WTNV141 – Validation of an evolutionary loading for a saturated hydraulic problem

Summary:

The purpose of this test is to validate the keyword EVOL_CHAR in THM in 2D and 3D. One has an analytical solution.

1 Problem of reference

1.1 Geometry

In 2D, a square on side is considered $1\mathrm{m}$. In 3D, a cube on side is considered $1\mathrm{m}$.

1.2 Properties of material

Intrinsic permeability: $\kappa = 0.05\, m^2.\, Pa^{-1}.\, s^{-1}$
Coefficient of Biot: $b = 1.$
Young modulus: $E = 2.5\, Pa$
Poisson's ratio: $\nu = 0.25$

There are thus the following coefficients of Lamé: $\lambda = \mu = 1.0\,\mathrm{Pa}$

1.3 Boundary conditions and loadings

The boundary conditions are the conditions of Dirichlet corresponding to the analytical solution one has.

In 2D, one defines $A = 2\times\pi^2\times\kappa$ and the voluminal mechanical loading is defined:

$$f(t, x, y) = \begin{bmatrix} \cos(\pi x)\sin(\pi y) \\ \sin(\pi x)\cos(\pi y) \end{bmatrix} \pi e^{-At} \left[-(\lambda+2\mu)+b\right]$$

In 3D, one defines $A = 3\times\pi^2\times\kappa$ and the voluminal mechanical loading is defined:

$$f(t, x, y, z) = \begin{bmatrix} \cos(\pi x)\sin(\pi y)\sin(\pi z) \\ \sin(\pi x)\cos(\pi y)\sin(\pi z) \\ \sin(\pi x)\sin(\pi y)\cos(\pi z) \end{bmatrix} \pi e^{-At} \left[-(\lambda+2\mu)+b\right]$$

1.4 Initial conditions

The initial conditions correspond to the analytical solution one has.

2 Reference solution

2.1 Method of calculating

One points out the system of equations which one solves:

$$\begin{cases} -\nabla\cdot\sigma(u)+b\nabla p & =f \\ \partial_t(\nabla\cdot u)-\kappa\Delta p & =0 \end{cases}$$

where $\sigma(u)=\lambda\nabla\cdot u I_d+2\mu\varepsilon(u)$ and I_d indicate the matrix identity in dimension d .

In 2D, there is the following analytical solution:

$$p(t,x,y)=e^{-At}\sin(\pi x)\sin(\pi y)$$

$$u(t,x,y)=-\begin{bmatrix}\cos(\pi x)\sin(\pi y)\\ \sin(\pi x)\cos(\pi y)\end{bmatrix}\frac{e^{-At}}{2\pi}$$

In 3D, there is the following analytical solution:

$$p(t,x,y)=e^{-At}\sin(\pi x)\sin(\pi y)\sin(\pi z)$$

$$u(t,x,y)=-\begin{bmatrix}\cos(\pi x)\sin(\pi y)\sin(\pi z)\\ \sin(\pi x)\cos(\pi y)\sin(\pi z)\\ \sin(\pi x)\sin(\pi y)\cos(\pi z)\end{bmatrix}\frac{e^{-At}}{3\pi}$$

2.2 Sizes and results of reference

One compares the digital solution with the analytical solution in 3 points distinct from the grid (in displacements and pressure).

2.3 Uncertainties on the solution

Analytical solution

3 Modeling A

3.1 Characteristics of modeling

Modeling used is D_PLAN_HMS. The duration of simulation is fixed at $T_1 = 0.1\,s$ with 10 pas de time.

3.2 Characteristics of the grid

2048 meshs TRIA6.

3.3 Sizes tested and results

One tests the values of the digital solution at the final moment $T_1 = 0.1\,s$ in 3 nodes.

Name node	X (m)	Y (m)	Analytical solution		Relative error
$N25$	0.75	0.75	PRE1	4.53E-1	0.7%
			DX	7.21E-2	0.2%
			DY	7.21E-2	0.2%
$N40$	0,875	0,125	PRE1	1.33E-1	0.75%
			DX	5.10E-2	0.2%
			DY	-5.10E-2	0.2%
$N35$	0,375	0,625	PRE1	7.73E-1	0.8%
			DX	-5.10E-2	0.2%
			DY	5.10E-2	0.2%

Table 3.3-1

3.4 Remarks

The digital results are in very good agreement with the analytical results.

4 Modeling B

4.1 Characteristics of modeling

Modeling used is 3D_HMS. Durea of simulation is fixed at $T_1=0.01\,s$ with 4 pas de time.

4.2 Characteristics of the grid

1000 meshs HEXA20.

4.3 Sizes tested and results

One tests the values of the digital solution at the final moment $T_1=0.1\,s$ in 3 distinct nodes.

Name node	X (m)	Y (m)	Z (m)	Analytical solution		Relative error
N1948	0.8	0.2	0.2	PRE1	2.00E-1	1.2%
				DX	2.92E-2	0.2%
				DY	-2.92E-2	0.2%
				DZ	-2.92E-2	0.2%
N1900	0.2	0.8	0.2	PRE1	2.00E-1	1.2%
				DX	-2.92E-2	0.2%
				DY	2.92E-2	0.2%
				DZ	-2.92E-2	0.2%
N2380	0.2	0.2	0.8	PRE1	2.00E-1	1.2%
				DX	-2.92E-2	0.2%
				DY	-2.92E-2	0.2%
				DZ	2.92E-2	0.2%

Table 4.3-1

4.4 Remarks

The digital results are in very good agreement with the analytical results.

5 Summary of the results

The digital results are in perfect agreement with the analytical solutions.