# WTNP124 – Case test of Liakopoulos: Drainage of a water column by the only force of gravity

**Summary:**

This case test is the simulation of the experiment of Liakopoulos of 1965 who represents the drainage of a sand column saturated with water. The latter goes then désaturer by the only force of gravity. To represent that, one considers a column of $1\,m$ of height subjected to the top with an atmospheric gas pressure.

It is about a miscible purely hydraulic calculation. The geometry represented corresponds to a vertical bar. The terms of transfer are described by a model of Mualem Van-Genuchten. With the problem is dealt by the various diagrams available for the modeling of the diphasic flows: classical finite elements, them Volumes Finis Décentrés Arête , Volumes Finis Décentrés Maille and Volumes Finis Centrés.

# 1 Problem of reference

This CAS-test represents the desaturation of a column initially saturated with water by gravitating effect (experiment of Liakopoulos). Here we consider that the air can dissolve in water (version proposed by VAUNAT in 1997).

## 1.1 Geometry

The field is a bar of size $[0\mathrm{m}\,;0{,}1\,m]\times[0\mathrm{m}\,;1\mathrm{m}]$.

## 1.2 Properties of materials

One gives here only the properties whose solution depends, knowing that the command file contains other data of material which do not play any part in the solution of with the dealt problem.

| | | |
|---|---|---|
| Liquid water | Density $(kg.m^{-3})$ | $1000$ |
| | Molar mass $(kg.mol^{-1})$ | $10^{-2}$ |
| | Viscosity $(kg.m^{-1}.s^{-1})$ | $10^{-3}$ |
| | compressibility | $0{,}5\,10^{-9}$ |
| Gas | Density $(kg.m^{-3})$ | $8\,10^{-2}$ |
| | Molar mass $(kg.mol^{-1})$ | $28{,}96\,10^{-3}$ |
| | Viscosity $(kg.m^{-1}.s^{-1})$ | $1{,}8\,10^{-5}$ |
| Dissolved gas | Coefficient of Henry $(Pa.mol^{-1}.m^{3})$ | $2\,10^{-6}$ |
| Vapor | Density $(kg.m^{-3})$ | $18\,10^{-3}$ |
| Homogenized parameters | Permeability $k$ $(m^2)$ | $10^{-12}$ |
| | Porosity | $0{,}2975$ |
| | Fick gas $(m^2.s^{-1})$ | $0$ |
| | Liquid Fick $(m^2.s^{-1})$ | $0$ |
| Parameters of Van-Genuchten | $N$ | $2$ |
| | $P_r$ $MPa$ | $10^4$ |
| | $S_{rl}$ | $0$ |
| | $S_{gr}$ | $0$ |
| | $Smax$ | $0{,}999$ |
| Initial state | Liquid pressure | $P_l^0=\rho\,g\,Y+1\,atm-\rho\,g$ |
| | Gas pressure | $P_g^0=1\,atm$ |

**Table 1.2-1 : Properties of materials**

The curves of saturation and permeabilities obey the Mualem-Van-Genuchten model (HYDR_VGM). It is thus necessary to define in materials the parameters $n$ , $Pr$ , $Sr$ , $Smax$ .
It is pointed out that these models are: and

$$S_{le}=\frac{S_l-S_{lr}}{1-S_{lr}} \qquad m=1-\frac{1}{n}$$

$$S_{we}=\frac{1}{\left[1+\left(\frac{P_c}{P_r}\right)^n\right]^m}$$

The permeability relating to water is expressed by integrating the model of prediction proposed by Mualem (1976) in the model of capillarity of Van Genuchten: $k_r^l=\sqrt{S_{le}}(1-(1-S_{le}^{\frac{1}{m}})^m)^2$

The permeability to gas is formulated in a similar way: $k_r^g=\sqrt{(1-S_{le})}(1-S_{le}^{\frac{1}{m}})^{2m}$

One recalls that for $S>Smax$ , these curves are interpolated by a polynomial of degree 2 $C1$ in $Smax$ .

## 1.3 Boundary conditions and initial

The limiting conditions are the following ones:

- conditions of Neumann on the flat rims and left of the field:

$$(\boldsymbol{F}_l^w+\boldsymbol{F}_g^w).\boldsymbol{n}=0$$
$$(\boldsymbol{F}_l^c+\boldsymbol{F}_g^c).\boldsymbol{n}=0$$

- conditions of Dirichlet on the high part of the field (surface to open air):

$$P_g(x,y=1,t)=10^5 \quad Pa$$

- conditions of Dirichlet on the low part of the field (saturated medium; water runs out):

$$P_g(x,y=0,t)=10^5 \quad Pa$$
$$P_l(x,y=0,t)=10^5 \quad Pa$$

The initial state corresponds in a state saturated with water with hydrostatic balance. The initial conditions are the following ones:

$$P_l(x,y,t=0)=\rho g(Y-1)+10^5$$
$$P_g(x,y,t)=0=10^5 \quad Pa$$

## 1.4 Time of simulation

Calculation is carried out over one year $(3.1536.10^7 s)$ .

# 2 Modeling A

## 2.1 Characteristics of modeling A

Modeling `D_PLAN_HH2SUDM`. This modeling corresponds to modeling Volume Finis Décentrés Maille. Coupling `LIQU_AD_GAZ`. One uses a grid made up of 50 elements `QUAD8`.

## 2.2 Results

One traces the profiles of capillary pressure and gas pressure at various moments:

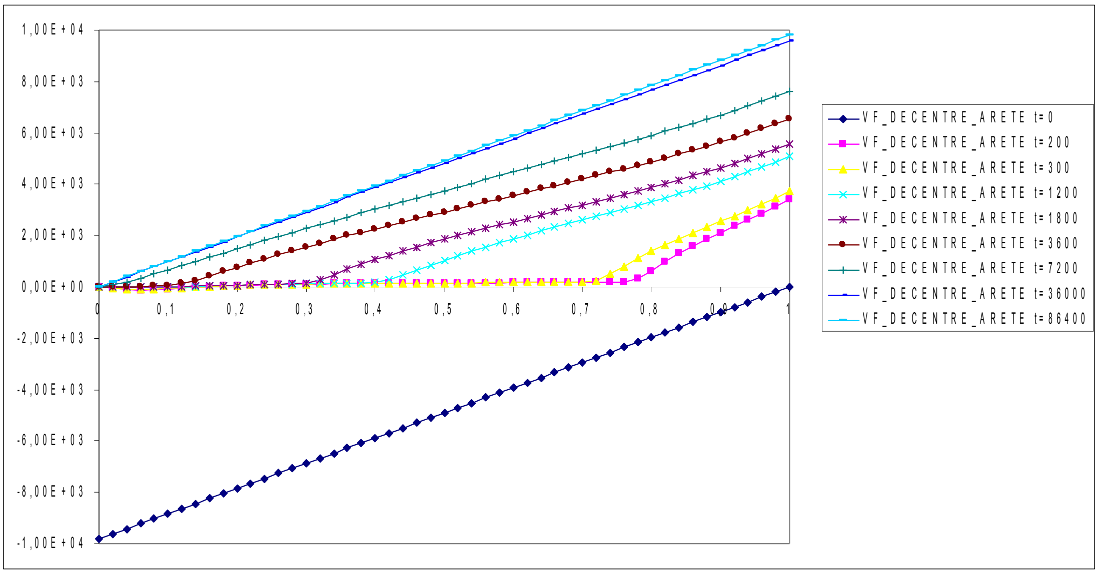


**Illustration 1: Capillary pressure along the column**

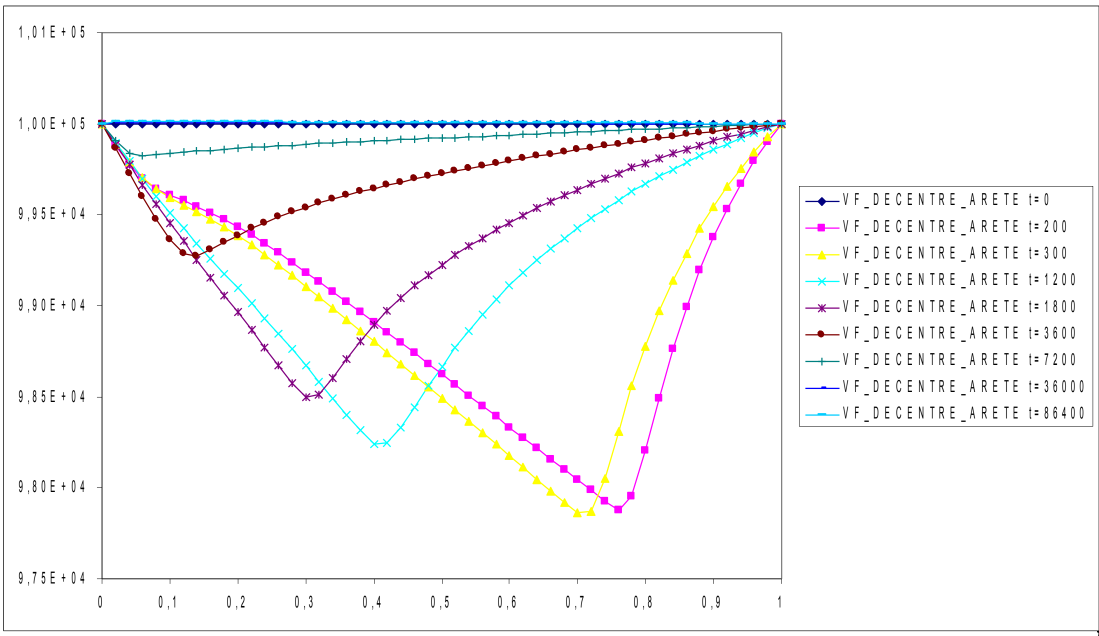


**Illustration 2: Gas pressure along the column**

One observes well a progressive desaturation of the medium (when the capillary pressure becomes positive). The desaturation on all the column and the permanent state are obtained at the end of one day ( $86400\,s$ ). The profiles of gas pressure have a pace in "point" whose change of incline corresponds to the face of saturation. In saturated zone, the gas pressure is controlled by the quantity of dissolved air; above face of saturation, the flow of air is primarily related to the movement of dry air. This shape of curve is typical of this kind of problem and corresponds well so that one finds in the literature.

## 2.3 Values tested

This CAS-test does not have a value of reference precise, one thus makes of them a case of nonregression.

One carries out tests on 6 values:

| **Points** $(x, y)$ | **Time** $(s)$ | **PRE1** ***Aster*** | **PRE2** ***Aster*** |
|---|---|---|---|
| $(0{,}05 ; 1)$ $N103$ | 200s | 3377. | 1.00E+05 |
| | 86400 S | 9811. | 1.00E+05 |
| $(0{,}05 ; 0{,}5)$ $N229$ | 86400 S | 4911. | 1.00E+05 |

**Table 2.3-1 : Values tested**

# 3 Modeling B

## 3.1 Characteristics of modeling B

Modeling `D_PLAN_HH2SUDA`. This modeling corresponds to modeling Volume Finis Décentrés Arêtes. Coupling `LIQU_AD_GAZ`. One uses a grid made up of 50 elements `QUAD8`.

## 3.2 Results

The results are very close to those obtained with modeling volumes finished eccentric on the mesh.

## 3.3 Values tested

This case test does not have a value of reference, one thus makes of them a case of nonregression.

One carries out tests on 6 values:

| **Points** $(x, y)$ | **Time** $(s)$ | **PRE1** ***Aster*** | **PRE2** ***Aster*** |
|---|---|---|---|
| $(0{,}05 ; 1)$ *N103* | 200s | 60.15. | 98259. |
| | 86400 S | 9810. | 1.00E+05 |
| $(0{,}05 ; 0{,}5)$ *N229* | 86400 S | 4912. | 1.00E+05 |

**Table 3.3-1 : Values tested**

# 4 Modeling C

## 4.1 Characteristics of modeling C

Modeling D_PLAN_HH2SUC. This modeling corresponds to modeling Volume Finis Centrés. Coupling LIQU_AD_GAZ. One uses a grid made up of 50 elements QUAD8.

## 4.2 Results

The results are very close to those obtained with modeling volumes finished eccentric on the mesh.

## 4.3 Values tested

This case test does not have a value of reference, one thus makes of them a case of nonregression.

One carries out tests on 6 values:

| **Points** $(x, y)$ | **Time** $(s)$ | **PRE1** ***Aster*** | **PRE2** ***Aster*** |
|---|---|---|---|
| $(0{,}05 ; 1)$ *N103* | 200s | 3384. | 1.00E+05. |
| | 86400 S | 9811. | 1.00E+05 |
| $(0{,}05 ; 0{,}5)$ *N229* | 86400 S | 4911. | 1.00E+05 |

Table 4.3-1 : Values tested

# 5 Modeling D

## 5.1 Characteristics of modeling D

Modeling D_PLAN_HH2S. This modeling corresponds to modeling Finite elements. Coupling LIQU_AD_GAZ. One uses a grid made up of 50 elements QUAD8.

## 5.2 Results

The results are very close to those obtained with modeling volumes finished eccentric on the mesh.

## 5.3 Values tested

This case test does not have a value of reference, one thus makes of them a case of nonregression.

| **Points** $(x, y)$ | **Time** $(s)$ | **PRE1** ***Aster*** | **PRE2** ***Aster*** |
|---|---|---|---|
| $(0; 0{,}049)$ *N51* | 200s | -124.6 | 99689. |
| | 1800s | -9,475 | 99659. |
| | 86400 S | 600.1 | 1.00E+05 |
| $(0; 1)$ *N2* | 200s | 3445. | 1.00E+05 |
| | 1800s | 5604. | 1.00E+05 |
| | 86400 S | 9811. | 1.00E+05 |

**Table 5.3-1 : Values tested**

# 6 Summary of the results

This case test proposes a modeling of the very classical experiment of Liakopoulos. Simulations reproduce perfectly the expected results, even if we do not have digital results of reference. This test enables us to validate the good taking into account of gravity by the various digital diagrams. With this problem is indeed dealt with the 4 digital diagrams available for the modeling of the diphasic flows:

- the 3 diagrams finished volumes: centered, decentred edge, decentred mesh
- classical finite elements

The got results are the same ones. In term of performance and reliability, one will strongly privilege the diagrams Finished volumes Decentred Edge (*_HH2SUDA).