WTNP106 - Heating of a porous environment désaturé with dissolved air

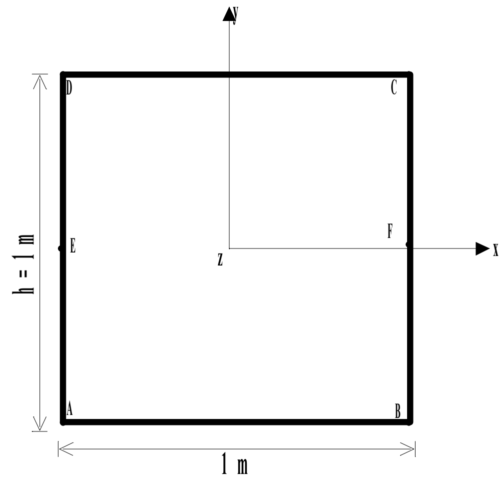
Summary:

One heats a porous environment of which the pores are filled with a mixture of water (liquid and vapor) and of air (dry and dissolved in water). Initial saturation in liquid is of 50%, the loading is a uniform heat flux on the edges of the field. The modeling made by only one element corresponds to the modeling of a homogeneous problem in space.

The reference solution is an approximate analytical solution.

1 Problem of reference

1.1 Geometry



Coordinates of the points (m) :

A	-0.5	-0.5	C	0.5	0.5
B	0.5	-0.5	D	-0.5	0.5

1.2 Properties of material

One gives here only the properties whose solution depends, knowing that the command file contains other data of material (moduli of elasticity, thermal conductivity…) who finally do not play any part in the solution of with the dealt problem.

Liquid water	Density ($kg.m^{-3}$)	10^3
	Heat with constant pressure ($J.K^{-1}$)	4180
	thermal dilation coefficient of the liquid (K^{-1})	0.
	Dynamic viscosity of liquid water ($Pa.s$)	0,001
	Permeability relating to water	$kr_w(S)=1$
Vapor	Specific heat ($J.K^{-1}$)	1900
	Initial enthalpy (latent heat of vaporization) J/Kg	2,5E6.
	Molar mass ($kg.mol^{-1}$)	0.018
Gas	Specific heat ($J.K^{-1}$)	1900
	Molar mass ($kg.mol^{-1}$)	0.018
	Permeability relating to gas	$kr_{gz}(S)=1$
	Viscosity of gas ($kg.m^{-1.s^{-1}}$)	1.8^{E}-5

Dissolved air	Specific heat ($J.K^{-1}$)	1900
	Constant of Henry ($Pa.m^{3.} mol^{-1}$)	50000
Skeleton	Heat-storage capacity with constant constraint ($J.K^{-1}$)	1050
Initial state	Porosity	0.3
	Temperature (K)	300
	Gas pressure (Pa)	1E5
	Steam pressure (Pa)	3700
	Initial saturation in liquid (Pa)	0.5
Constants	Constant of perfect gases	8.315
Homogenized coefficients	Homogenized density ($kg.m^{-3}$)	2200
	Isotherm of sorption	$S(P_c)=0.5-10^{-12}(P_c-P_{vp}^0-P_c^0)$ With $P_{vp}^0=3700$ $P_c^0=0$

1.3 Boundary conditions and loadings

On all the edges:

Heat flux $\mathbf{q}_{ext}.\mathbf{n}=10^6$

Hydraulic flow no one

2 Reference solution

2.1 Method of calculating

2.1.1 Calculation of the steam pressure starting from the temperature

We suppose the linear curve of saturation. She is thus written:

$$S = S_0 + S' \Delta P_c \qquad \text{éq 2.1.1-1}$$

The equation [éq 2.2.3.3 - 2] of the reference document [R7.01.11] gives then:

$$\begin{aligned}
\Delta m_w &= \rho_w \phi S' \Delta P_c \\
\Delta m_{vp} &= \left(\rho_{vp} - \rho_{vp}^0\right)\phi^0 (1 - S_0) - S' \rho_{vp}^0 \phi^0 \Delta P_c \\
\Delta m_{ad} &= \left(\rho_{ad} - \rho_{ad}^0\right)\phi^0 S_0 + S' \rho_{ad}^0 \phi^0 \Delta P \\
\Delta m_{as} &= \left(\rho_{as} - \rho_{as}^0\right)\phi^0 (1 - S_0) - S' \rho_{as}^0 \phi^0 \Delta P_c
\end{aligned} \qquad \text{éq 2.1.1-2}$$

It is written that the total water mass and the total mass of air are preserved (because there is no gas water flow nor at the edge) and one obtains:

$$\begin{aligned}
&\Delta m_w + \Delta m_{vp} = 0 \quad \Rightarrow \\
&\left(\rho_w - \rho_{vp}\right) S' \Delta P_c + \left(\rho_{vp} - \rho_{vp}^0\right)(1 - S_0) = 0
\end{aligned} \qquad \text{éq 2.1.1-3}$$

$$\begin{aligned}
&\Delta m_{ad} + \Delta m_{as} = 0 \quad \Rightarrow \\
&\left(\rho_{ad} - \rho_{as}\right) S' \Delta P_c + \left(\rho_{as} - \rho_{as}^0\right)(1 - S_0) + \left(\rho_{ad} - \rho_{ad}^0\right) S_0 = 0
\end{aligned} \qquad \text{éq 2.1.1-4}$$

[R7.01.11] [éq 4.1.4-1] gives in addition:

$$\ln\left(\frac{p_{vp}}{p_{vp}^0}\right) = \frac{M_{vp}^{ol}}{\rho_w^0}\left(\frac{1}{RT} - \frac{1}{K_H}\right)\left(p_{gz} - p_{gz}^0\right) + \frac{M_{vp}^{ol}}{\rho_w^0 K_H}\left(p_{vp} - p_{vp}^0\right) - \frac{M_{vp}^{ol}}{\rho_w^0 R T^+}\left(p_c - p_c^0\right) + \frac{M_{vp}^{ol} R}{\rho_w^0 K_H}\left(p_{vp} - p_{gz}\right)\ln\left(\frac{T}{T^0}\right) + \frac{M_{vp}^{ol}}{R}\int_{T^0}^{T}\left(h_{vp}^m - h_w^m\right)\frac{dT}{T^2} \qquad \text{éq 2.1.1-5}$$

Coupling of the equations [éq 2.1.1-3], [éq 2.1.1-4] and [éq 2.1.1-5], for which it the equation of perfect gases for the vapor, dry air and dissolved air are necessary to add as well as the law of Henry is a strongly nonlinear system which we will solve in small disturbances, which makes it possible to linearize it.

All done calculations, one obtains:

$$\Delta P_{vp}\left[(\rho_w-\rho_{vp}^0)S+\frac{(1-S_0)M_{vp}^{ol}}{RT^0}\right]-(\rho_w-\rho_{vp}^0)S\Delta P_w+\Delta P_{as}\left[(\rho_w-\rho_{vp}^0)S\left(1-\frac{RT^0}{K_H}\right)\right]+$$

$$\left[-(\rho_w-\rho_{vp}^0)S\frac{RP_{as}}{K_H}-\frac{M_{vp}^{ol}P_{vp}^0}{RT^{0\,2}}(1-S_0)\right]\Delta T=0$$

$$\Delta P_{vp}((\rho_{ad}^0-\rho_{as}^0)S)-(\rho_{ad}^0-\rho_{as}^0)S\Delta P_w+\Delta P_{as}\left[(\rho_{ad}^0-\rho_{as}^0)S\left(1-\frac{RT^0}{K_H}\right)+M_{vp}^{ol}\left(\frac{S_0}{K_H}+\frac{(1-S_0)}{RT^0}\right)\right]+$$

$$\left[-(\rho_{ad}^0-\rho_{as}^0)S\frac{RP_{as}}{K_H}-\frac{M_{vp}^{ol}P_{as}}{RT^{0\,2}}(1-S_0)\right]\Delta T=0$$

$$\Delta P_{vp}\left[-\frac{1}{P_{vp}^0}\right]+\frac{M_{vp}^{ol}}{\rho_w^0RT}\Delta P_w+$$

$$\left[\frac{M_{vp}^{ol}}{\rho_w^0}\frac{P_{as}}{K_HT^0}(1-R)+\frac{M_{vp}^{ol}}{R}\frac{h_{vp}^m-h_w^m}{T^{0\,2}}\right]\Delta T=0$$

éq 2.1.1-6

2.1.2 Calculation of the temperature

The equation [éq 3.2.4.3 - 1] of the reference document [R7.01.11] gives:

$$\Delta Q'=-3\alpha_{gz}^mT\Delta p_{gz}+C_\varepsilon^0\Delta T \qquad \text{éq 2.1.2-1}$$

(since the other dilation coefficients are worthless).

The equation [éq 3.2.4.3 - 2] gives:

$$\alpha_{gz}^m=\frac{\phi(1-S_{lq})}{3T} \qquad \text{éq 2.1.2-2}$$

One thus obtains:

$$\Delta Q'=-\phi(1-S_{lq})(\Delta p_{vp}+\Delta p_{as})+C_\varepsilon^0\Delta T \qquad \text{éq 2.1.2-3}$$

In this problem, $\Delta Q'$ is anything else only the heat brought per unit of volume.

While calling Vol the total volume of the part and $Surf$ its side surface and Δt the time of application of flows:

$$\Delta Q'=\Delta t\frac{Surf}{Vol}\mathbf{q}_{ext}.\mathbf{n} \qquad \text{éq 2.1.2-4}$$

2.1.3 System to be solved

$$
\begin{pmatrix}
\left(\rho_w - \rho_{vp}^0\right)S' + \dfrac{(1-S_0)M_{vp}^{ol}}{RT^0} & -\left(\rho_w - \rho_{vp}^0\right)S' & -(1-S_0)p_{vp}^0\dfrac{M_{vp}^{ol}}{RT^{0^2}}.-\left(\rho_w - \rho_{vp}^0\right)S'\dfrac{RP_{as}^0}{K_H} & \left(\rho_w - \rho_{vp}^0\right)S'(1-\dfrac{RT^0}{K_H}) \\
\left(\rho_{ad}^0 - \rho_{as}^0\right)S' & -\left(\rho_{ad}^0 - \rho_{as}^0\right)S' & -\left(\rho_{ad}^0 - \rho_{as}^0\right)S'\dfrac{RP_{as}^0}{K_H} - \dfrac{M_{vp}^{ol}P_{as}^0}{RT^{0^2}}(1-S_0) & \left(\rho_{ad}^0 - \rho_{as}^0\right)S'\left(1-\dfrac{RT^0}{K_H}\right) + M_{vp}^{ol}.\left(\dfrac{S_0}{K_H} + \dfrac{(1-S_0)}{RT^0}\right) \\
-\dfrac{1}{P_{vp}^0} & \dfrac{M_{vp}^{ol}}{\rho_w^0 RT} & \dfrac{M_{vp}^{ol}P_{as}^0}{\rho_w^0 K_H T^0}(1-R) + \dfrac{M_{vp}^{ol}}{R}\dfrac{h_{vp}^m - h_w^m}{T^{0^2}} & 0 \\
-\phi\left(1-S_{lq}\right) & 0 & C_\varepsilon^0 & -\phi\left(1-S_{lq}\right)
\end{pmatrix}
\times
\begin{pmatrix} \Delta P_{vp} \\ \Delta P_w \\ \Delta T \\ \Delta P_{as} \end{pmatrix}
=
\begin{pmatrix} 0 \\ 0 \\ \Delta t \dfrac{Surf}{Vol}\mathbf{q}_{ext}.\mathbf{n} \\ 0 \end{pmatrix}
$$

éq 2.1.2-5

S_0	S'	T^0	p_{vp}^0	h_{vp}^0	ρ_{vp}^0 (calculated)	ρ_{lq}
5,00E-01	-1,00E-12	3,00E+02	3,70E+03	2,50E+06	2,67E-02	1,00E+03

r_0	ϕ^0	ρ_s (calculated)	C_σ^s	C_{lq}^p L	C_{vp}^p	C_ε^0 (calculated)
2,20E+03	3,00E-01	2,93E+03	1,05E+03	4,18E+03	1,90E+03	2,78E+06

$\mathbf{q}_{ext}.\mathbf{n}$	Δt	$Surf$	Vol
1,00E+06	10	400	1,00E+04

The following results are got:

After resolution of this system, one obtains:

$$
\begin{pmatrix} \Delta P_{vp} \\ \Delta P_w \\ \Delta T \\ \Delta P_{as} \end{pmatrix}
=
\begin{pmatrix} 29.4 \\ -99500 \\ 0.144 \\ 45.7 \end{pmatrix}
$$

What gives in term of result Aster (increment):

$PRE1$	$PRE2$	DT	$PVP(V3)$
9.95 E4	7.5E1	1.44E-1	2.94E1

2.2 Uncertainties

Uncertainties are rather large because the analytical solution is a solution approached because of linearization of the equations.

3 Modeling A

3.1 Characteristics of modeling A

Modeling in plane deformations. An element $Q8$.
Discretization in time: only one step of time: $10\,s$.

3.2 Values tested

Node	Field	Component	Moment (s)	Reference (analytical)
$NO1$	DEPL	$TEMP$	10	0.1440
$NO1$	DEPL	$PRE1$	10	$9.95\,10^4$
$NO1$	DEPL	$PRE2$	10	75
$NO1$	VARI_ELNO	$V3$	10	29.4

4 Summary of the results

The solution is in very good agreement with the analytical solution except for the gas pressure. The weak differences are due to the linearization.