ZZZZ359 – Validation of the macro-order POST_CZM_FISS/OPTION = 'TRIAXIALITE'

Summary

This test validates the functionality available under the keyword OPTION = 'TRIAXIALITE' macro-order POST_CZM_FISS [U4.86.02]. The macro-order provides in this case a map dimensioned to the number of meshs carrying of the cohesive elements (joined or interfaces), containing the value of the rate of triaxiality realised in the elements of the solid mass directly connected to each one of these meshs. The objective is then to use this map as variable of order in the operator AFFE_MATERIAU (AFFE_VARC/CHAM_GD) [U4.43.03] in order to vary the parameters of the cohesive law with the triaxiality.

One validates this functionality on a simple problem of uniaxial traction, for which one has an analytical solution: it is about a linear elastic bar requested in traction, with the center of which a discontinuity governed by a cohesive behavior is laid out. One chooses for this material a null Poisson's ratio in order to preserve a uniaxial state since modelings of this test are carried out in dimension 2 and 3. Two modelings of this test have the following characteristics:

- modeling a: modeling 3D for the solid mass, modeling 3D_INTERFACE for the cohesive interface, the bar is entirely modelled;
- modeling b: modeling D_PLAN for the solid mass, modeling PLAN_JOINT for the cohesive interface, one models only half of the bar (condition of symmetry).

1 Problem of reference

1.1 Geometry

One considers a unidimensional bar length $2L=199\,mm$, with the center of which a discontinuity governed by a cohesive behavior is laid out.

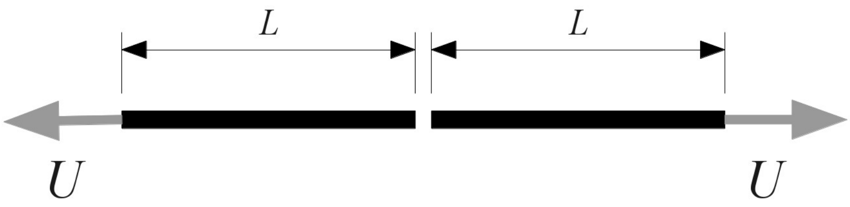

Figure 1.1-a: Bar unidimensional in traction

One notes $\hat{x}$ the variable of space, and one locates the center of the bar by $\hat{x}=0$.

1.2 Properties of material

The behavior of the ground is elastic linear:

- Young modulus: $E=30\,000\,MPa$ *
- Poisson's ratio: $\nu=0$

The cohesive behavior is described by a law closely connected in softening mode (`CZM_TAC_MIX` for the elements of interface used in modeling A, and `CZM_LIN_REG` for the elements of joint used in the modeling B), whose parameters are the following:

- Critical stress: $\sigma_c=3\,MPa$
- Energy of rupture: $G_c=0.1\,N/mm$

The values of the additional digital parameters are the following ones:

- $\mathrm{PENA\_LAGR}=45000$ for modeling A (`CZM_TAC_MIX`)
- $\mathrm{PENA\_ADHERENCE}=10^{-4}$ for modeling B (`CZM_LIN_REG`)

Note:
When one models only one half of the bar (as it is the case for modeling B) by imposing a condition of symmetry, the value of the energy of rupture is divided by 2.

1.3 Boundary conditions and loadings

The problem is unidimensional. The bar is requested in traction, by imposing the same displacement of intensity U with each one of its ends.

One makes grow the displacement imposed according to a slope active of $U=0$ with $U_{test}=2L\dfrac{\sigma_c}{E}=0.0199\,mm$

2 Reference solution

2.1 Method of calculating

Notations:

u : displacement;
σ : constraint;
ε : deformation;
$\delta=[\![u]\!]$: jump of displacement through cohesive discontinuity;
$\delta_c=\frac{2G_c}{\sigma_c}$: jump of displacement corresponding to the rupture of the bar (forced worthless).

It is about an analytical solution. The problem being unidimensional, all the quantities are scalar and the constraint is constant in space. The problem is symmetrical and can thus be restricted with the interval $[0,L]$. One has the set of equations then according to:

In the bar:

(eq1)
$$\sigma=E\varepsilon$$

(eq2)
$$\frac{du}{d\hat{x}}=\varepsilon$$

On the level of discontinuity (cohesive law refines in softening mode):

(eq3)
$$\delta=\delta_c\left(1-\frac{\sigma}{\sigma_c}\right)$$

While integrating (eq2) on the interval $[0,L]$ and while using (eq1), one obtains the following relation:

$u(\hat{x}=L)-u(\hat{x}=0)=L\varepsilon$, that is to say $U-\frac{\delta}{2}=L\frac{\sigma}{E}$

(eq4)

One notes U_c and U_f imposed displacements which correspond respectively to the levels of constraint σ_c (threshold of opening of discontinuity) and 0 (broken material). While applying (eq4), it comes:

$U_c=L\frac{\sigma_c}{E}$ and $U_f=\frac{\delta_c}{2}$

(eq5)

It is supposed that the values chosen for the parameters materials E , σ_c and G_c , and for the length of the bar L lead to a stable answer of the bar (absence of “snap-backs”). One makes moreover the assumption of a monotonous loading growing, one thus has $U_c\leqslant U\leqslant U_f$ in nonlinear mode. The values of the parameters materials and geometrical must thus check the following inequality:

$$L\frac{\sigma_c}{E}<\frac{\delta_c}{2} \quad \text{, that is to say} \quad \frac{L}{E}<\frac{G_c}{\sigma_c^2}$$

(eq6)

When this inequality is checked, one can then express the constraint according to imposed displacement:

$$\sigma=\frac{\frac{\delta_c}{2}-U}{\frac{\delta_c}{2\sigma_c}-\frac{L}{E}}, \forall U\in[\frac{L\sigma_c}{E},\frac{\delta_c}{2}]$$

(eq7)

2.2 Sizes and results of reference

At the moment corresponding to imposed displacement $U_{test}=0.0199\,mm$, the constraint is tested σ (constant in space) as well as displacement u at the point $\hat{x}=0^+$ (either the half jump of displacement).

$$\sigma=\frac{\frac{\delta_c}{2}-2\mathrm{L}\frac{\sigma_c}{E}}{\frac{\delta_c}{2\sigma_c}-\frac{L}{E}}$$

digital application: $\sigma=1.72344975053\,MPa$

$$u(\hat{x}=0^+)=2\delta_c(1-\frac{\sigma}{\sigma_c})$$

digital application : $u(\hat{x}=0^+)=0.0141838916607\,mm$

Identification	Type of reference	Value of reference
Under the loading $U_{test}=0.0199\,mm$, in any point: σ	'ANALYTICAL'	$1.72344975053\,MPa$
Under the loading $U_{test}=0.0199\,mm$, in $\hat{x}=0^+$: u	'ANALYTICAL'	$0.0141838916607\,mm$

3 Principle of the test

The test consists in defining two materials $MAT1$ and $MAT2$:

- for $MAT1$, one defines via DEFI_MATERIAU/RUPT_FRAG constant cohesive parameters: $\text{GC}=G_c$ and $\text{SIGM\_C}=G_c$;
- for $MAT2$, one defines via DEFI_MATERIAU/RUPT_FRAG_FO cohesive parameters which are functions of the triaxiality (noted α): $\text{GC}=f(\alpha).G_c$ and $\text{SIGM\_C}=f(\alpha).G_c$;

In uniaxial traction the triaxiality is worth $\alpha=1/3$, one then chooses to define for $MAT2$ a linear dependence with the triaxiality with a slope being worth 3 : $f(\alpha)=3\alpha$. In this manner: values of the parameters of the cohesive law in $MAT2$ must remain constant and identical to those selected in $MAT1$ throughout the history of the loading.

The test proceeds then in the following way:

1. definition of material $MAT1$;
2. first call to STAT_NON_LINE with $MAT1$ up to a level of imposed displacement lower than U_{test} ;
3. call to POST_CZM_FISS on the mechanical result previously got;
4. definition of material $MAT2$ by using the map of triaxiality obtained with POST_CZM_FISS like variable of order;
5. with material $MAT1$, continuation of calculation with STAT_NON_LINE starting from the mechanical state obtained as in point 2 until the loading U_{test} ;
6. with material $MAT2$, continuation of calculation with STAT_NON_LINE starting from the mechanical state obtained as in point 2 until the loading U_{test} .

It is ensured whereas the results got under the loading U_{test} at stages 5 and 6 are in agreement with the analytical solution presented previously.

4 Modeling a: elements of interface in dimension 3

4.1 Characteristics of modeling

The entirety of the bar is modelled. Modeling is chosen `3D` for the solid mass, and modeling `3D_INTERFACE` for the cohesive interface. The behavior of the cohesive interface is governed by the law of behavior `CZM_TAC_MIX`.

4.2 Characteristics of the grid

The bar is modelled with a regulated grid which comprises:

- 96 `PENTA15` and 64 `HEXA20` in the part which corresponds to the solid mass (in blue in the figure below);
- 12 `PENTA15` and 8 `HEXA20` in the part which corresponds to the cohesive interface (in red in the figure below);

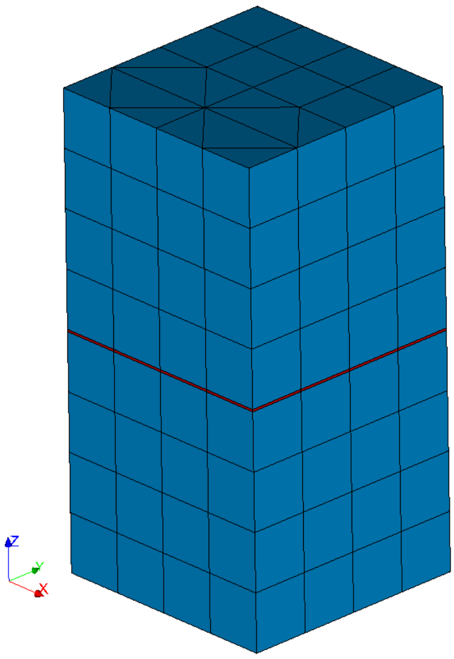

Figure 4.2-a: Grid A

4.3 Sizes tested and results

The axis of the bar corresponds to the axis z, the constraint σ and displacement u whose expressions were established previously correspond respectively to the components `SIZZ` and `DZ` fields `SIEF_ELGA` and `DEPL` (other components of these fields being worthless). One then tests these components under the loading U_{test} :

- in an unspecified point of Gauss of the solid mass for `SIZZ` ;
- for `DZ` , in an unspecified node among those which are connected at the same time to the upper part of the solid mass and the cohesive interface.

Identification	Type of reference	Value of reference
`SIZZ`	`'ANALYTICAL'`	$1.72344975053\,MPa$
`DZ`	`'ANALYTICAL'`	$0.0141838916607\,mm$

5 Modeling b: elements of joint in dimension 2

5.1 Characteristics of modeling

One models only half of the bar. Modeling is chosen D_PLAN for the solid mass, and modeling PLAN_JOINT for the cohesive interface. The behavior of the cohesive interface is governed by the law of behavior CZM_LIN_REG.

5.2 Characteristics of the grid

The bar is modelled with a regulated grid which comprises:

- 80 TRIA3 and 40 QUAD4 in the part which corresponds to the solid mass (in blue in the figure below);
- 8 QUAD4 in the part which corresponds to the cohesive interface (in red in the figure below);

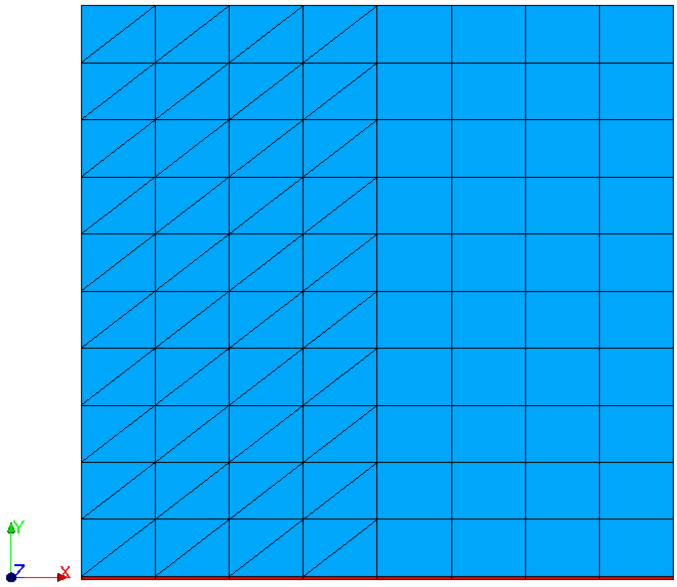

Figure 5.2-a: Grid B

5.3 Sizes tested and results

The axis of the bar corresponds to the axis y, the constraint σ and displacement u whose expressions were established previously correspond respectively to the components SIYY and DY fields SIEF_ELGA and DEPL (other components of these fields being worthless). One then tests these components under the loading U_{test} :

- in an unspecified point of Gauss of the solid mass for SIYY ;
- for DY , in an unspecified node among those connected at the same time to the upper part of the solid mass and the cohesive interface.

Identification	Type of reference	Value of reference
SIYY	'ANALYTICAL'	$1.72344975053\ MPa$
DY	'ANALYTICAL'	$0.0141838916607\ mm$

6 Summaries of the results

The goal of this test is achieved: to validate the functionality available under the keyword OPTION = 'TRIAXIALITE' macro-order POST_CZM_FISS.