WTNV114 - Hydrous flow on a saturated porous environment

Summary:

One studies the hydraulic behavior of a saturated porous environment. Nine modelings are carried out: two two-dimensional modelings (modelings A and B) and seven three-dimensional modelings (modelings C, D, E, F, G, H, I).

The distinction between modelings lies in the law of behavior of the fluid (`LIQU_SATU` for modelings A, C, G, H, I and `LIQU_GAZ_ATM` for modelings B, D, E, F).

This test consists in applying a hydrous flow to the higher face of the model and studying the effect of this flow on the distribution of the pressure of the fluid in the saturated medium. It is about an evolutionary problem.

The first eight studied models are 2D plans (`HM_DPQ8`) and 3D voluminal (`HM_HEXA20` or `HM_PYRAM13`) with a linear behavior. The last model is HM-XFEM.

The reference solution is unidimensional because it depends only on the vertical coordinate.

1 Problem of reference

One studies in this case test the hydraulic behavior of a saturated porous environment consisted only one fluid: water in its liquid phase. It acts in *Code_Aster* of a modeling HM . The associated law of behavior of the fluid is according to modelings is of type LIQU_SATU (modelings A and C) is of type LIQU_GAZ_ATM (modelings B and D).

1.1 Geometry

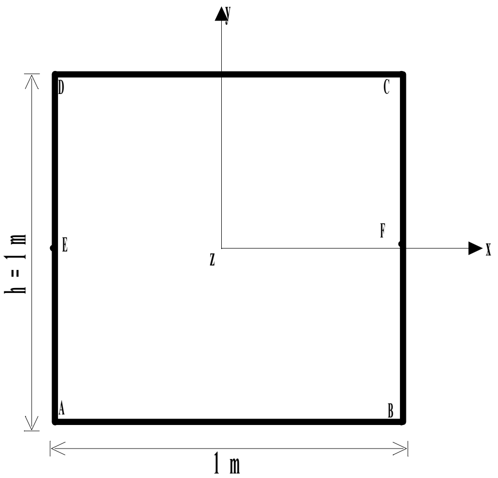


Illustration 1: Geometry

Coordinates of the points (m) :

$A(-0.5,-0.5)$; $C(0.5,0.5)$
$B(0.5,-0.5)$; $D(-0.5,0.5)$

1.2 Properties of material

Liquid water	Density $(kg.m^{-3})$ Dynamic viscosity of liquid water $(Pa.s)$ 1/Compressibility (Pa^{-1})	10^3 0.001 $K_e=3.77 10^{-9}$
Solid	Drained Young modulus $E(Pa)$ Poisson's ratio	$225 10^6$ 0
Initial state	Porosity Temperature (K) Liquid pressure (Pa) Steam pressure (Pa)	0.4 273 0 1
Constants	Constant of perfect gases	8.32
Homogenized coefficients	Homogenized density $(kg.m^{-3})$ Isotherm of sorption Coefficient of Biot Intrinsic permeability (m^2)	1600 $S(P_c)=1$ 1 $K_{int}=10^{-18}$

Table 1.2-1: Data materials

The gravity of water is neglected.

1.3 Boundary conditions and loadings

Complete element:
blocked displacements $u_x=u_y=u_z=0$
(it is here about a purely hydraulic case mechanics being blocked).
Higher face:
hydrous flow: $Q_{lq}=0.005\,kg.s^{-1}.m^{-2}$
Lower face:
hydrous flow $Q_{lq}=0$

Side faces:
Elsewhere: null flow

1.4 Initial conditions

The fields of displacement, of pressure of liquid are initially worthless and the temperature of reference is worth $T_0=273\,°K$.

2 Reference solution

2.1 Method of calculating

The reference solution is unidimensional because it depends only on the vertical coordinate.

The conservation equation of the water mass m_l is given by the following expression:

$$\frac{d\,m_l}{dt}+Div\,M_l=0 \qquad (1)$$

with M_l the water flow such as, if gravity is neglected:
$M_l=\rho_l\lambda_l(-\nabla\,p_l)$ with p_l pressure of liquid, ρ_l its density and λ_l its hydraulic conductivity such as $\lambda_l=\frac{K_{int}}{\mu}$. K_{int} is the intrinsic permeability and μ the viscosity of the liquid.
By considering an indeformable solid, one can write that
$\frac{d\,m_l}{dt}=\rho_l\frac{\phi}{K_l}\frac{d\,p_l}{dt}$ with ϕ porosity and K_l the compressibility of water. One notes $N=\frac{\phi}{K_l}$.
The equation (1) becomes then:

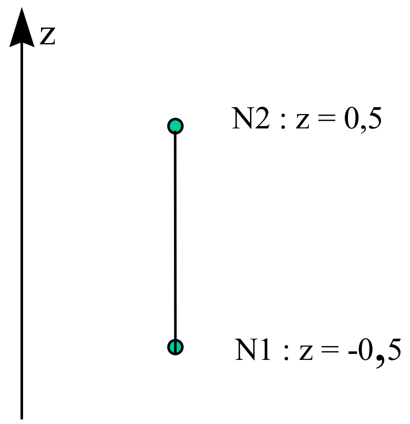
$$\rho_l\,N\frac{d\,p_l}{dt}+Div\left(\rho_l\lambda_l(-\nabla\,p_l)\right)=0$$

whose variational formulation is the following one:

$$\int_\Omega N\frac{d\,p_l}{dt}.\hat{p}_l+\int_\Omega\lambda_l\nabla\,p_l\nabla\,\hat{p}_l=-\int_{\partial\Omega}\frac{M^{ext}}{\rho_l}\hat{p}_l \qquad (2)$$

with M^{ext} the external loading.

To establish the solution one places oneself in the unidimensional case and one adopts a discretization corresponding to a single element of degree 1 since in modeling THM the hydraulic part is treated on linear elements. It is also supposed that nonthe linearities are low in this case and that coefficients N and ρ_l are constant what supposes a variation relatively weak of the pressure.
That is to say a linear element:



One then writes the pressure on the basis of function of forms such as:

$$p_l(z,t)=\sum_{i=1}^{i=2}p^i(t)\lambda_i(z)$$

with

$$\lambda_1(z)=0{,}5(1+2z)$$
$$\lambda_2(z)=0{,}5(1-2z)$$

The following matrices then are introduced:

$$[A]=[A_{ij}]\,;\,A_{ij}=\int_{-0,5}^{0,5}\lambda_i\lambda_j\,dz$$
$$[B]=[B_{ij}]\,;\,B_{ij}=\int_{-0,5}^{0,5}\frac{\partial\lambda_i}{\partial z}\frac{\partial\lambda_j}{\partial dz}dz$$

what leads to

and

$$[A]=\frac{1}{6}\begin{bmatrix}2&1\\1&2\end{bmatrix}\qquad[B]=\begin{bmatrix}1&-1\\-1&1\end{bmatrix}$$

One notes then classically:

$$\{p_l\}=\begin{Bmatrix}P_l^1\\P_l^2\end{Bmatrix}\quad\{M^{ext}\}=\begin{Bmatrix}M^{ext1}\\M^{ext2}\end{Bmatrix}$$

Knowing that one injects a water Q flow on the higher face, one thus has

$$\{M^{ext}\}=\begin{Bmatrix}0\\-Q\end{Bmatrix}$$

The equation (2) becomes

$$\frac{N}{\lambda_l}[A]\{\frac{dp_l}{dt}\}+[B]\{p_l\}=\frac{-1}{\lambda_l\,\rho}\{M^{ext}\}$$

Considering that for variations of short times ΔT, the evolution of p is almost linear, one can write that

$$\{\frac{dp_l}{dt}\}=\frac{1}{\Delta t}\begin{Bmatrix}P_l^1-p_0^1\\P_l^2-p_0^2\end{Bmatrix}$$

and like one leaves an initial state here worthless pressure, one will have

$$\{\frac{dp_l}{dt}\}=\frac{1}{\Delta t}\begin{Bmatrix}P_l^1\\P_l^2\end{Bmatrix}$$

Finally the system of two equations to two unknown factors is obtained:

$$p_l^1(\frac{N}{\lambda_l\Delta t}+6)-6p_l^2=\frac{-2Q}{\lambda_l\,\rho_l}$$
$$p_l^2(\frac{N}{\lambda_l\Delta t}+6)-6p_l^1=\frac{-4Q}{\lambda_l\,\rho_l}$$

There is then the following result:

$$\begin{Bmatrix}P_l^1\\P_l^2\end{Bmatrix}=\begin{bmatrix}\frac{N}{\lambda_l\Delta t}+6&-6\\-6&\frac{N}{\lambda_l\Delta t}+6\end{bmatrix}^{-1}\cdot\begin{Bmatrix}\frac{-2Q}{\lambda_l\,\rho_l}\\\frac{-4Q}{\lambda_l\,\rho_l}\end{Bmatrix}$$

2.2 Sizes and results of reference

2.3 Uncertainties on the solution

3 Modeling A

3.1 Characteristics of modeling

- Modeling in plane deformations `D_PLAN_HM`.
- Hydraulic behavior `LIQU_SATU`.

3.2 Characteristic of the grid

1 elements $Q8$.

3.3 Sizes and results tested

Discretization in time: 7 pas de increasing time. The list of moment in seconds is: $(1,5,10,50,100,500,1000)$.
If one looks at the geometry of the problem node A (N1 number) corresponds to the pressure p^1 and the node C with the pressure p^2

Table of results at the various moments:

$N°$ **NODE**	**Sequence number**	$PRE1(Pa)$	**Tolerance** $(\%)$
$N1, A$	1	-6.631×10^3	5
	2	-3.315×10^4	5
	3	$-6,631\times10^4$	5
	4	-3.314×10^5	5
	7	$-6,553\times10^5$	5
$N3, C$	1	1.326×10^4	5
	2	6.631×10^4	5
	3	1.326×10^5	5
	4	6.629×10^5	5
	7	1.318×10^7	5

Table 1: Results

The results are in perfect agreement with the analytical solution.

4 Modeling B

It acts of the same modeling but with a hydraulic law of behavior `LIQU_GAZ_ATM`. There is thus a constant gas phase with 1atm (assumption of Richard). The results are logically exactly the same ones as above.

5 Modeling C

It is the same modeling as in A but on a cube 3D (1 element `HEXA20`). The conditions on the faces of front and from behind are worthless flows and displacements. It is thus the same case that in 2D and the results are logically identical.

6 Modeling D

It is the same modeling as out of C but with a hydraulic law of behavior LIQU_GAZ_ATM. There is thus a constant gas phase with 1atm (assumption of Richard). The results are logically exactly the same ones as above.

7 Modeling E

Exactly the same case as modeling D but by testing the option SYME=' OUI' of linear solvor.
The results are logically the same ones as for modeling D.

8 Modeling F

This modeling is the same one as modeling D but in selective modeling. Integration is thus different: the integration of the non stationary terms is done with the nodes and that of the other terms at the points of Gauss.
The results on an element thus differ logically.

Table of results at the various moments:

$N°$ **NODE**	**Sequence number**	$PRE1(Pa)$	**Tolerance** $(\%)$
$N20, A$	1	8.79×10^{-6}	5
	2	1.84×10^{-4}	5
	3	6.24×10^{-4}	5
	4	0.018	5
	7	6.20	5
$N1, C$	1	6.63×10^{3}	5
	2	3.31×10^{4}	5
	3	6.63×10^{4}	5
	4	3.31×10^{5}	5
	7	6.68×10^{6}	5

Table 8-1: Results

9 Modeling G

With modeling G, one undertakes to validate the mesh PYRAM13 for modelings HM . One compares the behavior of one HEXA20 modeling H compared to 6 PYRAM13.

9.1 Characteristics of modeling

- Modeling in plane deformations 3D_HM.
- Hydraulic behavior LIQU_SATU.

9.2 Characteristic of the grid

6 elements $P13$.

The grid is represented on the figure 9-1.

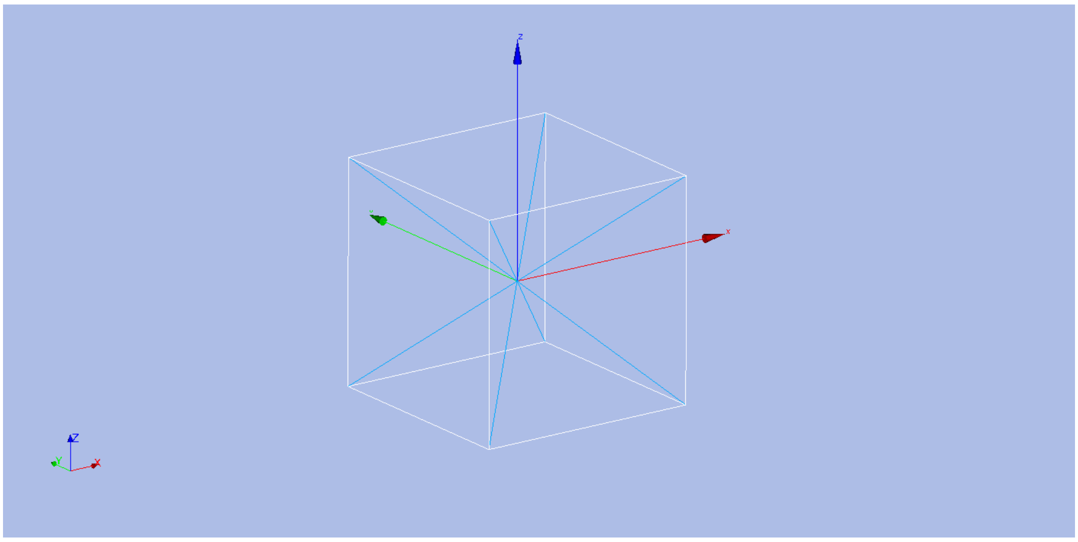


Figure 9-1 : Pyramidal grid

The transitory mode of establishment of flow being very sensitive to the structure, one decreases the reverse of the compressibility of the fluid which passes to $1.0E-18$ and one carries out the tests with the step of time 8 (10000) so as to exceed the transitory mode.

9.3 Sizes tested and results

One tests in not-regression the minimal and maximum values of the pressure of pore on the lower face and the higher face of the cube.

10 Modeling H

10.1 Characteristics of modeling

- Modeling in plane deformations 3D_HM.
- Hydraulic behavior LIQU_SATU.

10.2 Characteristic of the grid

The cube is composed of one HEXA20 instead of 6 PYRAM13.

10.3 Sizes tested and results

One tests in not-regression the minimal and maximum values of the pressure of pore on the lower face and the higher face of the cube.

These results, very close to those obtained in modeling G, ensure that the hydraulic behavior of the meshs HM_PYRAM13 is identical to that of the meshs HM_HEXA20.

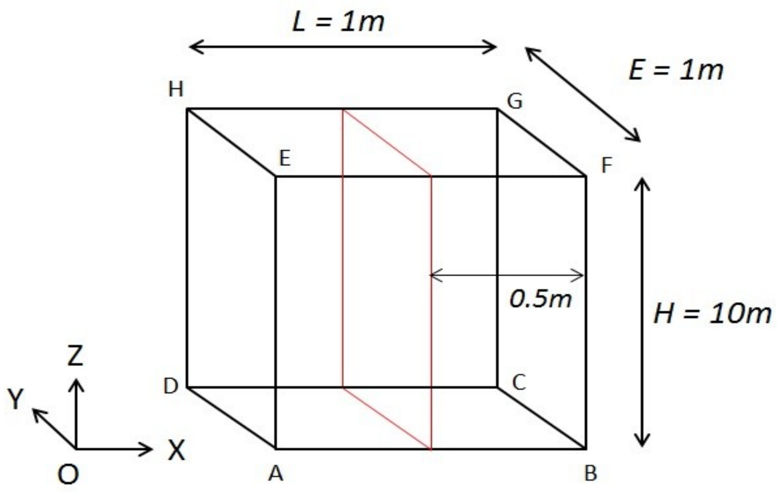
11 Modeling I

11.1 Characteristics of modeling

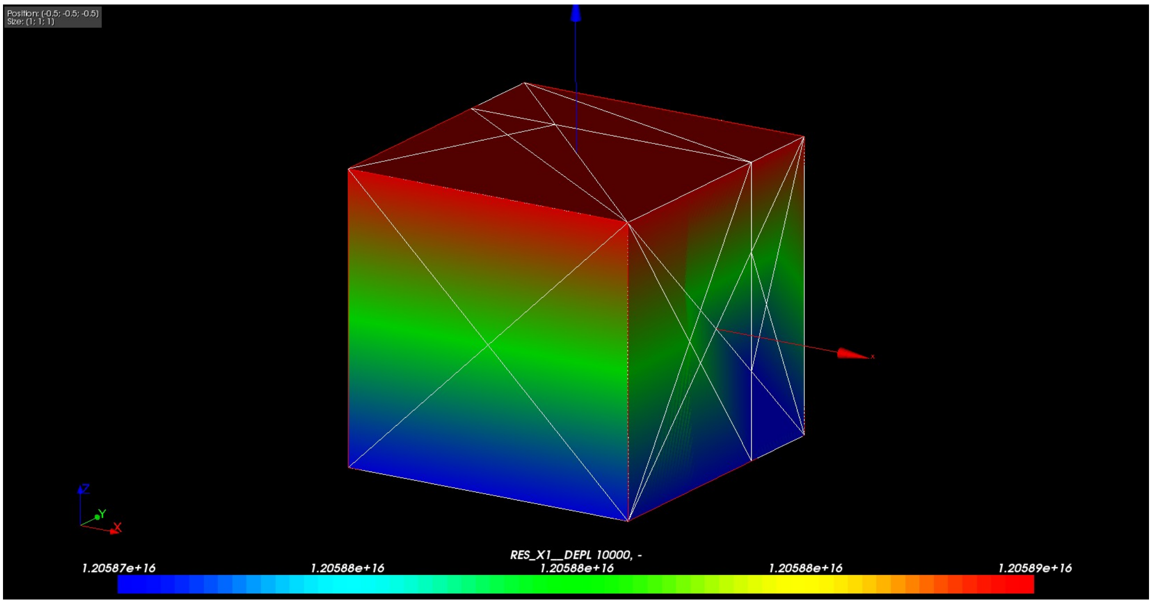
This modeling is a modeling HM-XFEM .
- Modeling in plane deformations 3D_HM ;
- Hydraulic behavior LIQU_SATU.

11.2 Characteristics of modeling

The initial grid is the same one as in modeling H but the cube is cut into two by a vertical crack.



One applies the same boundary conditions as in modeling H. In this modeling, the elements of edges located to the faces higher and lower of the cube and to which are applied the boundary conditions are elements of edge HM-XFEM nonin conformity with the crack. They are then cut out in subelements so as to be compatible with the crack. No loading is applied to the level of the crack.



11.3 Sizes and results tested

One tests in nonregression the minimal and maximum values of the pressure of pore on the lower face and the higher face of the cube.

These results, very close to those obtained in modeling H, ensure that the hydraulic behavior of elements HM-XFEM is similar to the hydraulic behavior of elements HM. In particular, the option CHAR_MECA_FLUX is validated for the elements HM-XFEM.

12 Summary of the results

The results are coherent physically. Modelings A with E admit an analytical solution which is checked perfectly.