ZZZZ293 – Validation of the position of the under-points of the multifibre beams

Summary:
This test validates the calculation of the position of the under-points of integration in the total reference mark for modeling POU_D_EM and POU_D_TGM. An elementary mechanical calculation is carried out in order to allow the creation of a table with CREA_TABLE starting from the result. Only the coordinates of some under-points are tested in the table.

1 Problem of reference

1.1 Geometry

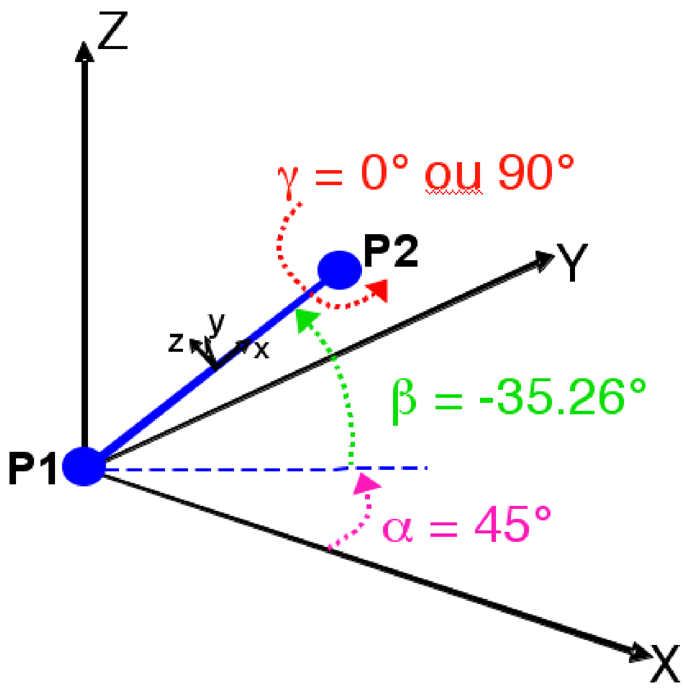

Figure 1.1-a : orientation of the element.

The beam is directed in space as indicated on the Figure 1.1-a.
Total coordinates of the points $P1$ and $P2$:

$$X_{P1}=0.0\,;Y_{P1}=0.0\,;Z_{P1}=0.0$$
$$X_{P2}=2.0\,;Y_{P2}=2.0\,;Z_{P2}=2.0$$

Length: $L=2\cdot\sqrt{3}\,m$

1.2 Properties of materials

Concrete:

Young modulus $E=3.7272^{10}\,Pa$

Poisson's ratio $\nu=0.0$

1.3 Boundary conditions and loadings

On the point $P1$ one blocks displacements according to X,Y,Z and rotation around the axes X,Y,Z :

$$D_X^{P1}=0.0\,;D_Y^{P1}=0.0\,;D_Z^{P1}=0.0\,;DR_X^{P1}=0.0\,;DR_Y^{P1}=0.0\,;DR_Z^{P1}=0.0$$

On the point $P2$ one applies a loading according to X,Y,Z:

$$F_X=100.0\,N\,;F_Y=100.0\,N\,;F_Z=-100.0\,N$$

2 Reference solution

2.1 Method of calculating

One calculates the position of the nodes, points of integration and under-points of integration from their cordonnées in the local axes of the beam and the matrices of passage between the local axes and the total axes.
By defaults, the local axes and the total axes coincide (Figure 2.1-a).

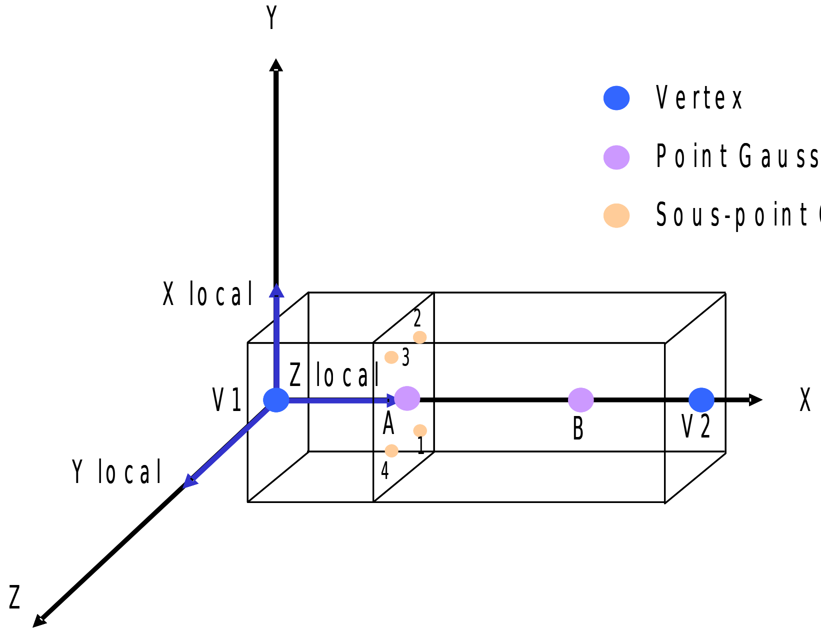

Figure 2.1-a : position by default

One applies two rotations (see Figure 1.1-a) to direct the axis of the beam, and the third rotation to position the cross-section:

- $\alpha=45°$ around Z
- $\beta=-35{,}26°$ around the new axis $Y1$
- $\gamma=0°$ or $90°$ around the new axis $X2$

Note:

one uses conventions of the nautical angles of Code_Aster (see the keyword `ORIENTATION` of `AFFE_CARA_ELEM`)

Rotation around the axis Z (α) is made starting from the following matrix:

$$Tz(\alpha)=\begin{bmatrix}\cos(\alpha) & -\sin(\alpha) & 0\\ \sin(\alpha) & \cos(\alpha) & 0\\ 0 & 0 & 1\end{bmatrix}$$

Rotation around the new axis $Y1$ (β) is made starting from the following matrix:

$$Ty1(\beta)=\begin{bmatrix}\cos(\beta) & 0 & \sin(\beta)\\ 0 & 1 & 0\\ -\sin(\beta) & 0 & \cos(\beta)\end{bmatrix}$$

Rotation around the new axis $X2$ (γ) is made starting from the following matrix:

$$Tx2(\gamma)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos(\gamma) & -\sin(\gamma)\\ 0 & \sin(\gamma) & \cos(\gamma)\end{bmatrix}$$

Therefore, for any point of coordinates (X,Y,Z) before rotations, one can calculate his coordinates (X',Y',Z') after rotations with the following transformation:

$$\begin{bmatrix}X'\\ Y'\\ Z'\end{bmatrix}=[Tx2(\gamma)][Tz(\alpha)][Ty(\beta)]\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}$$

2.2 Sizes and results of reference

One calculates the position of some under-points of integration in the total reference mark knowing their position in the local axes.

With the angles chosen, the digital application gives:

$$Ty1(\beta)=\begin{bmatrix}0.8165 & 0 & -0.5774\\ 0 & 1 & 0\\ 0.5774 & 0 & 0.8165\end{bmatrix} \quad Tz(\alpha)=\begin{bmatrix}0.7071 & -0.7071 & 0\\ 0.7071 & 0.7071 & 0\\ 0 & 0 & 1\end{bmatrix}$$

and

$$Tx2(\gamma)=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \text{ or } Tx2(\gamma)=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}$$

For an element length $L=2\cdot\sqrt{3}\,m$, the distance from the second point of Gauss compared to the first node is:

For the elements POU_D_EM (modelings A and B) which have two points of Gauss:

$$\left(\frac{1}{2}+\frac{1}{2\sqrt{3}}\right)L=1+\sqrt{3}=2.7320508075688772\,m$$

For the elements POU_D_TGM (modelings C and D) which have three points of Gauss:

$$\frac{L}{2}=\sqrt{3}=1.7320508075688772\,m$$

The section of the beam $(0,2\,m\times 0,1\,m)$ in 4 quadrilaterals (Figure is discretized 2.2-a).

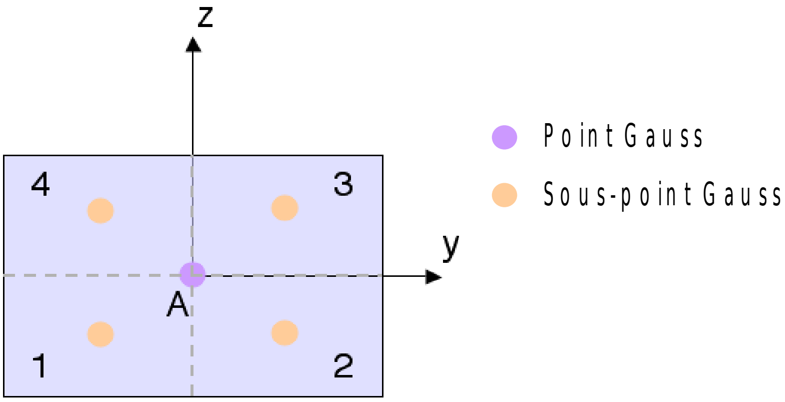

Figure 2.2-a : position of the under-point in the section

The position of the under-points chosen in the initial reference mark for modelings A and B (POU_D_EM) is thus :

Not	Under-point	x	y	z
2	1	2.732050807568877	-0.05	-0,025
2	2	2.732050807568877	0.05	-0,025
2	3	2.732050807568877	0.05	0,025
2	4	2.732050807568877	-0.05	0,025

And the position of the under-points chosen in the initial reference mark for modelings C and D (POU_D_TGM) is:

Not	Under-point	x	y	z
2	1	1.732050807568877	-0.05	-0,025
2	2	1.732050807568877	0.05	-0,025
2	3	1.732050807568877	0.05	0,025
2	4	1.732050807568877	-0.05	0,025

2.3 Uncertainties on the solution

No, exact solution.

3 Modeling A

3.1 Characteristics of the grid

The grid of the beam is composed of an element SEG2.
The grid of the section is composed of 4 quadrilateral elements.
The group of nodes ENC is composed of the node $P1$.
The group of nodes CHA is composed of the node $P2$.

3.2 Characteristics of modeling

It is about a modeling POU_D_EM (elements at 2 points of Gauss). The angle of gimlet is equal to $0°$.

Modeling:

```
MOPOU=AFFE_MODELE (MAILLAGE=MAPOU,
    AFFE=_F (TOUT=' OUI', PHENOMENE=' MECANIQUE',
        MODELISATION=' POU_D_EM',),
)
```

Boundary conditions:

```
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
    DDL_IMPO=_F (GROUP_NO=' ENC',
        DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0,),
)
```

Mechanical loading:

```
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
    FORCE_NODALE=_F (GROUP_NO=' CHA', FX = 100, FY = 100, FZ = 100.),
)
```

Assignment of the characteristics of the elements:

```
POUCA_0=AFFE_CARA_ELEM (MODELE=MOPOU, GEOM_FIBRE=GF,
    POUTRE=_F (GROUP_MA= ('BEAM'),
        SECTION=' RECTANGLE', CARA= ('HY', 'HZ'), VALE= (0.2, 0.1),
        PREC_AIRE=5., PREC_INERTIE=10.,),
    ORIENTATION=_F (GROUP_MA= ('BEAM'), CARA=' ANGL_VRIL', VALE=0.0,),
    MULTIFIBRE=_F (GROUP_MA= ('BEAM'), GROUP_FIBRE= ('SBET',),),
)
```

3.3 Values tested and results

Coordinate mesh SG01	Not integration	Under-point	Reference
COOR_X	2	1	0.377088184
COOR_X	2	2	0.447798863
COOR_X	2	3	0.468211277
COOR_X	2	4	0.397500599
COOR_Y	2	1	0.447798863
COOR_Y	2	2	0.377088184
COOR_Y	2	3	0.397500599
COOR_Y	2	4	0.468211277
COOR_Z	2	1	0.443062145
COOR_Z	2	2	0.443062145
COOR_Z	2	3	0.402237316
COOR_Z	2	4	0.402237316

All the tolerance are those by default: $1.0\text{E}-3$

4 Modeling B

4.1 Characteristics of the grid

The grid of the beam is composed of an element SEG2.
The grid of the section is composed of 4 quadrilateral elements.
The group of nodes ENC is composed of the node $P1$.
The group of nodes CHA is composed of the node $P2$.

4.2 Characteristics of modeling

It is about a modeling POU_D_EM (elements at 2 points of Gauss). The angle of gimlet is of $90°$.

Modeling:

```
MOPOU=AFFE_MODELE (MAILLAGE=MAPOU,
    AFFE=_F (TOUT=' OUI', PHENOMENE=' MECANIQUE',
        MODELISATION=' POU_D_EM',),)
```

Boundary conditions:

```
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
    DDL_IMPO=_F (GROUP_NO=' ENC',
        DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0,),
)
```

Mechanical loading:

```
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
        FORCE_NODALE=_F (GROUP_NO=' CHA', FX = 100, FY = 100, FZ = 100.),
)
```

Assignment of the characteristics of the elements:

```
POUCA_0=AFFE_CARA_ELEM (MODELE=MOPOU, GEOM_FIBRE=GF,
    POUTRE=_F (GROUP_MA= ('BEAM'), SECTION=' RECTANGLE',
        CARA= ('HY', 'HZ'), VALE= (0.2, 0.1),
        PREC_AIRE=5., PREC_INERTIE=10.,),
    ORIENTATION=_F (GROUP_MA= ('BEAM'), CARA=' ANGL_VRIL', VALE=90.0,),
    MULTIFIBRE=_F (GROUP_MA= ('BEAM'), GROUP_FIBRE= ('SBET',),),
)
```

4.3 Values tested and results

Coordinate mesh SG01	Not of integration	under-point	Reference
COOR_X	2	1	0.419914986
COOR_X	2	2	0.460739815
COOR_X	2	3	0.425384476
COOR_X	2	4	0.384559647
COOR_Y	2	1	0.384559647
COOR_Y	2	2	0.425384476
COOR_Y	2	3	0.460739815
COOR_Y	2	4	0.419914986
COOR_Z	2	1	0.463474560
COOR_Z	2	2	0.381824902
COOR_Z	2	3	0.381824902
COOR_Z	2	4	0.463474560

All the tolerance are those by default: $1.0E-3$.

5 Modeling C

5.1 Characteristics of the grid

The grid of the beam is composed of an element SEG2.
The grid of the section is composed of 4 quadrilateral elements.
The group of nodes ENC is composed of the node $P1$.
The group of nodes CHA is composed of the node $P2$.

5.2 Characteristics of modeling

It is about a modeling POU_D_TGM (elements at 3 points of Gauss). The angle of gimlet is of $0°$.

Modeling:

```
MOPOU=AFFE_MODELE (MAILLAGE=MAPOU,
    AFFE=_F (TOUT=' OUI', PHENOMENE=' MECANIQUE',
        MODELISATION=' POU_D_TGM',),
)
```

Boundary conditions:

```
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
    DDL_IMPO=_F (GROUP_NO=' ENC',
        DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0,),
)
```

Mechanical loading:

```
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
    FORCE_NODALE=_F (GROUP_NO=' CHA', FX = 100, FY = 100, FZ = 100.),
)
```

Assignment of the characteristics of the elements:

```
POUCA_0=AFFE_CARA_ELEM (MODELE=MOPOU, GEOM_FIBRE=GF,
    POUTRE=_F (GROUP_MA= ('BEAM'),
        SECTION=' RECTANGLE', CARA= ('HY', 'HZ'), VALE= (0.2, 0.1),
        PREC_AIRE=5., PREC_INERTIE=10.,),
    ORIENTATION=_F (GROUP_MA= ('BEAM'), CARA=' ANGL_VRIL', VALE=0.0,),
    MULTIFIBRE=_F (GROUP_MA= ('BEAM'), GROUP_FIBRE= ('SBET',),
)
```

5.3 Values tested and results

Coordinate mesh SG01	Not of integration	Under-point	Reference
COOR_X	2	1	0.954438454
COOR_X	2	2	1.025149132
COOR_X	2	3	1.045561546
COOR_X	2	4	0.974850868
COOR_Y	2	1	1.025149132
COOR_Y	2	2	0.954438454
COOR_Y	2	3	0.974850868
COOR_Y	2	4	1.045561546
COOR_Z	2	1	1.020412415
COOR_Z	2	2	1.020412415
COOR_Z	2	3	0.979587585
COOR_Z	2	4	0.979587585

All the tolerance are those by default: $1.0E-3$

6 Modeling D

6.1 Characteristics of the grid

The grid of the beam is composed of an element SEG2.
The grid of the section is composed of 4 quadrilateral elements.
The group of nodes ENC is composed of the node $P1$.
The group of nodes CHA is composed of the node $P2$.

6.2 Characteristics of modeling

It is about a modeling POU_D_TGM (elements at 3 points of Gauss). The angle of gimlet is of $90°$.

Modeling:

```
MOPOU=AFFE_MODELE (MAILLAGE=MAPOU,
    AFFE=_F (TOUT=' OUI', PHENOMENE=' MECANIQUE',
        MODELISATION=' POU_D_TGM',),
)
```

Boundary conditions:

```
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
    DDL_IMPO=_F (GROUP_NO=' ENC',
        DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0,),
)
```

Mechanical loading:

```
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
    FORCE_NODALE=_F (GROUP_NO=' CHA', FX = 100, FY = 100, FZ = 100.),
)
```

Assignment of the characteristics of the elements:

```
POUCA_0=AFFE_CARA_ELEM (MODELE=MOPOU, GEOM_FIBRE=GF
    POUTRE=_F (GROUP_MA= ('BEAM'),
        SECTION=' RECTANGLE',
        CARA= ('HY', 'HZ'), VALE= (0.2, 0.1),
        PREC_AIRE=5., PREC_INERTIE=10.,),
    ORIENTATION=_F (GROUP_MA= ('BEAM'), CARA=' ANGL_VRIL', VALE=0.0,),
    MULTIFIBRE=_F (GROUP_MA= ('BEAM'), GROUP_FIBRE= ('SBET',),)
)
```

6.3 Values tested and results

Coordinate mesh SG01	Not of integration	Under-point	Reference
COOR_X	2	1	0.997265255
COOR_X	2	2	1.038090084
COOR_X	2	3	1.002734745
COOR_X	2	4	0.961909916
COOR_Y	2	1	0.961909916
COOR_Y	2	2	1.002734745
COOR_Y	2	3	1.038090084
COOR_Y	2	4	0.997265255
COOR_Z	2	1	1.040824829
COOR_Z	2	2	0.959175171
COOR_Z	2	3	0.959175171
COOR_Z	2	4	1.040824829

All the tolerance are those by default: $1.0E-3$.

7 Summary of the results

The purpose of this test is principal to check if the positions of under points of integration of modelings POU_D_EM and POU_D_TGM is well calculated.

For these modelings, the maximum error found is of $1.3\mathrm{E}-07\%$.