WTNV112 – Gravitating flow in a porous environment unsaturated

Summary:

One studies the hydraulic behavior of a porous environment unsaturated. Five modelings are carried out: one is three-dimensional (modeling B) and the four others are two-dimensional (modelings A, C, D, E)

This test consists in studying the influence of a gravitating flow on the distribution of the pressure of the fluids (liquid and gas) of the medium unsaturated.

The studied models are $2D$ plans (DPQ8 and DPTR6) and $3D$ voluminal HEXA20 with a linear behavior, it is about an evolutionary problem.

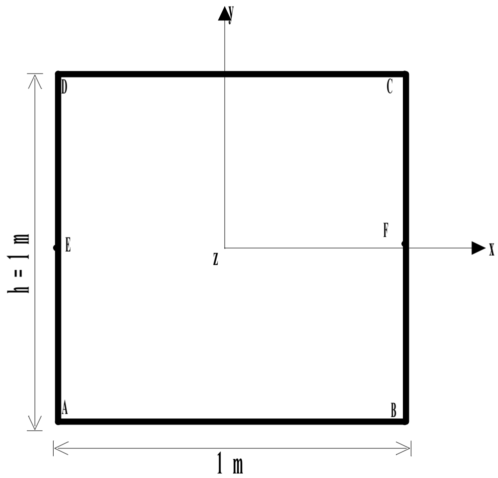
The reference solution is unidimensional because it depends only on the vertical coordinate.

1 Problem of reference

1.1 Presentation

One studies in this case test the hydraulic behavior of a porous environment unsaturated made up by two fluids: water in its liquid phase and dry air. It acts in *Code_Aster* of a modeling HHM. The associated law of behavior of the fluids is of type LIQU_GAZ .

1.2 Geometry



Coordinates of the points (m) :

$A: -0,5 -0,5$	$C: \quad 0,5\ 0,5$
$B: \quad 0,5 -0,5$	$D: -0,5\ 0,5$

1.3 Properties of material

Fluid (liquid water)	Density $(kg.m^{-3})$	10^3
	Compressibility of the liquid (Pa)	10^7
	Dynamic viscosity of liquid water $(Pa.s)$	10^{-3}
	Derived from the viscosity of the fluid compared to the temperature	$0.$
Gas (dry air)	Molar mass $(kg.Pa.K^{-1})$	1.8×10^{-3}
	Viscosity of gas $(Pa.s)$	10^{-5}
	Derived from the viscosity of gas compared to the temperature	$0.$
Coefficients of homogenisation	Coefficient of $Biot$	$1.$
	Porosity	0.14
Homogenized coefficients	Constant of perfect gases	8.315
	Homogenized density $(kg.m^{-3})$	1.6×10^3
	Saturation	0.5
	Derived from saturation compared to the pressure	$0.$
	Gravity according to X	$0.$
	Gravity according to Y	-10 in 2D, 0 in 3D
	Gravity according to Z	-10 in 3D, 0 in 2D
	Intrinsic permeability (m^2)	10^{-18}
	Permeability relating to the liquid (m^2)	$1.$
	Permeability relating to gas (m^2)	$1.$

1.4 Boundary conditions and loadings

- Complete element:
- displacements $u_x=0.0\,m, u_y=0.0\,m, u_z=0.0\,m$.

1.5 Initial conditions

The fields of displacement, of capillary pressure are initially worthless, the air pressure dryness is equal to the atmospheric pressure and the temperature of reference is worth $T_0=273\,°K$

2 Reference solution

2.1 Method of calculating used for the reference solution

The conservation equation of the fluid mass is given by the following expression:

$$\frac{dm_i}{dt}+DivM_i=0 \quad i \text{ varying } 1 \text{ with the number of components} \tag{1}$$

In our example, the model consists of two fluids: liquid water (e) and dry air (a). The equation (1) is thus divided into two:

$$\begin{cases}\dfrac{dm_e}{dt}+DivM_e=0\\ \dfrac{dm_a}{dt}+DivM_a=0\end{cases} \tag{2}$$

Flows of fluid have as an expression:

$$\begin{cases}M_e=\rho_e\lambda_e(-\nabla p_e+\rho_e g)\\ M_a=\rho_a\lambda_a(-\nabla p_a+\rho_a g)\end{cases} \tag{3}$$

However the mass contribution of fluid is defined by the equations (4) where $N=\begin{bmatrix}N_{ee} & N_{ea}\\ N_{ae} & N_{aa}\end{bmatrix}$ is a symmetrical matrix of which the terms (equations (5)) depend on the degree of saturation S, porosity ϕ, coefficient of $Biot\, b$, permeability of the liquid and K_e elasticity of the solid matrix K_s.

$$\begin{cases}\dfrac{dm_e}{dt}=\rho_e N_{ee}\dfrac{dp_e}{dt}+\rho_e N_{ea}\dfrac{dp_a}{dt}\\ \dfrac{dm_a}{dt}=\rho_a N_{ae}\dfrac{dp_e}{dt}+\rho_a N_{aa}\dfrac{dp_a}{dt}\end{cases} \tag{4}$$

$$\begin{cases}N_{ee}=-\varphi\dfrac{\partial S}{\partial p_c}+S\left(\dfrac{\varphi}{K_e}+\dfrac{b-\varphi}{K_s}S\right)\\ N_{aa}=-\varphi\dfrac{\partial S}{\partial p_c}+(1-S)\left(\dfrac{\varphi}{p_a}+\dfrac{b-\varphi}{K_s}(1-S)\right)\\ N_{ea}=N_{ae}=\varphi\dfrac{\partial S}{\partial p_c}+(1-S)\left(\dfrac{b-\varphi}{K_s}S\right)\end{cases} \tag{5}$$

The variational formulation of the equations (2), by taking account of (3) and (4) is:

$$\begin{cases} \int_\Omega N_{ee} \frac{dp_e}{dt} p_e^* + \int_\Omega N_{ea} \frac{dp_a}{dt} p_e^* + \int_\Omega \lambda_e \nabla p_e . \nabla p_e^* = \int_\Omega \lambda_e \rho_e g . \nabla p_e^* - \int_{\partial\Omega} \frac{M_e^{ext}}{\rho_e} p_e^* \\ \int_\Omega N_{ea} \frac{dp_e}{dt} p_a^* + \int_\Omega N_{aa} \frac{dp_a}{dt} p_a^* + \int_\Omega \lambda_a \nabla p_a . \nabla p_a^* = \int_\Omega \lambda_a \rho_a g . \nabla p_a^* - \int_{\partial\Omega} \frac{M_a^{ext}}{\rho_a} p_a^* \end{cases} \quad (6)$$

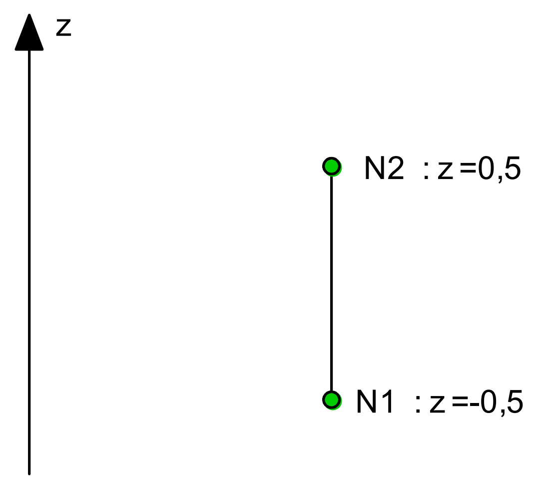
Discretization

For the calculation of the analytical solution, one places oneself in a unidimensional case with only one element of degree 1 .

Note:
In THM , all the grids must be quadratic, but in the hydraulic case integration is always linear (the nodes mediums are ignored).

One supposes in both cases that gravity is directed according to z negative.
It is supposed in addition that non-linearities are low and that coefficients N, λ, ρ are constant. It is necessary thus that the variations of pressure are sufficiently weak so that N and ρ can be presumedly constant.
In hydraulics, the discretization will be always linear.

- **Linear discretization:**



One will write:

$$p(z,t) = \sum_{i=1}^{2} p^i(t) \lambda_i(z) \quad (7)$$

With:

$$\begin{cases} \lambda_1 = \frac{1}{2} - z \\ \lambda_2 = \frac{1}{2} + z \end{cases} \tag{8}$$

By introducing the matrices and vectors then:

$$\begin{cases} [A]=[A_{ij}] \quad ; \quad A_{ij} = \int_{-1/2}^{1/2} \lambda_i \lambda_j \, dz \\ [B]=[B_{ij}] \quad ; \quad B_{ij} = \int_{-1/2}^{1/2} \frac{d\lambda_i}{dz} \frac{d\lambda_j}{dz} dz \\ \{F_g\}=\{F_{gi}\} \quad ; \quad F_{gj} = \int_{-1/2}^{1/2} \frac{d\lambda_i}{dz} dz \end{cases} \tag{9}$$

And while noting:

$$\{p_e\} = \begin{Bmatrix} P_e^1 \\ p_e^2 \end{Bmatrix} \quad ; \quad \{p_a\} = \begin{Bmatrix} P_a^1 \\ p_a^2 \end{Bmatrix} \tag{10}$$

$$\{M_e^{ext}\} = \begin{Bmatrix} M_{e\ 1}^{ext} \\ M_{e\ 2}^{ext} \end{Bmatrix} \quad ; \quad \{M_a^{ext}\} = \begin{Bmatrix} M_{a\ 1}^{ext} \\ M_{a\ 2}^{ext} \end{Bmatrix} \tag{11}$$

The equations (6) become:

$$\begin{cases} \frac{N_{ee}}{\lambda_e}[A]\begin{Bmatrix} \frac{dp_e}{dt} \end{Bmatrix} + \frac{N_{ea}}{\lambda_e}[A]\begin{Bmatrix} \frac{dp_a}{dt} \end{Bmatrix} + [B]\{p_e\} = \rho_e\{F_g\} - \frac{1}{\lambda_e \rho_e}\{M_e^{ext}\} \\ \frac{N_{ae}}{\lambda_a}[A]\begin{Bmatrix} \frac{dp_e}{dt} \end{Bmatrix} + \frac{N_{aa}}{\lambda_a}[A]\begin{Bmatrix} \frac{dp_a}{dt} \end{Bmatrix} + [B]\{p_a\} = \rho_a\{F_g\} - \frac{1}{\lambda_a \rho_a}\{M_a^{ext}\} \end{cases} \tag{12}$$

The calculation of the matrices $[A]$ and $[B]$ and of the vector $\{F\}$ give:

$$[A] = \frac{1}{3}\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix} \quad ; \quad [B] = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad ; \quad \{F_g\} = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} \tag{13}$$

One defines then $\{v_1\},\{v_2\}$ clean vectors of $[A]^{-1}[B]$.

There are the properties:

$$\{v_i\}^T [A]\{v_j\} = \{v_i\}^T [B]\{v_j\} = 0 \quad si \quad i \neq j \tag{14}$$

And one poses:

$$a_i=\{v_i\}^T[A]\{v_i\} \ , \ b_i=\{v_i\}^T[B]\{v_i\} \ , \ f_i=\{v_i\}^T\{F_g\} et\ M^i=\{v_i\}^T\{M^{ext}\} \quad (15)$$

One finds:

$$\{v_1\}=\begin{Bmatrix}1\\1\end{Bmatrix} \quad ; \quad \{v_2\}=\begin{Bmatrix}-1\\1\end{Bmatrix} \quad (16)$$

$$\begin{cases} a_1=1 \ ; \ b_1=0 \ ; \ f_1=0 \\ a_2=\dfrac{1}{3} \ ; \ b_2=4 \ ; \ f_2=-2g \end{cases} \quad (17)$$

One breaks up then $\{p_e\}$ and $\{p_a\}$ on the basis of $\{v_i\}$

$$\{p_e\}=\sum_{i=1}^{2}\alpha_e^i\{v_i\} \quad ; \quad \{p_a\}=\sum_{i=1}^{2}\alpha_a^i\{v_i\} \quad (18)$$

Taking into account the properties of orthogonality (14), the system of equations (12) is written:

$$\begin{cases} \dfrac{N_{ee}}{\lambda_e}a_i\dfrac{d\alpha_e^i}{dt}+\dfrac{N_{ea}}{\lambda_e}a_i\dfrac{d\alpha_a^i}{dt}+b_i\alpha_e^i=\rho_e f_i-\dfrac{1}{\lambda_e\rho_e}M_e^i \\ \dfrac{N_{ae}}{\lambda_a}a_i\dfrac{d\alpha_e^i}{dt}+\dfrac{N_{aa}}{\lambda_a}a_i\dfrac{d\alpha_a^i}{dt}+b_i\alpha_a^i=\rho_a f_i-\dfrac{1}{\lambda_a\rho_a}M_a^i \end{cases} \quad (19)$$

Posing:

$$\{\alpha^i\}=\begin{Bmatrix}\alpha_e^i\\\alpha_a^i\end{Bmatrix} \quad ; \quad [N]=\begin{bmatrix}N_{ee} & N_{ea}\\ N_{ae} & N_{aa}\end{bmatrix} \quad ; \quad [L]=\begin{bmatrix}\lambda_e & 0\\ 0 & \lambda_a\end{bmatrix} \quad (20)$$

The equation (19) is written:

$$[N]=\left[\frac{d\alpha^i}{dt}\right]+\frac{b_i}{a_i}[L]\{\alpha^i\}=\frac{f_i}{a_i}\begin{Bmatrix}\rho_e\lambda_e\\ \rho_a\lambda_a\end{Bmatrix}-\begin{Bmatrix}M_e^i/\rho_e a_i\\ M_a^i/\rho_a a_i\end{Bmatrix} \quad (21)$$

Initial conditions

It is supposed that:

$$\begin{matrix} p_a(x,t=0)=p_a^0 \\ p_e(x,t=0)=p_a^0-p_c^0 \end{matrix} \text{ uniforms in space;}$$

Taking into account the values of the vectors $\{v_1\},\{v_2\}$ (equations (16)), it is seen easily that:

$$\begin{matrix} \alpha_a^1(t=0)=p_a^0 \ ; \ \alpha_e^1(t=0)=p_a^0-p_c^0 \\ \alpha_a^2(t=0)=\alpha_e^2(t=0)=0 \end{matrix} \quad (22)$$

One places oneself in a case where the equations of hydraulics are uncoupled $\left(N_{ea}=N_{ae}=0\right)$ and in which flows of fluid are worthless $\left(\{M_e^{ext}\}=\{M_a^{ext}\}=0\right)$.

Taking into account (21), of $f_1=f_3=0$ (equations (17)), the system of equations (21) has as a solution:

$$\left\{\begin{array}{l} \alpha_e^1=P_a^0-p_c^0 \\ \alpha_e^2=\frac{f_2}{b_2}\rho_e\left(1-\exp\left(-\frac{b_2}{a_2}\frac{\lambda_e}{N_{ee}}t\right)\right) \\ \alpha_a^1=P_a^0 \\ \alpha_a^2=\frac{f_2}{b_2}\rho_a\left(1-\exp\left(-\frac{b_2}{a_2}\frac{\lambda_a}{N_{aa}}t\right)\right) \end{array}\right\} \quad (23)$$

One finds while returning to the nodal variables:

$$\begin{Bmatrix} P_1 \\ p_2 \end{Bmatrix}=\begin{Bmatrix} \alpha_1-\alpha_2 \\ \alpha_1+\alpha_2 \end{Bmatrix}$$

$$\begin{Bmatrix} P_1 \\ p_2 \end{Bmatrix}_{eau}=\begin{Bmatrix} P_a^0-p_c^0+\frac{\rho_e g}{2}\left(1-\exp\left(-12\frac{\lambda_e}{N_{ee}}t\right)\right) \\ P_a^0-p_c^0-\frac{\rho_e g}{2}\left(1-\exp\left(-12\frac{\lambda_e}{N_{ee}}t\right)\right) \end{Bmatrix} \quad (24)$$

$$\begin{Bmatrix} P_1 \\ p_2 \end{Bmatrix}_{air}=\begin{Bmatrix} P_a^0+\frac{\rho_a g}{2}\exp\left(-12\frac{\lambda_a}{N_{aa}}t\right) \\ P_a^0-\frac{\rho_a g}{2}\exp\left(-12\frac{\lambda_a}{N_{aa}}t\right) \end{Bmatrix} \quad (25)$$

and the definite capillary pressure like the difference between the air pressure and the pressure of water has as a value:

$$\begin{Bmatrix} P_1 \\ p_2 \end{Bmatrix}_{capillaire}=\begin{Bmatrix} P_1 \\ p_2 \end{Bmatrix}_{air}-\begin{Bmatrix} P_1 \\ p_2 \end{Bmatrix}_{eau}$$

We considered the following calculation case:

$$S\neq 1 \quad ; \quad \frac{\partial S}{\partial p_c}=0 \quad ; \quad K_s=\infty$$

$$N_{ee}=S\frac{\varphi}{K_e} \quad ; \quad N_{aa}=(1-S)\frac{\varphi}{p_a}$$

2.2 Reference variable

1) Evolution of the capillary pressure and the air pressure dryness according to time at the points
   * $C, D(z=h)$
   * $A, B(z=0)$
1) For the quadratic discretization, checking of the constant value of the pressure to the nodes $E, F(z=\frac{h}{2})$.

2.3 Uncertainties

Analytical solution on the equations of negligible hydraulics thus uncertainties for modelings A, B, C. Caution these analytical solutions do not apply to selective or lumpées modelings (D and E). Indeed, in this last case, integrations are made with the nodes and either at the points of Gauss. Indeed integration by point of Gauss is exact in $1\mathrm{D}$ for polynomial of degree lower or equal to 3 and thus for all the integrals presented in the equation (9). On the other hand the method of integration at the top is not exact that for the polynomials of degree 1. It is thus seen that the terms of the matrix $[A]$ under will be integrated. It is thus logical that on a unit grid as here the results got here are not exact. One however preserves these tests but with a result in “not regression”.

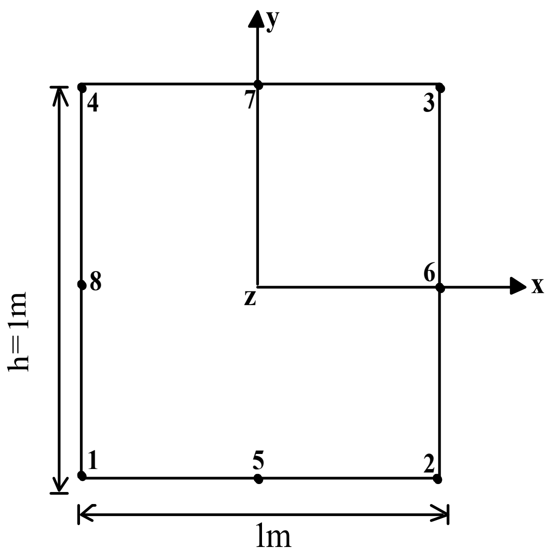
2.4 Bibliographical references

1 Thermo-hydro-mechanics of the porous environments in Code_Aster – Note EDF, HI-74/99/011/A

3 Modeling A

3.1 Characteristics of modeling A

Plane modeling: D_PLAN_HHM



1 mesh DPQ8 modeling D_PLAN_HHM : HHM_ DPQ8

3.2 Result of modeling A

Discretization in time: Several steps of time (16) to study the evolution of the pressure during the transitional stage until stabilizing itself. The diagram in time is implicit $(\theta=1)$.

List of the moments of calculation in seconds:
$1, 5, 10, 50, 100, 500, 10^3, 5.\times 10^3, 10^4, 5.\times 10^4, 10^5, 5.\times 10^5, 10^6, 5.\times 10^6, 10^7, 10^{10}$.

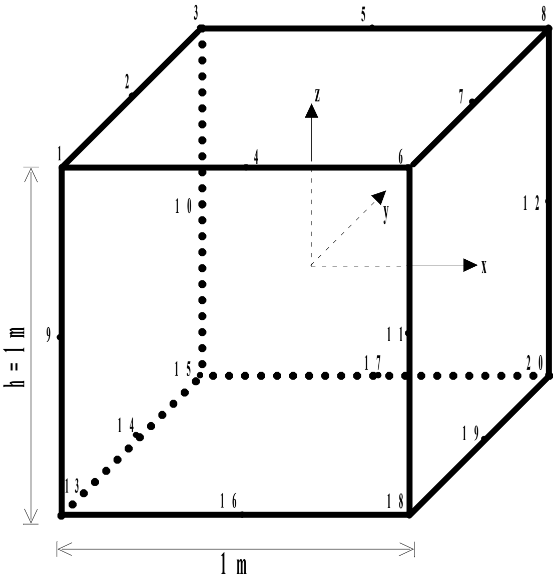
Nodal unknown factors: pressures of fluids evaluated in *Code_Aster* are variations compared to the initial pressures, this is why this table present of the variations of pressure in our comparison between calculation *Code_Aster* and the reference solution. Moreover variables of pressure used in *Code_Aster* to evaluate the laws of behavior are the total pressure of gas and the capillary pressure.

Node/not	Urgent sequence number/ (s)	Value	Pressure (Pa)	Tolerance
$1,2,5/A,B$	$1(t=1s)$	$PRE1$	-8.565 .10 $^{-3}$	10^{-4}
	$2(t=5s)$	$PRE1$	-4,282.10 $^{-2}$	10^{-4}
	$3(t=10s)$	$PRE1$	-8,565.10 $^{-2}$	10^{-4}
	$4(t=50s)$	$PRE1$	-4,282.10 $^{-1}$	1%
	$8(t=5.10^3 s)$	$PRE1$	-4,26.10 $^{+1}$	1%
	$16(t=10^{10} s)$	$PRE1$	-4,996.10 $^{+3}$	1%
	$1(t=1s)$	$PRE2$	6,796.10 $^{-6}$	10^{-4}
	$2(t=5s)$	$PRE2$	3,398.10 $^{-5}$	10^{-4}
	$3(t=10s)$	$PRE2$	6,796.10 $^{-5}$	10^{-4}
	$4(t=50s)$	$PRE2$	3,398.10 $^{-4}$	10^{-4}
	$8(t=5.10^3 s)$	$PRE2$	3,384.10 $^{-2}$	10^{-4}
	$16(t=10^{10} s)$	$PRE2$	3.964	10^{-3}
$3,4,7/C,D$	$1(t=1s)$	$PRE1$	8.565 .10 $^{-3}$	10^{-4}
	$2(t=5s)$	$PRE1$	4,282.10 $^{-2}$	10^{-4}
	$3(t=10s)$	$PRE1$	8,565.10 $^{-2}$	10^{-4}
	$4(t=50s)$	$PRE1$	4,282.10 $^{-1}$	1%
	$8(t=5.10^3 s)$	$PRE1$	4,26.10 $^{+1}$	1%
	$16(t=10^{10} s)$	$PRE1$	4,996.10 $^{+3}$	1%
	$1(t=1s)$	$PRE2$	-6,796.10 $^{-6}$	10^{-4}
	$2(t=5s)$	$PRE2$	-3,398.10 $^{-5}$	10^{-4}
	$3(t=10s)$	$PRE2$	-6,796.10 $^{-5}$	10^{-4}
	$4(t=50s)$	$PRE2$	-3,398.10 $^{-4}$	10^{-4}
	$8(t=5.10^3 s)$	$PRE2$	-3,384.10 $^{-2}$	10^{-4}
	$16(t=10^{10} s)$	$PRE2$	-3.964	10^{-3}

4 Modeling B

4.1 Characteristics of modeling B

Voluminal modeling: 3D_HHM



1 mesh HEXA20 modeling 3D_HHM : HHM_HEXA20

4.2 Result of modeling B

Discretization in time: Several steps of time (16) to study the evolution of the pressure during the transitional stage until stabilizing itself. The diagram in time is implicit $(\theta=1)$.

List of the moments of calculation in seconds:
$1, 5, 10, 50, 100, 500, 10^3, 5.\times10^3, 10^4, 5.\times10^4, 10^5, 5.\times10^5, 10^6, 5.\times10^6, 10^7, 10^{10}.$

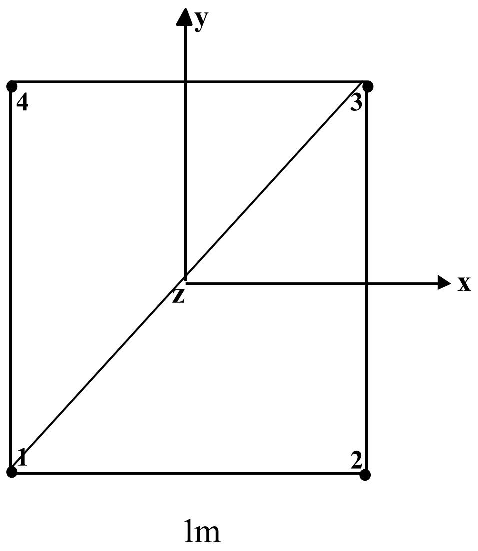
Nodal unknown factors of pressure of fluid evaluated in *Code_Aster* are variations compared to the initial pressures, this is why this table present of the variations of pressure in our comparison between calculation *Code_Aster* and the reference solution. Moreover variables of pressure used in *Code_Aster* to evaluate the laws of behavior are the total pressure of gas and the capillary pressure.

Node/not	**Urgent sequence number/** (s)	**Value**	**Pressure** (Pa)	**Tolerance**
13 to 20/ A and B	$1(t=1\,s)$	*PRE1*	-8.565 .10 $^{-3}$	10^{-4}
	$2(t=5\,s)$	*PRE1*	-4,282.10 $^{-2}$	10^{-4}
	$3(t=10\,s)$	*PRE1*	-8,565.10 $^{-2}$	10^{-4}
	$4(t=50\,s)$	*PRE1*	-4,282.10 $^{-1}$	1%
	$8(t=5.10^{3}s)$	*PRE1*	-4,26.10 $^{+1}$	1%
	$16(t=10^{10}s)$	*PRE1*	-4,996.10 $^{+3}$	1%
	$1(t=1\,s)$	*PRE2*	6,796.10 $^{-6}$	10^{-4}
	$2(t=5\,s)$	*PRE2*	3,398.10 $^{-5}$	10^{-4}
	$3(t=10\,s)$	*PRE2*	6,796.10 $^{-5}$	10^{-4}
	$4(t=50\,s)$	*PRE2*	3,398.10 $^{-4}$	10^{-4}
	$8(t=5.10^{3}s)$	*PRE2*	3,384.10 $^{-2}$	10^{-4}
	$16(t=10^{10}s)$	*PRE2*	3.964	10^{-3}
1 to 8/ C and D	$1(t=1\,s)$	*PRE1*	8.565 .10 $^{-3}$	10^{-4}
	$2(t=5\,s)$	*PRE1*	4,288.10 $^{-2}$	10^{-4}
	$3(t=10\,s)$	*PRE1*	8,565.10 $^{-2}$	10^{-4}
	$4(t=50\,s)$	*PRE1*	4,282.10 $^{-1}$	1%
	$8(t=5.10^{3}s)$	*PRE1*	4,26.10 $^{+1}$	1%
	$16(t=10^{10}s)$	*PRE1*	4,996.10 $^{+3}$	1%
	$1(t=1\,s)$	*PRE2*	-6,796.10 $^{-6}$	10^{-4}
	$2(t=5\,s)$	*PRE2*	-3,398.10 $^{-5}$	10^{-4}
	$3(t=10\,s)$	*PRE2*	-6,796.10 $^{-5}$	10^{-4}
	$4(t=50\,s)$	*PRE2*	-3,398.10 $^{-4}$	10^{-4}
	$8(t=5.10^{3}s)$	*PRE2*	-3,384.10 $^{-2}$	10^{-4}
	$16(t=10^{10}s)$	*PRE2*	-3.964	10^{-3}

5 Modeling C

5.1 Characteristics of modeling C

Plane modeling: `D_PLAN_HHM`



2 meshs `DPTR6` modeling `D_PLAN_HHM` : `HHM_ DPTR6`

5.2 Result of modeling C

Discretization in time: Several steps of time (16) to study the evolution of the pressure during the transitional stage until stabilizing itself. The diagram in time is implicit $(\theta=1)$.

List of the moments of calculation in seconds:
$1, 5, 10, 50, 100, 500, 10^3, 5.\times 10^3, 10^4, 5.\times 10^4, 10^5, 5.\times 10^5, 10^6, 5.\times 10^6, 10^7, 10^{10}$.

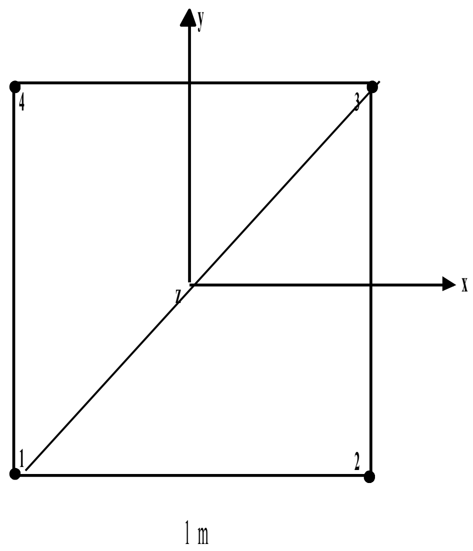
Nodal unknown factors: pressures of fluids evaluated in *Code_Aster* are variations compared to the initial pressures, this is why this table present of the variations of pressure in our comparison between calculation *Code_Aster* and the reference solution. Moreover variables of pressure used in *Code_Aster* to evaluate the laws of behavior are the total pressure of gas and the capillary pressure.

Node/not	**Urgent sequence number/** (s)	**Value**	**Pressure** (Pa)	**Tolerance**
1.2/ A and B	$1(t=1\,s)$	$PRE1$	-8.565 .10 $^{-3}$	10^{-4}
	$2(t=5\,s)$	$PRE1$	-4,282.10 $^{-2}$	10^{-4}
	$3(t=10\,s)$	$PRE1$	-8,565.10 $^{-2}$	10^{-4}
	$4(t=50\,s)$	$PRE1$	-4,282.10 $^{-1}$	1%
	$8(t=5.10^{3}s)$	$PRE1$	-4,26.10 $^{+1}$	1%
	$16(t=10^{10}s)$	$PRE1$	-4,996.10 $^{+3}$	1%
	$1(t=1\,s)$	$PRE2$	6,796.10 $^{-6}$	10^{-4}
	$2(t=5\,s)$	$PRE2$	3,398.10 $^{-5}$	10^{-4}
	$3(t=10\,s)$	$PRE2$	6,796.10 $^{-5}$	10^{-4}
	$4(t=50\,s)$	$PRE2$	3,398.10 $^{-4}$	10^{-4}
	$8(t=5.10^{3}s)$	$PRE2$	3,384.10 $^{-2}$	10^{-4}
	$16(t=10^{10}s)$	$PRE2$	3.964	10^{-3}
3.4/ C and D	$1(t=1\,s)$	$PRE1$	8.565 .10 $^{-3}$	10^{-4}
	$2(t=5\,s)$	$PRE1$	4,282.10 $^{-2}$	10^{-4}
	$3(t=10\,s)$	$PRE1$	8,565.10 $^{-2}$	10^{-4}
	$4(t=50\,s)$	$PRE1$	4,282.10 $^{-1}$	1%
	$8(t=5.10^{3}s)$	$PRE1$	4,26.10 $^{+1}$	1%
	$16(t=10^{10}s)$	$PRE1$	4,996.10 $^{+3}$	1%
	$1(t=1\,s)$	$PRE2$	-6,796.10 $^{-6}$	10^{-4}
	$2(t=5\,s)$	$PRE2$	-3,398.10 $^{-5}$	10^{-4}
	$3(t=10\,s)$	$PRE2$	-6,796.10 $^{-5}$	10^{-4}
	$4(t=50\,s)$	$PRE2$	-3,398.10 $^{-4}$	10^{-4}
	$8(t=5.10^{3}s)$	$PRE2$	-3,384.10 $^{-2}$	10^{-4}
	$16(t=10^{10}s)$	$PRE2$	-3.964	10^{-3}

6 Modeling D

6.1 Characteristics of modeling D

Plane modeling: D_PLAN_HHMS



1 mesh DPQ8 modeling D_PLAN_HHMS : HHM_DPQ8S

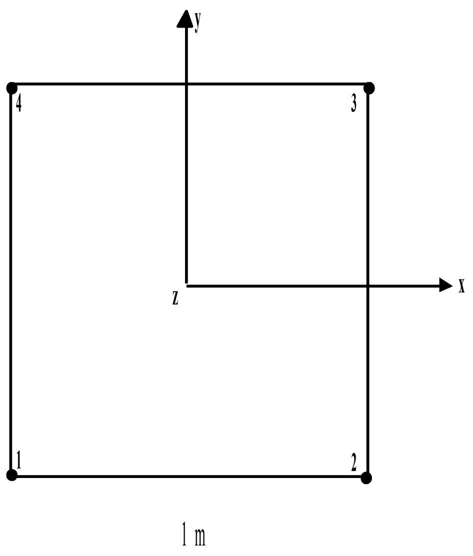
6.2 Result of modeling D

Node/not	**Urgent sequence number/** (s)	**Value**	**Pressure** (Pa)	**Tolerance**
1.2/ A and B	$1(t=1\,s)$	*PRE1*	$-2{,}8549\times10^{-3}$	10^{-4}
	$2(t=5\,s)$	*PRE1*	$-0{,}01427$	10^{-4}
	$3(t=10\,s)$	*PRE1*	$-0{,}028549$	10^{-4}
	$4(t=50\,s)$	*PRE1*	$-0{,}1427$	10^{-2}
	$8(t=5.10^{3}s)$	*PRE1*	$-14{,}24$	10^{-2}
	$16(t=10^{10}s)$	*PRE1*	$-4995{,}0$	10^{-2}
	$1(t=1\,s)$	*PRE2*	$2{,}2656\times10^{-6}$	10^{-4}
	$2(t=5\,s)$	*PRE2*	$1{,}1328\times10^{-5}$	10^{-4}
	$3(t=10\,s)$	*PRE2*	$2{,}2656\times10^{-5}$	10^{-4}
	$4(t=50\,s)$	*PRE2*	$1{,}133\times10^{-4}$	10^{-4}
	$8(t=5.10^{3}s)$	*PRE2*	$0{,}011301$	10^{-4}
	$16(t=10^{10}s)$	*PRE2*	$3{,}9647$	10^{-3}
3.4/ C and D	$1(t=1\,s)$	*PRE1*	$2{,}8549\times10^{-3}$	10^{-4}
	$2(t=5\,s)$	*PRE1*	$0{,}01427$	10^{-4}
	$3(t=10\,s)$	*PRE1*	$0{,}028549$	10^{-4}
	$4(t=50\,s)$	*PRE1*	$0{,}1427$	10^{-2}
	$8(t=5.10^{3}s)$	*PRE1*	$14{,}24$	10^{-2}
	$16(t=10^{10}s)$	*PRE1*	$4997{,}0$	10^{-2}
	$1(t=1\,s)$	*PRE2*	$-2{,}2656\times10^{-6}$	10^{-4}
	$2(t=5\,s)$	*PRE2*	$-1{,}1328\times10^{-5}$	10^{-4}
	$3(t=10\,s)$	*PRE2*	$-2{,}2656\times10^{-5}$	10^{-4}
	$4(t=50\,s)$	*PRE2*	$-1{,}133\times10^{-4}$	10^{-4}
	$8(t=5.10^{3}s)$	*PRE2*	$-0{,}0113$	10^{-4}
	$16(t=10^{10}s)$	*PRE2*	$-3{,}9647$	10^{-3}

7 Modeling E

7.1 Characteristics of modeling E

Plane modeling: D_PLAN_HHMD



1 mesh DPQ8 modeling D_PLAN_HHM : HHM_DPQ8D

7.2 Result of modeling E

Node/not	Urgent sequence number/ (s)	Value	Pressure (Pa)	Tolerance
1.2/ A and B	$1(t=1\,s)$	*PRE1*	$-2{,}85486\times10^{-3}$	10^{-4}
	$2(t=5\,s)$	*PRE1*	$-0{,}0142743$	10^{-4}
	$3(t=10\,s)$	*PRE1*	$-0{,}0285487$	10^{-4}
	$4(t=50\,s)$	*PRE1*	$-0{,}14274$	10^{-4}
	$8(t=5.10^{3}\,s)$	*PRE1*	$-14{,}2406$	10^{-4}
	$16(t=10^{10}\,s)$	*PRE1*	$-4995{,}06$	10^{-4}
	$1(t=1\,s)$	*PRE2*	$-2{,}26558\times10^{-6}$	10^{-4}
	$2(t=5\,s)$	*PRE2*	$1{,}13279\times10^{-5}$	10^{-4}
	$3(t=10\,s)$	*PRE2*	$2{,}26557\times10^{-5}$	10^{-4}
	$4(t=50\,s)$	*PRE2*	$1{,}132764\times10^{-4}$	10^{-4}
	$8(t=5.10^{3}\,s)$	*PRE2*	$0{,}0113012$	10^{-4}
	$16(t=10^{10}\,s)$	*PRE2*	$3{,}96734$	10^{-4}
3.4/ C and D	$1(t=1\,s)$	*PRE1*	$2{,}85488\times10^{-3}$	10^{-4}
	$2(t=5\,s)$	*PRE1*	$-0{,}0142743$	10^{-4}
	$3(t=10\,s)$	*PRE1*	$0{,}0285487$	10^{-4}
	$4(t=50\,s)$	*PRE1*	$0{,}14274$	10^{-3}
	$8(t=5.10^{3}\,s)$	*PRE1*	$14{,}2407$	10^{-4}
	$16(t=10^{10}\,s)$	*PRE1*	$4996{,}93$	10^{-4}
	$1(t=1\,s)$	*PRE2*	$-2{,}26557\times10^{-6}$	10^{-4}
	$2(t=5\,s)$	*PRE2*	$-1{,}13279\times10^{-5}$	10^{-4}
	$3(t=10\,s)$	*PRE2*	$-2{,}26557\times10^{-5}$	10^{-4}
	$4(t=50\,s)$	*PRE2*	$-1{,}13276\times10^{-4}$	10^{-4}
	$8(t=5.10^{3}\,s)$	*PRE2*	$-0{,}0113012$	10^{-4}
	$16(t=10^{10}\,s)$	*PRE2*	$-3{,}96734$	10^{-4}

8 Summary of the results

Values of *Code_Aster* are in concord with the values of reference.