WTNV132 - Construction of a column of ground with the law of Hujeux

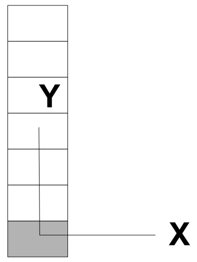
Summary

One wishes to numerically model the successive installation of the layers of a unidimensional column of ground, by taking into account at the same time the hydraulic coupling and the plasticization of the ground due to his non-linear behavior (by the law of Hujeux [R7.01.23]). The calculated solutions are compared with results resulting from the code finite elements GEFDYN Central School Paris.

1 Problem of reference

1.1 Geometry

The model consists of $N+1$ elements on the whole: it is a question of posing N soil horizons ($N=10$ in the CAS-test) on a porous elastic substratum infinitely rigid represented by one $0^{ème}$ sleep. Each layer consists of an element of grid (quadratic), a height of $2\mathrm{m}$ each one. The column of ground once built measurement thus $20\mathrm{m}$ on the whole. By principle, the problem is two-dimensional (the plane deformations occur in a vertical plan): indeed, one places oneself on the assumption of an invariance of the ground by horizontal adjustment, which imposes that the mechanical deformations and hydraulic flows are worthless in the horizontal direction (model œdometric).



1.2 Properties of materials

The elastic, unelastic properties and hydraulics of the layers are given hereafter:

	Parameters		Values
ELASTIC PROPERTIES	E	Young modulus (for the substratum, one takes $100\times E$)	$100\,MPa$
	ν	Poisson's ratio	0,3
	ρ_h	homogenized density	$2105\,kg/m^3$
PROPERTIES HUJEUX	n	exhibitor of the elastic law in power	0,89
	d		1,7
	b		1
	α	coefficient of dilatancy	1
	φ	angle of friction	21°
	ψ	angle of dilatancy	21°
	P_{co}	critical pressure or of consolidation	$25\,kPa$
	$P_{réf}$	pressure of reference	$1\,MPa$
	a_{mon}		0.005
	a_{cyc}		0.005
	c_{mon}		0.18
	c_{cyc}		0.18
	$r^m_{dév}$	elastic ray déviatoire monotonous	0.025
	r^m_{iso}	monotonous isotropic elastic ray	0.01
	$r^c_{dév}$	elastic ray déviatoire cyclic	0.025
	r^c_{iso}	cyclic isotropic elastic ray	0.01
	r_{hys}		0.1

	r_{mob}		0.5
	x_m		2
HYDRAULIC PROPERTIES	ϕ	porosity	0.35
	ρ_e	density of water	$1000\,kg/m^3$
	B	coefficient of Biot	1
	K^{-1}	opposite of the compressibility of water	$9{,}35\times 10^{-8}\,Pa^{-1}$
	K_{int}	intrinsic permeability of water	10^{-12}
	v	viscosity of water	$0{,}001\,Pa.s$
	Dv/DT	derived from viscosity by the temperature	$0\,Pa.s.K^{-1}$

1.3 Boundary conditions and loadings

In the model considered, the limiting conditions apply to $n+1$ layers present at the stage n of calculation. They are the same ones as for a oedometer (the column of ground is a sample of an infinite space by horizontal adjustment):

Conditions of horizontal invariance:

- $u_x=0$ on the side meshs;

A condition of blocking of the $0^{ème}$ sleep (presumedly rigid):

- $u_y=0$ on the mesh of bottom;

A condition of worthless water pressure at the free surface of the column:

- $PRE_1=0$ on the mesh of the top of $n+1^{ème}$ sleep (the last posed);

A state of stress *effective* initial *isotropic* and *not no one* in each layer posed, because of the aversion of the law of Hujeux for states of stress close to zero:

- $\sigma_{xx}'=\sigma_{yy}'=\sigma_{zz}'=\sigma_0'=-20.10^{+3}\,Pa$ in $n+1^{ème}$ sleep (the last posed);

Conditions of loading:

- the whole of the column is subjected to gravity (acceleration $g=9{,}81\,m/s^2$ and directed according to $-\vec{e}_y$);

The construction of the column is carried out by respecting a period of time $\Delta t=10^{+6}$ seconds between the beginning of the stage n and that of the stage $n+1$. During this period of time, there is diffusion of the fluid and consolidation of the column under the effect of its own weight (compressing). It is important to take care that this period of time is sufficient, by putting it in keeping with the value of permeability of porous material [1]. In particular, the product of Δt with this permeability a distance from diffusion of the fluid gives which must be sufficient here (about $10\,m$) compared to the dimension of the column ($20\,m$ of height).

	Elements of the model	Values
Boundary conditions	LOW	$DY=0$; **hydraulic flow no one**
	SIDE FACES	$DX=0$; **hydraulic flow no one**
	HIGH	$PRE1=0$
Initial conditions	SLEEP $n+1$ (at the stage n)	$SIXX=SIYY=SIZZ=20\,kPa$

1*One calculates initially the conductivity of the fluid starting from intrinsic conductivity:* $\lambda=\frac{K^{\text{int}}}{\upsilon}=10^{-9}\,kg^{-1}.m^3.s$ *; what gives finally for the permeability of porous material:* $K=\rho_e\times g\times\lambda=9{,}81\times 10^{-6}\,m.s^{-1}$

Loading	`ALL`	$PESANTEUR = 9,81\, m/s^2$

2 Reference solutions

2.1 Method of calculating

The method is used **multi-models** to carry out calculation. With each stage $n+1$ installation, one associates a model containing them strictly $n+1$ layers posed. The states of constraint, displacement and the internal variables at the conclusion of the preceding stage are transferred at the following stage by transfer operations from fields. The field of initial displacement of the soil horizon posed must vary linearly upwards, It varies indeed between the value of the compressing of the sub-bases and the geometrical coast to respect, associated with an initial displacement no one.

2.2 Sizes and results of reference

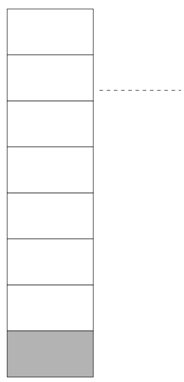
One post-draft solutions in terms of compressing. However, the use of the method multi-models does not give access directly to the compressing of the column: the vertical displacement of the last layer posed is the sum of really sudden compressing by it and compressing already carried out when it was not there: it is this last component which it is necessary to remove.
Let us consider for example layer 4 (cf appears hereafter). This one is posed at the moment $n=4$. Compressing has direction there only for $n\geq 4$. That is to say δu_4^n the increment of compressing enters the moments n and $n+1$ above layer 4. One defines the compressing of layer 4 in the moment n : Δu_4^n, as the accumulation of the increments of compressing undergone by the layer during the total process of construction of the column of ground, i.e. by the installation of the successive layers located above it ($n\geq 5$).
As follows: $\Delta u^{n\geq 4}=\sum_{i=4}^{n}\delta u_4^i$ with $\delta u_4^n=u_4^n-u_4^{n-1}$

Thus, by decomposition of the iterative process, one rewrites easily compressing as being:

$$\Delta u_4^{n\geq 4}=u_4^n-u_4^0 .$$

i.e. displacement above the 4ème sleep at the moment n (under the action of the layers located above it), *less its displacement during its installation* ($n=4$).

The validation is carried out by comparison with solutions GEFDYN provided by the Central School Paris.



At the moment $n\geq 4$:

$$\Delta \boldsymbol{u}_4^{n\geq 4}=\sum_{i=4}^{n}\delta \boldsymbol{u}_4^i$$
$$=\sum_{i=4}^{n}\boldsymbol{u}_4^i-\boldsymbol{u}_4^{i-1}$$
$$=\boldsymbol{u}_4^n-\boldsymbol{u}_4^4=\boldsymbol{u}_4^n-\boldsymbol{u}_{4,0}$$

For modeling C, one also carries out the elementary calculation of option `PDIL_ELGA` with an aim of validating the digital developments for modelings `HM`. The values obtained are tested in not-regression.

2.3 Uncertainties on the solution

The results established at the time of modeling with the software Finite elements GEFDyn of the Central School Paris are precise according to the levels of the convergence criteria used in this software. The definition of the convergence criteria is specified in the instruction manual of the software [1]. The value of the criteria relating to displacements and the water pressure is equal to 10^{-3} and the value of the criteria relating to mechanical imbalances (forces) and hydraulics (flow) is equal to 10^{-2} .

2.4 Bibliographical references

[1] D. Aubry, A. Modaressi. *GEFDyn, Scientific Maneul*. Central school Paris, LMSS-Chechmate, 1996.

3 Modeling A

3.1 Characteristics of modeling

Modeling **With** is *three-dimensional* and *quasi-static*. One is used *modeling with selective integration* (MODELING = 'HMS').

3.2 Characteristics of the grid

The grid used is composed of 11 HEXA20, that is to say an element for each layer of the column of ground (10 elements), and the rock on which puts back the column of ground.

3.3 Sizes tested and results

Compressing is calculated at the top of each layer and compared to solutions given GEFDYN:

Compressing (in millimetres) of the n°1 layer

Number of stage	Code_Aster	GEFDYN	relative error
2	-4,573	-4,648	-1,622%
3	-8,297	-8,391	-1,116%
4	-11,828	-11,940	-0,935%
5	-15,356	-15,440	-0,543%
6	-18,931	-18,980	-0,260%
7	-22,587	-22,650	-0,279%
8	-26,368	-26,470	-0,384%
9	-30,307	-30,490	-0,600%
10	-34,419	-34,730	-0,895%

Compressing (in millimetres) of the n°2 layer

Number of stage	Code_Aster	GEFDYN	relative error
3	-9,093	-8,409	8,136%
4	-16,386	-15,720	4,236%
5	-23,453	-22,780	2,956%
6	-30,555	-29,830	2,430%
7	-37,785	-37,030	2,039%
8	-45,222	-44,510	1,600%
9	-52,942	-52,360	1,112%
10	-60,992	-60,610	0,630%

Compressing (in millimetres) of the n°3 layer

Number of stage	Code_Aster	GEFDYN	relative error
4	-12,010	-11,980	0,251%
5	-22,813	-22,800	0,055%
6	-33,445	-33,410	0,105%
7	-44,202	-44,120	0,186%
8	-55,213	-55,150	0,115%
9	-66,589	-66,650	-0,091%
10	-78,419	-78,730	-0,395%

Compressing (in millimetres) of the n°4 layer

Number of stage	Code_Aster	GEFDYN	relative error
5	-16,280	-15,470	5,238%
6	-30,682	-29,840	2,821%
7	-44,984	-44,110	1,981%
8	-59,522	-58,650	1,486%
9	-74,471	-73,690	1,060%
10	-89,956	-89,430	0,588%

Compressing (in millimetres) of the n°5 layer

Number of stage	Code_Aster	GEFDYN	relative error
6	-20,109	-19,020	5,725%
7	-38,202	-37,050	3,109%
8	-56,300	-55,150	2,085%
9	-74,776	-73,700	1,460%
10	-93,833	-92,980	0,917%

Compressing (in millimetres) of the n°6 layer

Number of stage	Code_Aster	GEFDYN	relative error
7	-23,832	-22,680	5,080%
8	-45,723	-44,540	2,657%
9	-67,763	-66,650	1,669%
10	-90,345	-89,440	1,012%

Compressing (in millimetres) of the n°7 layer

Number of stage	Code_Aster	GEFDYN	relative error
8	-27,632	-26,500	4,271%
9	-53,465	-52,380	2,072%
10	-79,610	-78,720	1,131%

Compressing (in millimetres) of the n°8 layer

Number of stage	Code_Aster	GEFDYN	relative error
9	-31,369	-30,520	2,783%
10	-61,288	-60,630	1,085%

Compressing (in millimetres) of the n°9 layer

Number of stage	Code_Aster	GEFDYN	relative error
10	-35,435	-34,750	1,973%

3.4 Comments

The relative error is of to the maximum 9% , which is relatively satisfactory.

4 Modeling B

4.1 Characteristics of modeling

Modeling **B** is *Bidimensional* and *quasi-static*. One is used *modeling with classical integration* (MODELING = 'HM'). One superimposes on the hydro-mechanical problem "macroscopic" thus defined a model of *second gradient of dilation*, known as "microscopic", the parameters constitutive of the model of second gradient are given in order not to modify the basic solution of the macroscopic problem. In particular, the small coefficient of penalization is chosen: PENA_LAGR = 1,

The model of second gradient is used as a patch that one superimposes on the macroscopic problem: with the initial grid (here QUAD8), it is necessary to superimpose a grid (made up of QUAD9) on which the model of second gradient will be defined. It is what is made in this modeling where as starter a grid is given QUAD8 double, one of both is transformed into QUAD9 using the order: CREA_MAILLAGE → MODI_MAILLAGE → OPTION=' QUAD8_9',

The model of second gradient makes it possible to treat the phenomena of instability material related to the loss of ellipticity of the tensor stress-strain, and which result in a localization of the deformations, Here, one really does not seek to regularize the problem (since there is no material instability), but to implement the model of second gradient in a relatively representative CAS-test,

4.2 Characteristics of the grid

For modeling DPLAN_HM, each soil horizon is represented by an element QUAD8, that is to say 11 elements on the whole. The grid of superposition consists of elements QUAD9, that is to say also 11 elements on the whole.

One superimposes on the problem of hydraulic coupling macroscopic a model of second microscopic gradient of dilation, In the integration of the equilibrium equations, one thus asks for a reactualization of the tangent matrix, which is provided by the routines of the law of Hujeux and accelerates convergence appreciably. One also asks for the subdivision of the step of time (order DEFI_LIST_INST) to treat the situations of failure of the local integration of with increments of too large loading. *This functionality is largely recommended*.

4.3 Sizes tested and results

Unchanged compared to modeling **A.**

4.4 Comments

These results validate the capacity of the modeling of second gradient of dilation to being used for calculations with a loading of gravity.

5 Modeling C

5.1 Characteristics of modeling

Modeling **C** is *Bidimensional* and *quasi-static*. One is used *modeling with classical integration* (MODELING = 'HM'). The first difference with modeling **With** is dependent on the way of a modeling 3D_HM with a modeling DPLAN_HM. The second difference holds with the use of a secant matrix of rigidity for the total resolution of the balance of the structure between the internal forces and the external efforts applied.

5.2 Characteristics of the grid

Each layer is represented by an element QUAD8. The complete grid thus consists of 11 elements QUAD8.

5.3 Sizes tested and results

Compressing is calculated at the top of each layer and compared to solutions given GEFDYN:

Compressing (in millimetres) of the n°1 layer

Number of stage	GEFDYN	Tolerance (%)
2	-4,648	4.00
3	-8,391	3.00
4	-11,940	2.00
5	-15,440	2.00
6	-18,980	2.00
7	-22,650	2.00
8	-26,470	2.00
9	-30,490	2.00
10	-34,730	2.00

Compressing (in millimetres) of the n°2 layer

Number of stage	GEFDYN	Tolerance (%)
3	-8,409	3.00
4	-15,720	2.00
5	-22,780	2.00
6	-29,830	2.00
7	-37,030	2.00
8	-44,510	2.00
9	-52,360	2.00
10	-60,610	2.00

Compressing (in millimetres) of the n°3 layer

Number of stage	GEFDYN	Tolerance (%)
4	-11,980	2.00
5	-22,800	2.00
6	-33,410	2.00
7	-44,120	2.00
8	-55,150	2.00
9	-66,650	2.00

10	-78,730	2.00

Compressing (in millimetres) of the n°4 layer

Number of stage	GEFDYN	Tolerance (%)
5	-15,470	2.00
6	-29,840	2.00
7	-44,110	2.00
8	-58,650	2.00
9	-73,690	2.00
10	-89,430	2.00

Compressing (in millimetres) of the n°5 layer

Number of stage	GEFDYN	Tolerance (%)
6	-19,020	2.00
7	-37,050	2.00
8	-55,150	2.00
9	-73,700	2.00
10	-92,980	2.00

Compressing (in millimetres) of the n°6 layer

Number of stage	GEFDYN	Tolerance (%)
7	-22,680	2.00
8	-44,540	2.00
9	-66,650	2.00
10	-89,440	2.00

Compressing (in millimetres) of the n°7 layer

Number of stage	GEFDYN	Tolerance (%)
8	-26,500	2.00
9	-52,380	2.00
10	-78,720	2.00

Compressing (in millimetres) of the n°8 layer

Number of stage	GEFDYN	Tolerance (%)
9	-30,520	2.00
10	-60,630	2.00

Compressing (in millimetres) of the n°9 layer

Number of stage	GEFDYN	Tolerance (%)
10	-34,750	2.00

The elementary option of calculation INDL_ELGA is also tested in not-regression to validate its development in modeling DPLAN_HM. All the components are tested on the mesh $M2$, located at the base of the column, just above elastic rock.

Number of stage	Component INDL_ELGA	Type of reference	Reference	Tolerance (absolute)
10	INDEX	NON_REGRESSION	0.0	0,001
10	DIR1	NON_REGRESSION	0.0	0,001

10	DIR2	NON_REGRESSION	0.0	0,001
10	DIR3	NON_REGRESSION	0.0	0,001
10	DIR4	NON_REGRESSION	0.0	0,001

The elementary option of calculation PDIL_ELGA is tested in not-regression to validate its development in modeling DPLAN_HM. The component A1_LC2 is tested on the mesh $M2$, located at the base of the column, just above elastic rock.

Number of stage	Component INDL_ELGA	Type of reference	Reference	Tolerance (absolute)
10	A1_LC2	NON_REGRESSION	0.0	0,001

5.4 Comments

The relative error is of to the maximum 4% , which is relatively satisfactory. The results are overall closer to the results resulting from the reference, in comparison with modelings **With** and **B**. One can add that the algorithm used in this modeling to solve the balance of the structure is identical to that of the reference, which can explain these results.

6 Modeling D

6.1 Characteristics of modeling

Modeling **D** is *Bidimensional* and *quasi-static*. It is identical to modeling **C**, with three differences near:

- one is used *under-integrated modeling* (MODELING = 'D_PLAN_HM_SI') instead of classical modeling (MODELING = 'D_PLAN_HM');
- one carries out a redimensioning of the hydro-mechanical problem saturated by applying the factors with redimensioning $P_0=10^{+6}$ and $K_0=10^{-5}$;
- The convergence criteria used are RESI_REFE_RELA = 10^{-4}, with SIGM_REFE = 1 and FLUX_HYD1_REFE = 1;

6.2 Characteristics of the grid

Unchanged compared to modeling **C**.

6.3 Sizes tested and results

Unchanged compared to modeling **C**. Postprocessing must take account of the reverse transformation towards the units of origin.

6.4 Comments

These results validate under-integrated modeling and the method of redimensioning.

7 Modeling E

7.1 Characteristics of modeling

Modeling **E** is *Bidimensional* and *quasi-static*. It is identical to modeling **D**, except that the convergence criteria used are RESI_GLOB_RELA = 10^{-8}.

7.2 Characteristics of the grid

Unchanged compared to modeling **D**.

7.3 Sizes tested and results

Unchanged compared to modeling **D**.

7.4 Comments

These results validate under-integrated modeling and the method of redimensioning.

8 Summary of the results

One represents in **Figure 1** a comparison of compressings calculated along the column of ground by *Code_Aster* and *GEFDYN* (using modeling **With**), As one can note it, the coincidence of the results is rather satisfactory.

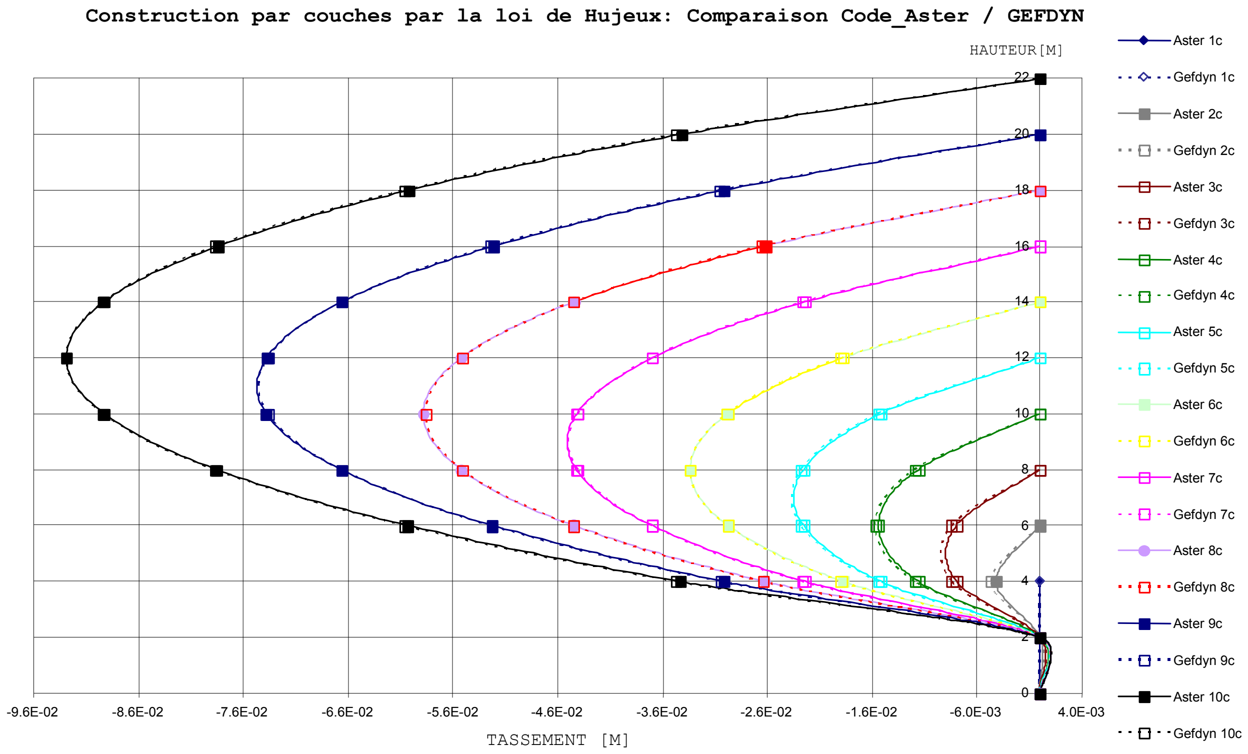


Figure 1 : *Compressing in the column of ground to each stage of calculation: comparison of the Code_Aster solutions and GEFDYN.*