

WTNP122 - Modeling of a bar saturated with compressible gas slightly non-linear (monophasic flow) subjected to a shock with pressure

Summary:

This case test aims to validate:

- the diagrams finished volumes developed for the modeling of the diphasic flows.
- the hydraulic modeling saturated with finite elements `D_PLAN_HS`

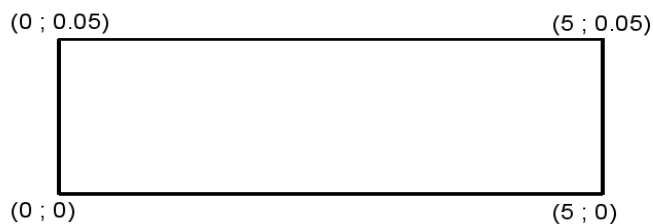
The diphasic problem here will be degenerated in a monophasic problem gas which one knows the analytical solution. It is the monodimensional modeling of a bar saturated with gas subjected to a shock with pressure.

1 Problem of reference

The objective of this case test is to compare the solution obtained with the various diagrams with an analytical solution.

1.1 Geometry

A bar is considered 1D of 5m of length. Concretely the field with a grid will make $[0m,5m] \times [0m;0,05m]$ (in the case of modeling in triangle, it is important not to have too "flattened" triangles, the choice height of the field is thus not pain-killer).



The duration of simulation is of 100s and the number of steps of time is of 100.

1.2 Properties of materials

One gives here only the properties whose solution depends, knowing that the command file contains other data of material which do not play any part in the solution of with the dealt problem.

Gas	Molar mass ($kg \cdot mol^{-1}$)	0,0001
	Viscosity ($kg \cdot m^{-1} \cdot s^{-1}$)	1
	Relative permeability (m^2)	1
Dissolved gas	Coefficient of Henry ($Pa \cdot mol^{-1} \cdot m^3$)	10000000000
Liquid	Relative permeability (m^2)	1
Homogenized parameters	Permeability $K_{int}(m^2)$	10^{-7}
	Porosity	1
	Fick gas ($m^2 \cdot s^{-1}$)	0
	Liquid Fick ($m^2 \cdot s^{-1}$)	0

Table 1.2-1 : Properties of materials

1.3 Boundary conditions and loadings

The limiting conditions are the following ones:

- conditions of Neumann on the right of field:

$$\frac{\partial(\delta P_g)}{\partial x}(t, x=5, y)=0 Pa$$

- condition of Dirichlet on the left part of the field:

$$P_g(t, x=0, y)=0 Pa$$

1.4 Initial conditions

The variation of initial gas pressure compared to the pressure of reference is of $\delta P_g(t=0, x, y)=10^4 Pa$.

One also has $P_g^{ref}(t=0, x, y)=10^4 Pa$ what amounts studying a problem slightly nonlinear (to be linear one would have to choose $P_g^{ref}(t=0, x, y)=10^{10} Pa$) (because one a: $P_g(t=0, x, y)=P_g^{ref} + \delta P_g(t=0, x, y)$).

2 Analytical solution

2.1 Method of calculating

The non stationary and monodimensional monophasic problem can be written in a general form of the type:

$$\begin{aligned} N \frac{\partial P}{\partial t} - K_{int} \Delta P &= 0 \\ P(t=0) &= P_0 \\ P(t, x=0) &= 0 \\ \frac{\partial P}{\partial x}(t, x, L) &= 0 \end{aligned}$$

This problem admits an analytical solution obtained by development in Fourier series.

$$P = \sum_{k=0}^K \frac{4P_0}{(2k+1)\pi} \exp\left(\frac{-K_{int}}{N} \omega_k^2 t\right) \sin(\omega_k x) \quad \text{with } \omega_k = \left(k + \frac{1}{2}\right) \frac{\pi}{L}$$

The number of terms K of this series is in the following way given:

That is to say n_x the number of points x_i where the solution is evaluated at one moment t .
One poses:

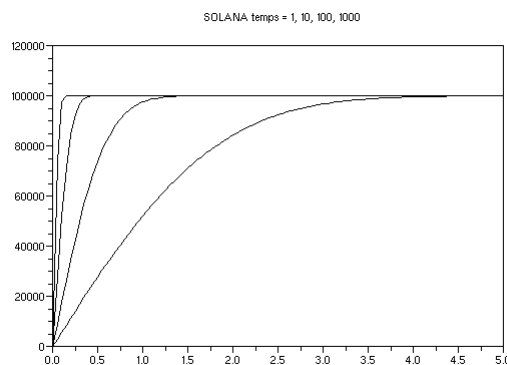
$$a_k^i = \frac{4}{(2k+1)\pi} \exp\left(\frac{-K_{int}}{N} \omega_k^2 t\right) \sin(\omega_k x_i)$$

So that the solution can be written: $P(x_i) = \sum_{k=0}^K P_0 \cdot a_k^i$

One chooses K such as: $\frac{1}{n_x} \sqrt{\sum_{i=1}^{n_x} (a_k^i)^2} < \epsilon$

In practice we took $\epsilon = 10^{-10}$.

The paces of the analytical solution at times 1,10,100,1000 are shown on the figure below:



The following table gives the number of terms according to time:

Time	Many series terms
1	194
10	64
100	22
1000	8

Table 2.1-1 : Representation amongst term according to time

2.2 Simplifying assumptions

It is considered that the medium is completely saturated with gas and one imposes in aster a pressure of worthless liquid on all the nodes. One imposes an initial gas pressure P_g^{ref} and one gives boundary conditions which corresponds to a variation of this pressure of reference, That is to say then δP_g this variation of pressure. The conservation equation of the gas mass will be written:

$$\frac{\partial(\varphi \delta P_g)}{\partial t} + \text{div}\left(\frac{K_{int} k_g}{\mu_g} (P_g^{ref} + \delta P_g) \text{div}(P_g^{ref} + \delta P_g)\right) = 0$$

While supposing δP_g small in front P_g^{ref} , this equation becomes:

$$\frac{\partial(\varphi \delta P_g)}{\partial t} + \frac{K_{int} k_g}{\mu_g} P_g^{ref} \Delta(\delta P_g) = 0$$

It is thus δP_g that one will identify with the solution of the model analytical equation.

In order to find the coefficients of the model problem, one will take:

$$\begin{aligned} \varphi &= 1 \\ k_g &= \mu_g = 1 \end{aligned}$$

and it will be made so that

$$K_{int} P_{ref} = 10^{-3}.$$

3 Modeling A

3.1 Characteristics of modeling

Modeling `D_PLAN_HH2SUDM`. This modeling corresponds with modeling Volume Finished with decentring on the mesh neighbor for terms of mobilities (the fickiens terms are realised). The hydraulic mixing rate is `LIQU_AD_GAZ_VAPE`.

3.2 Characteristics of the grid

One uses a grid made up of 100 elements `QUAD9`.

3.3 Sizes tested and results

One traces the profiles of gas pressure at various times as well as the analytical solution at these same moments. We are slightly nonlinear here, it is normal that the curves are slightly far away from the analytical solution.

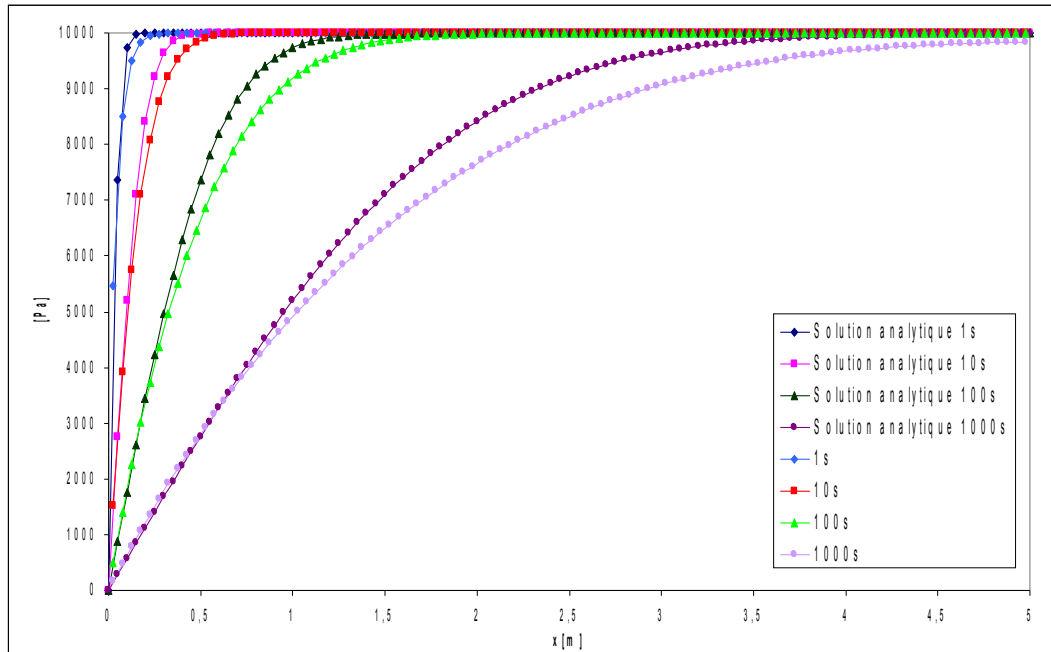


Illustration 1: Profiles of gas pressure

One for the first time carries out tests on 4 nodes with 1 moment by comparing the results with the analytical solution and second once by carrying out a test of nonregression.

Size	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance
PRE2	$(0,075; 0)$ N304	100	analytical	1331.0	7.0%
PRE2	$(0,075; 0)$ N304	100	not regression	1412.2251	1.0%
PRE2	$(0,075; 1)$ N293	100	analytical	1331.0	7.0%
PRE2	$(0,075; 1)$ N293	100	not regression	1412.2509021789	1.0%
PRE2	$(0,05; 0,05)$ N469	100	analytical	889.3176	7.5%
PRE2	$(0,05; 0,05)$ N469	100	not regression	955.70263804746	1.0%
PRE2	$(0,075; 0,5)$ NQ95	100	analytical	1331.0	9.0%
PRE2	$(0,075; 0,5)$ NQ95	100	not regression	1412.2509021788	1.0%

Table 3.3-1 : Values tested

4 Modeling B

4.1 Characteristics of modeling

Modeling D_PLAN_HH2SUDA. This modeling corresponds with modeling Volume Finished diecentered on the edges for mobilities (the fickiens terms are centered). The hydraulic mixing rate is LIQU_AD_GAZ_VAPE.

4.2 Characteristics of the grid

One uses a grid made up of 100 elements QUAD9.

4.3 Sizes tested and results

One for the first time carries out tests on 4 nodes with 1 moment by comparing the results with the analytical solution and second once by carrying out a test of nonregression.

Size	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	(0,075; 0) N304	100	analytical	1331.0	9.0%
PRE2	(0,075; 0) N304	100	not regression	1444.1	1.0%
PRE2	(0,075; 1) N293	100	analytical	1331.0	9.0%
PRE2	(0,075; 1) N293	100	not regression	1444.1	1.0%
PRE2	(0,05; 0,05) N469	100	analytical	889.3176	12.0%
PRE2	(0,05; 0,05) N469	100	not regression	990.9	1.0%
PRE2	(0,075; 0,5) NQ95	100	analytical	1331.0	9.0%
PRE2	(0,075; 0,5) NQ95	100	not regression	1444.1	1.0%

Table 4.3-1 : Values tested

5 Modeling C

5.1 Characteristics of modeling

Modeling D_PLAN_HH2SUC. This modeling corresponds with modeling Finished Volumes centered (for the darcéens terms and fickiens). The hydraulic mixing rate is LIQU_AD_GAZ_VAPE.

5.2 Characteristics of the grid

The grid consists of 100 elements QUAD9.

5.3 Sizes tested and results

The results are identical that those obtained with modeling Volumes finished eccentric on the mesh.

One for the first time carries out tests on 4 nodes with 1 moment by comparing the results with the analytical solution and second once by carrying out a test of nonregression.

Size	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	(0,075; 0) N304	100	analytical	1331.0	9.0%
PRE2	(0,075; 0) N304	100	not regression	1444.0876929334	1.0%
PRE2	(0,075; 1) N293	100	analytical	1331.0	9.0%
PRE2	(0,075; 1) N293	100	not regression	1444.0876929335	1.0%
PRE2	(0,05; 0,05) N469	100	analytical	889.3176	9.0%
PRE2	(0,05; 0,05) N469	100	not regression	1444.0876929335	1.0%
PRE2	(0,075; 0,5) NQ95	100	analytical	1331.0	12.0%
PRE2	(0,075; 0,5) NQ95	100	not regression	990.94043287847	1.0%

Table 5.3-1 : Values tested

6 Modeling D

6.1 Characteristics of modeling D

Modeling D_PLAN_HH2S. This modeling corresponds with modeling classical Finite elements. The hydraulic mixing rate is LIQU_AD_GAZ_VAPE.

6.2 Characteristics of the grid

The grid consists of 100 elements QUAD8.

6.3 Sizes tested and results

One for the first time carries out tests on 2 nodes with 1 moment by comparing the results with the analytical solution and second once by carrying out a test of nonregression.

Size	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	(0,05; 0) N104	100	analytical	889.3176	12.0%
PRE2	(0,05; 0) N104	100	not regression	992.8591566774	1.0%
PRE2	(0,05; 1) N103	100	analytical	889.3176	12.0%
PRE2	(0,05; 1) N103	100	not regression	992.85915667759	1.0%

Table 6.3-1 : Values tested

7 Modeling E

7.1 Characteristics of modeling

Modeling D_PLAN_HH2SUDM. This modeling corresponds with modeling Volume Finished with decentring on the mesh neighbor for terms of mobilities (the fickiens terms are realised). The hydraulic mixing rate is LIQU_AD_GAZ_VAPE. The goal is here to validate this diagram on meshes triangles.

7.2 Characteristics of the grid

The grid consists of 200 elements TRIA7.

7.3 Sizes tested and results

One traces the profiles of gas pressure at various times as well as the analytical solution at these same moments. We are slightly nonlinear here, it is normal that the curves are slightly far away from the analytical solution.

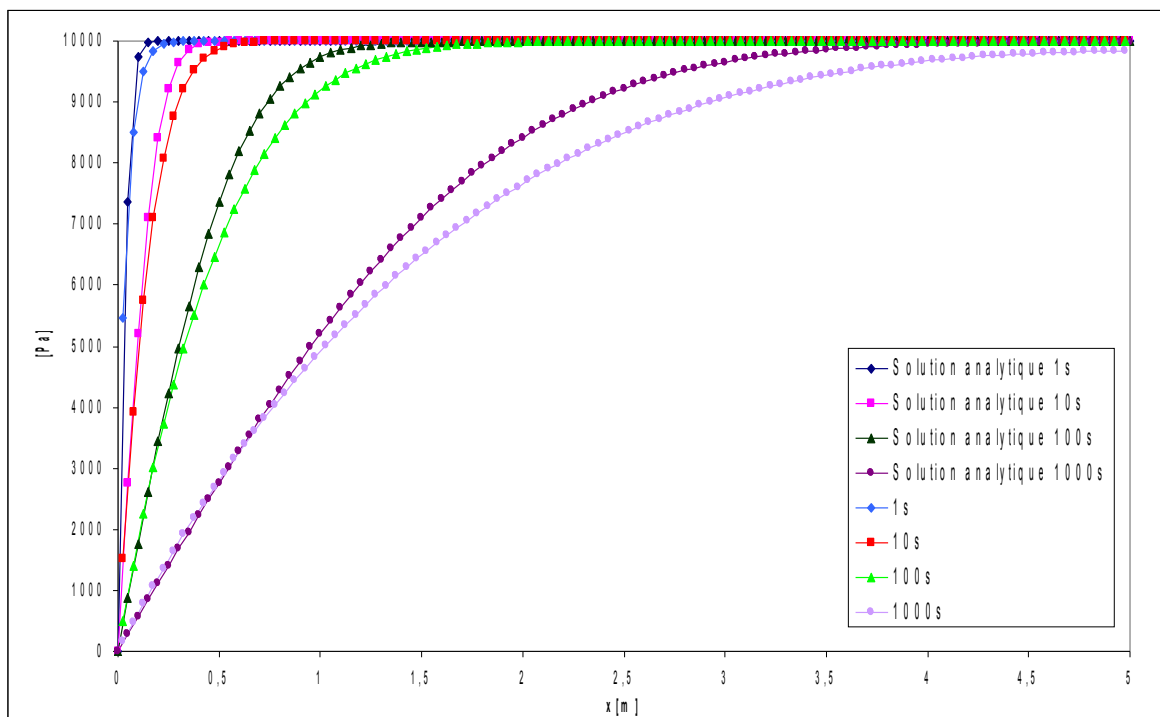


Illustration 2: Profiles of gas pressures

One for the first time carries out tests on 3 nodes with 1 moment by comparing the results with the analytical solution and second once by carrying out a test of nonregression.

Size	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance
PRE2	(0,075; 0) N360	100	analytical	1331.0	10.0%
PRE2	(0,075; 0) N360	100	not regression	1454.395434045	1.0%
PRE2	(0,075; 0,025) N505	100	analytical	1331.0	10.0%
PRE2	(0,075; 0,025) N505	100	not regression	1450.3938034156	1.0%
PRE2	(0,016; 0,0158) NT70	100	not regression	353.97760862167	1.0%

Table 7.3-1 : Values tested

8 Modeling F

8.1 Characteristics of modeling

Modeling D_PLAN_HH2SUDA. This modeling corresponds with modeling Finished Volumes diecentered on the edges for mobilities (the fickiens terms are centered). The hydraulic mixing rate is LIQU_AD_GAZ_VAPE.

8.2 Characteristics of the grid

The grid consists of 200 elements TRIA7.

8.3 Sizes tested and results

One carries out tests on 2 nodes with 1 moment by comparing the results with the analytical solution and on 3 nodes with 1 moment by carrying out a test of nonregression.

Size	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	(0,075; 0) N360	100	analytical	1331.0	11.0%
PRE2	(0,075; 0) N360	100	not regression	1454.4	1.0%
PRE2	(0,075; 0,025) N505	100	analytical	1331.0	10.0%
PRE2	(0,075; 0,025) N505	100	not regression	1450.4	1.0%
PRE2	(0,016; 0,0158) NT70	100	not regression	353,978	1.0%

Table 8.3-1: Values tested

9 Modeling G

9.1 Characteristics of modeling

Modeling D_PLAN_HH2SUC. This modeling corresponds with modeling Volume Finished centered (for the darcéens terms and fickiens). The hydraulic mixing rate is LIQU_AD_GAZ_VAPE.

9.2 Characteristics of modeling

The grid consists of 200 elements TRIA7.

9.3 Sizes tested and results

One carries out tests on 2 nodes with 1 moment by comparing the results with the analytical solution and on 3 nodes with 1 moment by carrying out a test of nonregression.

Size	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	(0,075; 0) N360	100	analytical	1473.0	10.0%
PRE2	(0,075; 0) N360	100	not regression	1473.3	1.0%
PRE2	(0,075; 0,025) N505	100	analytical	1472.0	10.0%
PRE2	(0,075; 0,025) N505	100	not regression	1472.4	1.0%
PRE2	(0,016; 0,0158) NT70	100	not regression	353,978	1.0%

Table 9.3-1 : Values tested

10 Modeling H

10.1 Characteristics of modeling

Modeling D_PLAN_HH2S. This modeling corresponds with modeling Finite elements. The hydraulic mixing rate is LIQU_AD_GAZ_VAPE.

10.2 Characteristics of the grid

The grid consists of 200 elements TRIA6.

10.3 Sizes tested and results

One for the first time carries out tests on 2 nodes with 1 moment by comparing the results with the analytical solution and second once by carrying out a test of nonregression.

Size	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	(0,05;0) N103	100	analytical	889.3176	12.0%
PRE2	(0,05;0) N103	100	not regression	992.699	1.0%
PRE2	(0,05;0,05) N203	100	analytical	889.3176	12.0%
PRE2	(0,05;0,05) N203	100	not regression	993.06	1.0%

Table 10.3-1 : Values tested

11 Modeling I

11.1 Characteristics of modeling

Modeling D_PLAN_HS. This modeling corresponds with modeling Finite elements. The hydraulic mixing rate is GAS.

11.2 Characteristics of the grid

The grid consists of 200 elements TRIA6.

11.3 Sizes tested and results

One carries out tests on 2 nodes at the moment $t=100\text{ans}$ for the first time by comparing the results with the analytical solution and second once by carrying out a test of nonregression.

Size	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	$(0,05; 0,05)$ N103	100	analytical	889.3176	12.0%
PRE2	$(0,05; 0,05)$ N103	100	not regression	992,699	1.0%
PRE2	$(0,05; 0)$ N203	100	analytical	889.3176	12.0%
PRE2	$(0,05; 0)$ N203	100	not regression	993.06	1.0%

Table 11.3-1 : Values tested

12 Summary of the results

This case test makes it possible to test the diagrams volumes finished in various configurations on a modeling of gas flow:

- the 3 diagrams finished volumes: centered, decentred edge, decentred mesh;
- on various types of meshes (triangles and rectangles).

These same cases are also carried out with the classical diagrams finite elements. All the results are very close to the analytical solution.

From a performance point of view, one will recommend the use of the eccentric diagram edge.