of the results

In conclusion, results *Code_Aster* are in agreement with the analytical reference solutions.