SDLD04 - Transitory answer of a system mass-springs subjected to an acceleration imposed

Summary

This test consists in calculating the not deadened transitory answer of a embed-free system mass-springs linear subjected to an imposed acceleration.

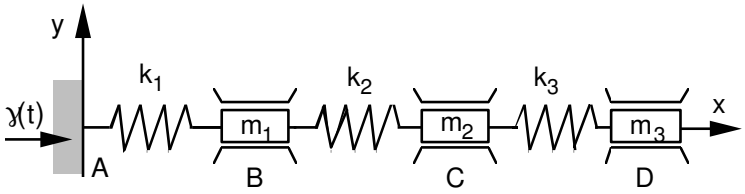
One tests the discrete element in traction and compression, the calculation of the clean modes, the static modes and the calculation of the transitory answer of a system subjected to an imposed acceleration. One compares the direct calculation of the answer to his calculation by modal recombination.

This case test is from guide VPCS. The reference solution is an analytical calculation. The errors on the got results are normal taking into account the step of time chosen for digital integration.

1 Problem of reference

1.1 Geometry

One calculates the response of a linear system composed of three masses and three springs to an acceleration imposed on the level of his point of anchoring (A) :



1.2 Properties of materials

- stiffnesses of connection: $k=k_1=k_2=k_3=1000\,N/m$;
- specific masses: $m=m_1=m_2=m_3=1\,kg$.

1.3 Boundary conditions and loadings

Boundary conditions

Only authorized displacements are the translations according to the axis x .
The point A is embedded: $dx=dy=dz=drx=dry=drz=0$.

Loading

The point of anchoring A is subjected to an acceleration, function increasing of time, according to the direction x : $\gamma(t)=2.10^5.t^2$ (t vary 0 with $0,1\,s$).

1.4 Initial conditions

The system is initially at rest: with $t=0$, $dx(0)=0$ and $dx/dt(0)=0$ in any point.

2 Reference solution

2.1 Method of calculating used for the reference solution

One calculates the Eigen frequencies initially f_i and clean vectors Φ_{Ni} associated standardized compared to the matrix of mass. One calculates then the generalized answer of the mono-excited system by solving analytically the integral of Duhamel [bib1]. Lastly, one restores on the physical basis displacement relating to the point D.

Calculation of the Eigen frequencies

The matrices of mass and stiffness are the following ones:

$$M=\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix},\ K=k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Eigen frequencies ω are solution of the equation $det[K-\lambda^2 M]=0$, that is to say $\lambda^3-5\lambda^2+6\lambda-1=0$ where $\lambda=\frac{\omega}{\omega_0}$ and $\omega=\sqrt{\frac{k}{m}}$.

Calculation of the generalized answer of the mono-excited system

$\gamma(t)=a.t^2$ with $a=2.10^5$.

In the absolute reference mark, the fundamental equation of the dynamics of the system mass-springs not deadened is written: $M\ddot{X}_a+K X_a=0$.

Absolute displacement X_a breaks up into a uniform displacement of training in translation X_e and in a relative displacement X_r: $X_a=X_r+X_e$.
The equation of the movement in the relative reference mark is written then: $M\ddot{X}_r+K X_r=-M\Psi\ddot{X}_s=Q$
with $\ddot{X}_s=\gamma(t)=a.t^2$ and $\Psi=\begin{bmatrix}1\\1\\1\end{bmatrix}$ and thus $Q=a.t^2 m\begin{bmatrix}1\\1\\1\end{bmatrix}$.

The equation of the movement projected on the basis of standardized dynamic mode compared to the matrix of mass is written:

$$\ddot{\alpha}_i(t)+\omega_i^2\alpha_i(t)=\frac{\Phi_i^T.M.\Psi}{\Phi_i^T.M.\Phi_i}\gamma(t)=-p_i(t)\gamma(t).$$

The answer of this linear system, at one moment t is given by the integral of Duhamel:

$$\ddot{\alpha}_i(t)=\frac{1}{\omega_i}\int_0^t -p_i(t)\gamma(t).\sin\omega_i(t-\tau)\,d\tau=-\frac{p_i(t)}{\omega_i}\int_0^t a.t^2\sin\omega_i(t-\tau)\,d\tau.$$

However, according to [bib1], $\int_0^t a.t^2 \sin \omega_i (t-\tau)\, d\tau = \frac{\alpha}{\omega_i}\left[t^2 + \frac{2}{\omega_i}(\cos \omega_i t - 1)\right]$.

Thus $X_r = \Phi_i \cdot \alpha_i = -\sum_i \frac{a.p_i(t).\Phi_i}{\omega_i^2}\left[t^2 + \frac{2}{\omega_i}(\cos \omega_i t - 1)\right]$.

2.2 Results of reference

One takes for results of reference the three Eigen frequencies of the system and relative displacement x_r at the point D, for various moments understood enters 0 and $0,1\ s$.

2.3 Uncertainty on the solution

No if one calculates the integral of Duhamel analytically [bib1], [bib2].

2.4 Bibliographical references

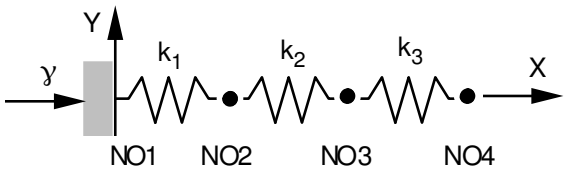
1) J.S. PRZEMIENIECKI: Theory of matrix structural analysis. New York, Mac Graw-Hill, 1968, p. 351-357

2) S.P. TIMOSHENKO, D.H. YOUNG and W. WEAVER: Vibrations problems in engineering 4th edition, New York, Wiley & Sounds, 1974, p. 284-321

3 Modeling A

3.1 Characteristics of modeling

The springs and specific masses are modelled by discrete elements with 3 degrees of freedom DIS_T:



The node $NO1$ is embedded and subjected to an imposed acceleration $\gamma(t)$. One calculates the relative displacement of the node $NO4$.

Calculations by modal synthesis
One considers the complete base of the clean modes. Temporal integration is carried out with the algorithms of Newmark, Euler and Devogelaere with a step of time of $0{,}001\,s$. Calculations are filed all the steps of time.
A reduced damping is considered ξ_i no one for the whole of the calculated modes.

The loading is taken into account in the form of vector project on the modal basis EXCIT : (VECT_GENE) or in the form of modal component EXCIT: (NUME_MODE) or both at the same time.

Direct calculations
Temporal integration is carried out either with the algorithm of Newmark or with the explicit algorithm of the differences centered with a step in time of $0{,}001\,s$. Calculations are filed all the ten steps of time.

Note:

> *As the diagram of the centered differences can be used only with one matrix of diagonal mass, one calculates the elementary matrices with the option MASS_MECA_DIAG in the operator CALC_MATR_ELEM.*

Taking into account of an initial state
In the two types of calculation, one checks that the relative displacement obtained of a calculation carried out in once is identical to that obtained in several times, i.e. while regarding as initial state, the result of the last step of calculated time:

ETAT_INIT =_F (RESULT…) for a calculation by modal synthesis;
ETAT_INIT =_F (DEPL…
QUICKLY.) for a direct calculation.

Taking into account of the modes neglected by static correction:
One considers a modal base made up of the first two clean modes and one has it complete by a mode corresponding to the static response of the system studied to a unit loading of type forces imposed in the direction $-x$ (keywords MODE_CORR and CORR_STAT in the operator DYNA_VIBRA).

3.2 Characteristics of the grid

Many nodes: 4
Many meshs and types: 3 DIS_T

3.3 Features tested

One tests in particular the taking into account of an initial state and static corrections.

3.4 Sizes tested and results

Eigen frequencies (in Hz) system:

Number of the mode	Analytical
1	2.239
2	6.275
3	9.069

Values of the relative displacement of the node $NO4$ for various moments:

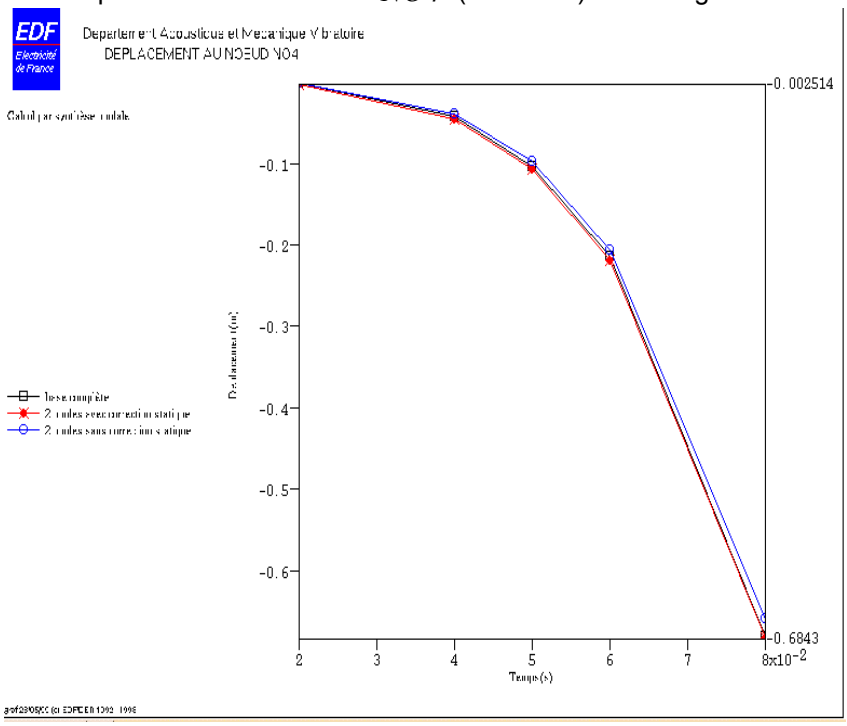
Transitory calculation by modal synthesis
One tests the taking into account of a loading in the form of vector project on the modal basis, in the form of modal component, vector project and of modal component simultaneously as well as the taking into account of the neglected modes.

Time (s)	Reference	*Code_Aster* *Loading of type generalized vector* *Algorithm of Newmark*	Relative error %	*Code_Aster* *Loading of modal the component type* *Algorithm of Euler*	Relative error %
0.02	– 2,700E-03	– 2,680E-03	– 0.741	– 2,660E-03	– 1.481
0.04	– 4,260E-02	– 4,272E-02	0.279	– 4,264E-02	0.091
0.05	– 1,041E-01	– 1,042E-01	0.134	– 1,041E-01	0.015
0.06	– 2,158E-01	– 2,161E-01	0.121	– 2,159E-01	0.038
0.08	– 6,813E-01	– 6,819E-01	0.094	– 6,816E-01	0.049
0.10	– 1,658E+00	– 1,659E+00	0.082	– 1,659E+00	0.055

Type of loading	Time (s)	Reference	*Code_Aster*	relative error %
Generalized vector and modal component simultanéement (Euler)	0.02	– 5,400E-03	– 5,320E-03	– 1.482
	0.04	– 8,520E-02	– 8,528E-02	0.091
	0.05	– 2,082E-01	– 2,082E-01	0.015
	0.06	– 4,316E-01	– 4,318E-01	0.038
	0.08	– 1,363E+00	– 1,363E+00	0.049
	0.10	– 3,316E+00	– 3,318E+00	0.055
Generalized vector Devogelaere (more correction statics)	0.02	– 4,000E-03	– 3,985E-03	– 0.373
	0.04	– 4,640E-02	– 4,640E-02	0.01
	0.05	– 1,085E-01	– 1,086E-01	0.084
	0.06	– 2,203E-01	– 2,204E-01	0.039
	0.08	– 6,842E-01	– 6,843E-01	0.021
	0.10	– 1,659E+00	– 1,659E+00	0.026

The results with incomplete modal base without static correction are not tested. One illustrates Ci - below the interest of the static correction:

Displacement of the node $NO4$ (in meters) according to time



Base complete — Base incomplete without static correction — Base incomplete with static correction

A test of not-regression is made so making sure of the good performance of the order POST_GENE_PHYS following the correction of a bug.

Time (s)	Identification	Type of Référence	Value of reference	Precision
0,1	Field DEPL, Component DX, Node N02	'NON_REGRESSION'	-	1E-6

Direct transitory calculation

One compares displacements calculated with the node $N04$ according to various diagrams of integration:

Time (s)	Reference	*Code_Aster* *Diagram of Newmark*	Relative error %	*Code_Aster* *Diagram of the centered differences*	Relative error %
0.02	– 2,700E-03	– 2,680E-03	– 0.741	– 2,660E-03	– 1.482
0.04	– 4,260E-02	– 4,272E-02	0.279	– 4,264E-02	0.091
0.05	– 1,041E-01	– 1,042E-01	0.134	– 1,041E-01	0.015
0.06	– 2,158E-01	– 2,161E-01	0.121	– 2,159E-01	0.038
0.08	– 6,813E-01	– 6,819E-01	0.094	– 6,745E-01	– 1.004
0.10	– 1,658E+00	– 1,659E+00	0.082	– 1,645E+00	– 0.803

Taking into account of an initial state:
As expected, the relative displacements calculated in once are strictly identical to those obtained by regarding as initial state the result of the last step of calculated time.

One also carries out tests of nonregression on the energy balance.

Energy	Moment (s)	*Code_Aster* *Diagram of Newmark*	*Code_Aster* *Diagram of the centered differences*
TRAV_EXT	0,06	2.9 7989 E+02	2.9 7,838 E+02
ENER_TOT	0,06	1.8 8902 E+01	1.8 8,688 E+01
ENER_CIN	0,06	2,79099E+02	2.79 132 E+02
TRAV_EXT	0.1 1	1,03435E+04	9.98152 E+0 3
ENER_TOT	0.1 1	1,71815E+03	1.67741 E+03
ENER_CIN	0.1 1	8,62533E+03	8.30528 E+03

4 Summary of the results

The reference solution is an analytical calculation. The errors on the got results are normal taking into account the step of time chosen for digital integration.