WTNA112 – Thermal pressurization of a not drained saturated cylindrical test-tube

Summary:

It is about a problem of THM saturated and elastic. One increases the temperature of a sample not drained maintained with constant containment (constant total constraint at the edge). The resulting water pressure varies then linearly with the temperature according to a thermal coefficient of pressurization which one calculates analytically. The solution obtained here is thus to compare with an analytical solution.

1 Problem of reference

1.1 Geometry

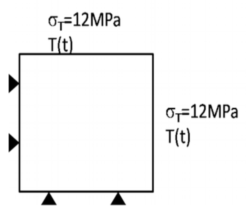
A cylinder of ray is considered $1\,cm$ and height $1\,cm$ (either a grid corresponding to a square field of $1\,cm\times 1\mathrm{cm}$, modeling being axisymmetric).

1.2 Properties of material

One chooses parameters here corresponding to a mudstone so as to obtain a coefficient of realistic thermal pressurization.

Liquid water	Density $(kg.m^{-3})$	10^3
	Specific heat with constant pressure $(J.K^{-1})$	4180
	Dynamic viscosity of liquid water $(Pa.s)$	0.001
	Thermal dilation coefficient of the liquid (K^{-1})	1.10^{-4}
	Compressibility (Pa^{-1})	$K_e=5.10^{-10}$
Solid	Drained Young modulus $E(Pa)$	$3{,}14\,10^9$
	Poisson's ratio	0.375
	Thermal dilation coefficient of the solid (K^{-1})	10^{-5}
State of reference	Porosity	0.18
	Temperature (K)	273
	Liquid pressure (Pa)	0
Homogenized coefficients	Homogenized density $(kg.m^{-3})$	2410
	Coefficient of Biot	0.6
	Intrinsic permeability (m^2)	$K_{\mathrm{int}}=10^{-21}$
	Thermal conductivity	$\lambda_T=1.61$

1.3 Boundary conditions and loadings



One imposes:

- On the low and left edges: worthless displacements, hydraulic flow no one, worthless heat fluxes. They are conditions of symmetry.

- On the edges high and right: Total constraint imposed on $12\,MPa$, hydraulic flow no one, imposed temperature function of time $T(t)$ according to a linear slope such as: $T(t)=T_0+\frac{\Delta T}{t_{sim}}$ where t_{sim} corresponds at the time of simulation (here $t_{sim}=1\mathrm{h}$) and ΔT temperature variation imposed during this time (here $\Delta T=40°C$).

1.4 Initial conditions

$P(x)=4\mathrm{MPa}$ and $T(x)=T_0=20°C$ everywhere.

2 Reference solution

One R call that the contribution of water mass is written: $m_w=\varphi.\rho_w.(1+\varepsilon_v)$, which one can derive in the following form: $dm_w=d\varphi\,\rho_w(1+\varepsilon_v)+d\rho_w\varphi(1+\varepsilon_v)+\rho_w\varphi\,d\varepsilon_v$ with φ porosity eulérienne.

If one places oneself in assumption of small displacements, one will thus have:

$$dm_w=d\varphi\,\rho_w+d\rho_w\varphi+\rho_w\varphi\,d\varepsilon_v \quad (1)$$

The variation of porosity is written according to the relation:

$$d\varphi=(b-\varphi)\left(d\varepsilon_V-3\alpha_0 dT+\frac{dp_w}{K_s}\right) \quad (2)$$

with α_0 the linear dilation of the skeleton (comparable to the porous environment). It is pointed out that the coefficient of Biot b and compressibility of the solid matter constituents modulates it K_s are connected to the “drained” module of compressibility of the porous environment K_0, such as:

$$b=1-\frac{K_0}{K_s}$$

In addition, the variation of the density of water is written:

$$\frac{d\rho_w}{\rho_w}=\frac{dp_w}{K_w}-3\alpha_w dT \quad (3)$$

with the module of compressibility of water K_w and its module of dilation α_w.

Lastly, if the law of behavior is elastic, it is pointed out that the deformation is connected to the effective constraint such as:

$$d\varepsilon_V=\frac{d\sigma'}{K_0}+3\alpha_0 dT \quad (4)$$

In addition, the formulation in total constraint, indicates to us that:

$d\sigma'=d\sigma+b\,dp_w$, considering here that the mediums is with constant containment, one thus has:

$d\sigma'=b\,dp_w$, which gives us with the final one

$$d\,\varepsilon_V=\frac{b\,dp_w}{K_0}+3\,\alpha_0\,dT \quad (5)$$

One can now inject (2), (3), (4) and (5) in the equation (1) and one obtains that:

$$\frac{dm_w}{\rho_w}=\left(\frac{b^2}{K_0}+\frac{(b-\varphi)}{K_s}+\frac{\varphi}{K_w}\right)dp_w+\varphi\left(3\,\alpha_0\text{-}3\,\alpha_w\right)dT \quad (6)$$

Considering that the medium is not drained one thus has:

$$\left(\frac{b^2}{K_0}+\frac{(b-\varphi)}{K_s}+\frac{\varphi}{K_w}\right)dp_w=\varphi\left(3\,\alpha_w\text{-}3\,\alpha_0\right)dT \quad (7)$$

What can be written in the form:

$$dp_w=\Lambda\,dT \quad (8)$$

With Λ the thermal coefficient of pressurization such as:

$$\Lambda=\frac{\varphi\left(3\,\alpha_w\text{-}3\,\alpha_0\right)}{\left(\frac{b^2}{K_0}+\frac{(b-\varphi)}{K_s}+\frac{\varphi}{K_w}\right)}$$

It is noticed that this coefficient revealed the thermal differential $(\alpha_w-\alpha_0)$

Digital application:

With the data defined previously, one obtains:

$\Lambda=2{,}25\,10^5\,Pa.\,K^{-1}$

What gives for a temperature variation $\Delta\,T=40\,°C$, a variation of pressure of $\Delta\,p=9{,}01\,Mpa$.

3 Modeling A

3.1 Characteristics of modeling A

- Plane modeling `'AXIS_THMS'`. Mechanical law `'ELAS'`. Coupling `'LIQU_SATU'`.
- 20×20 elements $Q4$ of equal width.

3.2 Results of modeling A

Discretization in time: 10 pas de time of $180 s$ each one. The solution calculated by Aster which takes account of the movements of the fluid and heat (diffusive phenomena), it is normal not to obtain the reference solution exactly. The differences remain very weak.

Result at the final moment $3600 s$ **:**

$N°$ **NODE**	$COOR\_X$	$COOR\_Y$	**Reference** $PRE1(MPa)$	**Aster** $PRE1(MPa)$	**Differences** $(\%)$	**Tolerance** $(\%)$
1	0	0	13,01	12,98	0,158	1
2	0	0.01	13,01	12,99	0,098	1

4 Modeling B

4.1 Characteristics of modeling B

- Plane modeling 'AXIS_THMS'. Mechanical law 'ELAS_ORTH'. Coupling 'LIQU_SATU'. 20×20 elements $Q4$ of equal width. Also anisotropic terms of couplings.

It acts of the same modeling as previously but via the modules of orthotropism (although the test-tube is regarded as isotropic and that the characteristics remain the same ones in each direction). In theory the results should give the same thing exactly.
The model differs however here slightly and the analytical solution indicated previously is not completely any more exact. Indeed instead of the relation used into isotropic:

$$d\varphi=(b-\varphi)\left(d\varepsilon_V-3\alpha_0 dT+\frac{dp_w}{K_s}\right)$$

The relation used in this case becomes tensorial (cf Doc. R7.01.11) and is:

$$d\varphi=\boldsymbol{B}:\boldsymbol{d}\varepsilon-\varphi\, d\varepsilon_V-3\alpha_\varphi dT+\frac{dp_{gz}-S_{lq}dp_c}{M_\varphi}$$

with

$$\frac{1}{M_\varphi}=(\boldsymbol{B}-\varphi\boldsymbol{\delta}):\boldsymbol{S}_0^S:\boldsymbol{\delta}$$

where $\boldsymbol{S}_0^S$ the matrix of flexibility DU skeleton, function of the Young modulus of the solid matrix E^S and of the Poisson's ratio of the solid matrix ν^S.
In addition porosity cannot be integrated here analytically and is thus in an explicit way (porosity taken at previous time). The resolution is thus here less precise.

This modeling aims to quantify the difference obtained via this modeling.

4.2 Results of modeling B

One tests the same results as previously initially on 10 pas de time as for modeling A

Result at the final moment $3600\,s$ **:**

$N°$ **NODE**	$COOR\_X$	$COOR\_Y$	**Reference** $PRE1(MPa)$	**Aster** $PRE1(MPa)$	**Differences** (%)	**Tolerance** (%)
1	0	0	13,01	12,258	3,25	5
2	0	0.01	13,01	12,259	2,80	5

The results got here are a little less precise than previously what is explained by the explicit treatment of porosity.

To ensure itself some one tests the same case but with 15 pas de time:

Result at the final moment $3600\,s$ **:**

$N°$ **NODE**	$COOR\_X$	$COOR\_Y$	**Reference** $PRE1(MPa)$	**Aster** $PRE1(MPa)$	**Differences** (%)	**Tolerance** (%)
1	0	0	13,01	12.994	0,04	1
2	0	0.01	13,01	13.001	0,01	1

One converges perfectly towards the analytical solution.

5 Summary of the results

The results are into coherent with the analytical solution.