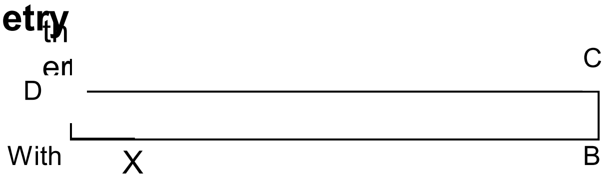
WTNP103 - Diffusion of air dissolved in water (plan)

Summary:

One considers here a problem at temperature and constant saturation. By boundary conditions suitable one imposes a water pressure and a steam pressure constants. A gas pressure is imposed on an edge of the field (worthless flows of the other with dimensions). Only the air pressures dryness and of dissolved air connected by the law of Henry evolve. This problem is brought back in an equation for the air pressure dryness of type "equation of heat". The reference solution will be then a thermal calculation ASTER.

1 Problem of reference

1.1 Geometry



Coordinates of the points (m) :

A	0	0	C	1	0.5
B	1	0	D	0	0.5

1.2 Properties of material

One gives here only the properties whose solution depends, knowing that the command file contains other data of material (moduli of elasticity, thermal conductivity…) who finally do not play any part in the solution of with the dealt problem.

Liquid water	Density ($kg.\,m^{-3}$)	10^3
	Specific heat with constant pressure ($J.K^{-1}$)	0.
	Dynamic viscosity of liquid water ($Pa.s$)	0,001
	thermal dilation coefficient of the liquid (K^{-1})	0.
	Permeability relating to water	$kr_w(S)=0.5$
Vapor	Specific heat ($J.K^{-1}$)	0.
	Molar mass ($kg.mol^{-1}$)	0.01
Gas	Specific heat ($J.K^{-1}$)	0.
	Molar mass ($kg.mol^{-1}$)	0.01
	Permeability relating to gas	$kr_{gz}(S)=0.5$
	Viscosity of gas ($kg.m^{-1}.s^{-1}$)	0,001
Dissolved air	Specific heat ($J.K^{-1}$)	0.
	Constant of Henry ($Pa.m^{3.}\,mol^{-1}$)	50000
Initial state	Porosity	1
	Temperature (K)	300
	Gas pressure (Pa)	1.01E5
	Steam pressure (Pa)	1000
	Capillary pressure (Pa)	1.^{E}6
	Initial saturation in liquid	0.4
Constants	Constant of perfect gases	8.32
Homogenized coefficients	Homogenized density ($kg.m^{-3}$)	2200

Isotherm of sorption	$S(P_c)=0.4$
Coefficient of Biot	0
Fick Vapor ($m^{2.} s^{-1}$)	$FV=0$
Fick dissolved air ($m^{2.} s^{-1}$)	$FA=6\ .E-10$
Intrinsic permeability (m^2)	$Kint=1.\mathrm{E}-19$

1.3 Boundary conditions and loadings

On the whole of the field, one wants:

$$p_w = cte = p_w^0$$
$$\frac{1}{K_w} = 0 \Rightarrow \rho_w = cte = \rho_w^0$$
$$p_{vp} = cte = p_{vp}^0$$
$$F_{vp} = 0$$
$$S(p_c) = cte = S_0$$
$$T = cte = T^0$$
$$\phi = 1$$
$$M_{as}^{ol} = M_{vp}^{ol} = M_{ad}^{ol}$$

On all the edges: Hydraulic flows and worthless thermics.

One now will linearize p_{vp} according to p_w.

Writing of p_{vp} linear function of p_w :

Section 4.2.3 of the reference document *Aster* [R7.01.11] the relation gives us: $\frac{dp_{vp}}{p_{vp}} = \frac{M_{vp}^{ol}}{RT} \frac{dp_w}{\rho_w}$.

If this expression is linearized one obtains: $p_{vp} = \frac{p_{vp}^0}{RT} \frac{M_{vp}^{ol}}{\rho_w^0} p_w + \left(p_{vp}^0 - \frac{p_{vp}^0}{RT} \frac{M_{vp}^{ol}}{\rho_w^0} p_w^0 \right)$ that one can write in the form:

$$p_{vp} = A p_w + B \qquad \text{éq 1.3-1}$$

with $A = \frac{p_{vp}^0}{RT} \frac{M_{vp}^{ol}}{\rho_w^0}$ and $B = p_{vp}^0 - \frac{p_{vp}^0}{RT} \frac{M_{vp}^{ol}}{\rho_w^0} p_w^0$

Sur le bord AB : $p_{gz} = 115000$
$pc = 10E6$

2 Reference solution

2.1 Method of calculating

2.1.1 Calculation of the conservation of the mass of air

The conservation of the gas mass is written:

$$\frac{dm_{air}}{dt} + div(\mathbf{M}_{as} + \mathbf{M}_{ad}) = 0 \qquad \text{éq 2.1.1-1}$$

It is written that the total water mass and the total mass of air are preserved (because there is no gas water flow nor at the edge) and one obtains:

$$m_{air} = m_{as} + m_{ad} = S_0(\rho_{ad} - \rho_{ad}^0) + (1 - S_0)(\rho_{as} - \rho_{as}^0)$$

thus

$$d(m_{as} + m_{ad}) = S_0 d\rho_{ad} + (1 - S_0) d\rho_{as} \qquad \text{éq 2.1.1-2}$$

$d\rho_{as} = \frac{M_{as}^{ol}}{RT} dP_{as}$ and $d\rho_{ad} = \frac{M_{ad}^{ol}}{K_H} dP_{as}$

$$\frac{dm_{air}}{dt} = \left[S_{0.} \frac{M_{as}^{ol}}{K_H} + (1 - S_0) \frac{M_{as}^{ol}}{RT} \right] \frac{dP_{as}}{dt}$$

Calculation speeds:

$$\frac{\mathbf{M_{as}}}{\rho_{as}} = \lambda_{gz}(-\nabla P_{as}) \qquad \text{éq 2.1.1-3}$$

since $F_{vp} = 0$ and $\nabla P_{vp} = 0$

and

$\mathbf{M}_{\mathbf{a}d} = \rho_{ad} \lambda_{lq}(-\nabla P_{lq}) - F_{ad} \nabla C_{ad}$ with $C_{ad} = \rho_{ad}$

Like $\nabla P_{lq} = \nabla P_w + \nabla P_{ad} = \nabla P_{ad} = \frac{RT}{K_H} \nabla P_{as}$

$$\mathbf{M}_{\mathbf{a}d} = \rho_{ad} \lambda_{lq} \frac{RT}{K_H}(-\nabla P_{as}) - .\frac{M_{ad}^{ol}}{K_H}.F_{ad} \nabla P_{as}$$

[éq 2.1.1-1] can then be simplified in the following form:

$$C \frac{dP_{as}}{dt} = L div(\nabla P_{as})$$

$$\text{with} \begin{cases} C = S_{0.} \frac{M_{as}^{ol}}{K_H} + (1 - S_0) \frac{M_{as}^{ol}}{RT} \\ L = \rho_{as}^0 \lambda_{gz} + \frac{RT}{K_H} \rho_{ad}^0 \lambda_{lq} + \frac{M_{as}^{ol}}{K_H}.F_{ad} \end{cases}$$

Equation of the heat whose one knows the result.

2.2 Results of reference

With the preceding digital values, one finds:

$$P_{as} = 10^5 \Rightarrow P_{ad}^0 = \frac{RT}{K_H} P_{as}^0 = 4992$$

$\rho_{as}^0 = \frac{M_{as}^{ol}}{RT} P_{as}^0 = 0.4$ and $\rho_{ad}^0 = \frac{M_{ad}^{ol}}{RT} P_{ad}^0 = 0.02$

$$\rho_{vp}^0 = \rho_{vp} = 4.10^{-3}$$

The constants of the equation of heat are then:

$$C = 2{,}4810^{-6}$$
$$L = 1{,}4.10^{-16}$$

2.3 Uncertainties

Uncertainties are rather large because the analytical solution is a solution approached because of linearization of the equations.

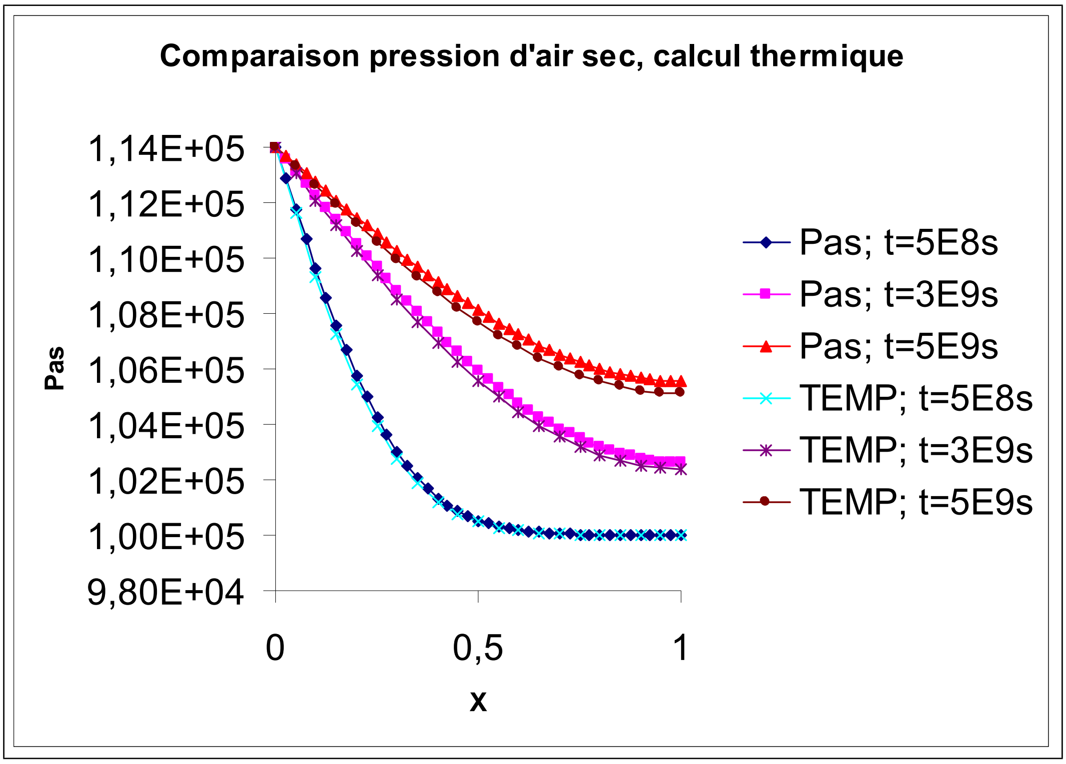
3 Modeling A

3.1 Characteristics of modeling A

Modeling in plane deformations. 20 elements `QUAD8`.

3.2 Sizes tested and results

$X(m)$	**Time** (s)	$PRE2$ **Aster**	$PRE2$ **thermal calculation**	**Relative error**
0.2	3,00E+009	1.128E4	1.120E4	0.73%
0.2	5,00E+009	1.127E4	1.224E4	0.24%



4 Summary of the results

The results are in very good agreement with the semi-analytical solution.