SDLD30 - Spectral seismic answer of a system 2 masses and 3 springs multimedia

Summary:

The problem consists in calculating the spectral response of a system 2 masses - 3 springs subjected to a multiple seismic excitation.

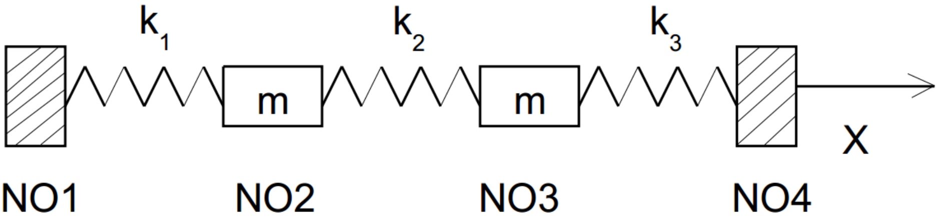
One tests the discrete element in traction, the calculation of the clean modes, the static modes and the spectral response by modal superposition via the operator `COMB_SISM_MODAL`. Various office pluralities are tested during the calculation of the answers of supports.

The got results are in very good agreement with the analytical results of reference.

1 Problem of reference

1.1 Geometry

The structure is modelled by a set of 3 springs and of 2 specific masses.



1.2 Material properties

Stiffness of connection: $k_1 = k_2 = k = 1000 \; N/m$; $k_3 = 10k = 10000 \; N/m$
specific mass: $m_2 = m_3 = m = 10 \; kg$.

1.3 Boundary conditions and loadings

- **boundary conditions**
  Only authorized displacements are the translations according to the axis x .
  Points $NO1$ and $NO4$ are embedded: DX=DY=DZ=DRX=DRY=DRZ=0.
  The other points are free in translation according to the direction x : DY=DZ=DRX=DRY=DRZ=0.

- **loading**
  The structure is subjected to a multiple spectral seismic excitation and differential displacements.
  The spectra of answers of oscillator in pseudonym acceleration are simplified. Only the values corresponding to the 2 Eigen frequencies of the system are mentioned. They do not depend on damping:
- with the node $NO1$:

$$SRO_{NO1}(f_1) = A_{11} = 7 \, m/s^2$$
$$SRO_{NO1}(f_2) = A_{21} = 5 \, m/s^2$$
$$DDS_{NO1} = D1 = -0.04 \, m$$

- with the node $NO4$:

$$SRO_{NO4}(f_1) = A_{12} = 12 \, m/s^2$$
$$SRO_{NO4}(f_2) = A_{22} = 6 \, m/s^2$$
$$DDS_{NO4} = D_2 = 0.06 \, m$$

1.4 Initial conditions

The system is initially at rest.

2 Reference solution

2.1 Method of calculating used for the reference solution

One calculates the spectral response by modal superposition of a system masses spring subjected to two distinct excitations. One determines the displacement of the masses and the reactions of support to the nodes $NO1$ and $NO4$ along the axis x .

One calculates analytically:

- Eigen frequencies f_i ,
- associated clean vectors φ_{Ni} standardized compared to the modal mass,
- static modes of supports ψ_j system,
- factors of modal participation P_{ij} relating to the supports,
- Rm_{ij} the maximum of answer of each mode starting from the spectra of excitation,
- Re_j the contribution of the movement of training of each support starting from differential displacements,
- Rc_j the static term of correction,
- primary and secondary components of the answer according to the adopted rules of office plurality.

2.2 Results of reference

- **matrix of rigidity** K

$$K=\begin{bmatrix} k & -k & 0 & 0 \\ -k & 2\text{k} & -k & 0 \\ 0 & -k & 11\text{k} & -10\text{k} \\ 0 & 0 & -10\text{k} & 10\text{k} \end{bmatrix}$$

$$K^p=\begin{bmatrix} 2\text{k} & -k & -k & 0 \\ -k & 11\text{k} & 0 & -10\text{k} \\ -k & 0 & k & 0 \\ 0 & -10\text{k} & 0 & 10\text{k} \end{bmatrix}$$

partitionnée matrix degrees of freedom of structure 2.3, degrees of freedom of support 1.4

$$K^p=\begin{bmatrix} k_{xx} & k_{xs} \\ k_{sx} & k_{ss} \end{bmatrix} \qquad k_{xx}=\begin{bmatrix} 2\text{k} & -k \\ -k & 11\text{k} \end{bmatrix} \qquad k_{xs}=\begin{bmatrix} -k & 0 \\ 0 & -10\text{k} \end{bmatrix}$$

$$k_{sx}=\begin{bmatrix} -k & 0 \\ 0 & -10\text{k} \end{bmatrix} \qquad k_{ss}=\begin{bmatrix} k & 0 \\ 0 & 10\text{k} \end{bmatrix}$$

- **matrix of mass** M

$$M=\begin{bmatrix}0&0&0&0\\0&m&0&0\\0&0&m&0\\0&0&0&0\end{bmatrix}$$

- **modal calculation in embedded base**

$$k_{xx}=\begin{bmatrix}2k&-k\\-k&11k\end{bmatrix}$$

$$\left|k_{xx}-\lambda_i m_{xx}\varphi_i\right|=0 \qquad \lambda_i=\omega_i^2$$

that is to say $\lambda_1=\frac{k}{2m}\left(13-\sqrt{85}\right)$ $\lambda_2=\frac{k}{2m}\left(13+\sqrt{85}\right)$

1) Eigen frequencies:

that is to say $f_1=\frac{\omega_1}{2\pi}$ $f_2=\frac{\omega_2}{2\pi}$

1) not normalized clean modes:

that is to say $\varphi_1=\begin{pmatrix}0\\1\\(-9+\sqrt{85})/2\\0\end{pmatrix}$ $\varphi_2=\begin{pmatrix}0\\-1\\(9+\sqrt{85})/2\\0\end{pmatrix}$

- generalized modal masses $\mu_i={}^T\varphi_i M\varphi_i$:

that is to say $\mu_1=\frac{m}{4}\left(170-18\sqrt{85}\right)$ $\mu_2=\frac{m}{4}\left(170+18\sqrt{85}\right)$

[1] own standards modes with the unit generalized modal mass φ_{Ni} :

that is to say $\varphi_{N1}=\frac{\varphi_1}{\sqrt{\mu_1}}$ $\varphi_{N2}=\frac{\varphi_2}{\sqrt{\mu_2}}$

- modal reactions Fm_i :

$$r_i=k_{sx}\varphi_{Nis} \qquad \varphi_{Ni}^p=\begin{pmatrix}\varphi_{Nix}\\\varphi_{Nis}\end{pmatrix} \qquad Fm_i^p=\begin{pmatrix}0\\r_i\end{pmatrix}$$

that is to say $Fm_1=\frac{k}{\sqrt{\mu_1}}\begin{pmatrix}-1\\0\\0\\5(9-\sqrt{85})\end{pmatrix}$ $Fm_2=\frac{k}{\sqrt{\mu_2}}\begin{pmatrix}1\\0\\0\\-5(9+\sqrt{85})\end{pmatrix}$

- factors of modal participation $P_{ij}={}^T\varphi_i M\,\psi_j$:

- contribution of the dynamic mode 1 to the movement imposed on the node $NO1$:

$$P_{11}={}^T\varphi_1 M\,\psi_1=\frac{m}{42\sqrt{\mu_1}}\left(13+\sqrt{85}\right)$$

- contribution of the dynamic mode 1 to the movement imposed on the node $NO4$:

$$P_{12}={}^T\varphi_1 M\,\psi_2=\frac{10\,m}{21\sqrt{\mu_1}}\left(-8+\sqrt{85}\right)$$

- contribution of the dynamic mode 2 to the movement imposed on the node $NO1$:

$$P_{21}={}^T\varphi_2 M\,\psi_1=\frac{m}{42\sqrt{\mu_2}}\left(-13+\sqrt{85}\right)$$

- contribution of the dynamic mode 2 to the movement imposed on the node $NO4$:

$$P_{22}={}^T\varphi_2 M\,\psi_2=\frac{10\,m}{21\sqrt{\mu_2}}\left(8+\sqrt{85}\right)$$

- factor of participation of the dynamic mode 1 in the direction X:

$$P_{1X}=P_{11}+P_{12}$$

- factor of participation of the dynamic mode 2 in the direction X:

$$P_{2X}=P_{21}+P_{22}$$

- **static modes of supports** ψ_j

- static solution with a unit displacement of the node $NO1$:

displacements: $\psi_1=\frac{1}{21}\begin{pmatrix}21\\11\\1\\0\end{pmatrix}$ nodal reactions: $Fs_1=K\,\psi_1=\frac{10}{21}k\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$

- static solution with a unit displacement of the node $NO4$:

displacements: $\psi_2=\frac{1}{21}\begin{pmatrix}0\\10\\20\\21\end{pmatrix}$ nodal reactions: $Fs_2=K\,\psi_2=\frac{10}{21}k\begin{pmatrix}-1\\0\\0\\1\end{pmatrix}$

- **answer of the mode** i **with the movement of the support** j

$Rm_{ij}=r_i P_{ij}\frac{A_{ij}}{\omega_i^2}$ with $r_i=\varphi_{Ni}$ or Fm_i

- **static correction**

- static modes u_j solution of $K\,u_j=M\,\psi_j$:

displacements: $u_1=\frac{m}{441\,k}\begin{pmatrix}0\\122\\13\\0\end{pmatrix}$ nodal reactions: $F\,u_1=\frac{m}{441}\begin{pmatrix}-122\\231\\21\\-130\end{pmatrix}$

displacements: $u_2=\frac{m}{441\,k}\begin{pmatrix}0\\130\\50\\0\end{pmatrix}$ nodal reactions: $F\,u_2=\frac{m}{441}\begin{pmatrix}-130\\210\\420\\-500\end{pmatrix}$

- static correction relating to the movement of the support j if mode 2 is not retained:

$$Rc_j=\left(ru_j-\frac{P_{1j}r_1}{\omega_1^2}\right)A_{1j} \text{ with: } ru_j=u_j \text{ ou } Fu_j \text{ and } r_1=\varphi_{NI} \text{ ou } Fm_1$$

- **contribution of the support j with the movement of training**

$$R\,e_j=r_j\,D_j \text{ with } r_j=\psi_j \text{ ou } Fs_j$$

These analytical calculations are described in the file Matlab sdld30a.55.

2.3 Uncertainty on the solution

No (exact analytical solution).

3 Modeling A

3.1 Characteristics of modeling

The system is modelled by:

- 3 discrete elements K_T_D_L,
- 2 discrete elements M_T_D_N.

3.2 Characteristics of the grid

The grid consists of 3 meshs SEG2.

4 Results of modeling A

4.1 Eigen frequencies

MODE	Reference
1	2,18815E+00
2	5,30484E+00

4.2 Total answer on complete modal basis

Modes 1 and 2 are taken into account. The components inertial (primary education) and statics (secondary) of the answer are directly cumulated on the level as of supports.

- **calculation n°1**

  COMB_MODE=' SRSS'

- answer of the support $j=1$ (node $NO1$): $R_1=\sqrt{Rm_1^2+Re_1^2}$ with $Rm_1=\sqrt{Rm_{11}^2+Rm_{21}^2}$ (office plurality on the modes)
- answer of the support $j=2$ (node $NO4$): $R_2=\sqrt{Rm_2^2+Re_2^2}$ with $Rm_2=\sqrt{Rm_{12}^2+Rm_{22}^2}$
- total answer: $R=\sqrt{R_1^2+R_2^2}$ (office plurality on the supports)

absolute displacements: DEPL

NODE	Reference
$NO1$	4,00000E-02
$NO2$	5,43820E-02
$NO3$	5,75544E-02
$NO4$	6,00000E-02

nodal reactions: REAC_NODA

NODE	Reference
$NO1$	5,36769E+01
$NO4$	7,44120E+01

4.3 Total answer on incomplete modal basis without static correction

Only mode 1 is taken into account. The components inertial (primary education) and statics (secondary) of the answer are directly cumulated on the level of the supports.

- **calculation n°1**

```
COMB_MODE=' SRSS'
```

- answer of the support $j=1$ (node $NO1$): $R_1=\sqrt{Rm_1^2+Re_1^2}$ with $Rm_1=Rm_{11}$
- answer of the support $j=2$ (node $NO4$): $R_2=\sqrt{Rm_2^2+Re_2^2}$ with $Rm_2=Rm_{12}$
- total answer: $R=\sqrt{R_1^2+R_2^2}$

absolute displacements: `DEPL`

NODE	Reference
$NO1$	4,00000E-02
$NO2$	5,43794E-02
$NO3$	5,73536E-02
$NO4$	6,00000E-02

nodal reactions: `REAC_NODA`

NODE	Reference
$NO1$	5,36743E+01
$NO4$	5,68312E+01

4.4 Total answer on incomplete modal basis with static correction

Only mode 1 intervenes in the calculation of the answer. The static contribution of neglected mode 2 is taken into account.

- **calculation n°1**

```
COMB_MODE=' SRSS'
```

- answer of the support $j=1$ (node $NO1$): $R_1=\sqrt{Rm_1^2+Rc_1^2+Re_1^2}$ with $Rm_1=Rm_{11}$
- answer of the support $j=2$ (node $NO4$): $R_2=\sqrt{Rm_2^2+Rc_2^2+Re_2^2}$ with $Rm_2=Rm_{12}$
- total answer: $R=\sqrt{R_1^2+R_2^2}$

absolute displacements: `DEPL`

NODE	Reference	Tolerance
$NO1$	4,00000E-02	0,001
$NO2$	0.054389658	0,001
$NO3$	0.058152653	0,001
$NO4$	6,00000E-02	0,001

nodal reactions: `REAC_NODA`

NODE	Reference	Tolerance
NO1	53.6846755	0,001
NO4	111.6190600	0,001

4.5 Partition of the components primary and secondary of the answer

The components inertial (primary education) and statics (secondary) are treated separately.

- **calculation n°1**
- primary answer on modal basis supplements (modes 1 and 2)
  `COMB_MODE=' SRSS'`

1) answer of the support $j=1$ (node $NO1$): $RI_1=\sqrt{Rm_{11}^2+Rm_{21}^2}$ (office plurality on modes)

- answer of the support $j=2$ (node $NO4$): $RI_2=\sqrt{Rm_{12}^2+Rm_{22}^2}$
- primary answer: $RI=\sqrt{RI_1^2+RI_2^2}$

relative displacements: `DEPL`

NODE	Reference
NO1	0,00000E+00
NO2	4,12562E-02
NO3	6,60152E-03
NO4	0,00000E+00

nodal reactions: `REAC_NODA`

NODE	Reference
NO1	4,12562E+01
NO4	6,60152E+01

- secondary answer
  `COMB_DEPL_APPUI=' QUAD'`

1) secondary answer: $RII=\sqrt{Re_1^2+Re_2^2}$

displacements of training: `DEPL`

NODE	Reference
NO1	4,00000E-02
NO2	3,54306E-02
NO3	5,71746E-02
NO4	6,00000E-02

nodal reactions: `REAC_NODA`

NODE	Reference
NO1	3,43386E+01
NO4	3,43386E+01

- **calculation n°2**

  - primary answer on incomplete modal basis without static correction
  Only mode 1 intervenes in the calculation of the answer
  `COMB_MODE=' SRSS'`

  - answer of the support $j=1$ (node $NO1$): $RI_1 = Rm_{11}$
  - answer of the support $j=2$ (node $NO4$): $RI_2 = Rm_{12}$
  - primary answer: $RI = \sqrt{RI_1^2 + RI_2^2}$

  relative displacements: `DEPL`

NODE	Reference
NO1	0,00000E+00
NO2	4,12528E-02
NO3	4,52841E-03
NO4	0,00000E+00

  nodal reactions: `REAC_NODA`

NODE	Reference
NO1	4,12528E+01
NO4	4,52841E+01

- secondary answer
`COMB_DEPL_APPUI=' LINE'`

- secondary answer: $RII = Re_1 + Re_2$

  - displacements of training: `DEPL`

NODE	Reference
$NO1$	-4,00000E-02
$NO2$	7,61905E-03
$NO3$	5,52381E-02
$NO4$	6,00000E-02

  nodal reactions: `REAC_NODA`

NODE	Reference
$NO1$	-4,76190E+01
$NO4$	4,76190E+01

- **calculation n°3**

  - primary answer on incomplete modal basis with static correction
  Only mode 1 intervenes in the calculation of the answer
  `COMB_MODE=' SRSS'`

  - answer of the support $j=1$ (node $NO1$): $RI_1 = \sqrt{Rm_{11}^2 + Rc_1^2}$

* answer of the support $j=2$ (node $NO4$): $RI_2=\sqrt{Rm_{12}^2+Rc_2^2}$
* primary answer: $RI=\sqrt{RI_1^2+RI_2^2}$

relative displacements: `DEPL`

NODE	**Reference**	**Tolerance**
$NO1$	0,00000E+00	-
$NO2$	4,1266282E-02	0,001
$NO3$	1.0620582E-02	0,001
$NO4$	0,00000E+00	-

nodal reactions: `REAC_NODA`

NODE	**Reference**	**Tolerance**
$NO1$	4,12662823E+001	0,001
$NO4$	1.0620581996E+02	0,001

* secondary answer
  `COMB_DEPL_APPUI=' ABS'`

* secondary answer: $RII=|Re_1|+|Re_2|$

displacements of training: `DEPL`

NODE	**Reference**
$NO1$	4,00000E-02
$NO2$	4,95238E-02
$NO3$	5,90476E-02
$NO4$	6,00000E-02

nodal reactions: `REAC_NODA`

NODE	Reference
NO1	4,76190E+01
NO4	4,76190E+01

- **calculation n°4**
- primary answer on incomplete modal basis with static correction

Only mode 1 intervenes in the calculation of the answer.
`COMB_MODE=' SRSS'`

- answer of the support $j=1$ (node $NO1$): $RI_1=\sqrt{Rm_{11}^2+Rc_1^2}$
- answer of the support $j=2$ (node $NO4$): $RI_2=\sqrt{Rm_{12}^2+Rc_2^2}$
- primary answer: $RI=\sqrt{RI_1^2+RI_2^2}$

- secondary answer: test office plurality of $DDSs$

5 loading cases are defined. The 5 associated elementary static answers are:

- case a: $DDSa_{NO1}=-0.04$ that is to say $R_a=r_1\times DDSa_{NO1}$
- case b: $DDSb_{NO4}=0.06$ that is to say $R_b=r_2\times DDSb_{NO4}$
- case C: $DDSc_{NO4}=0.03$ that is to say $R_c=r_2\times DDSc_{NO4}$
- case D: $DDSd_{NO1}=-0.07$ that is to say $R_d=r_1\times DDSd_{NO1}$
- case E: $DDSe_{NO4}=0.05$ that is to say $R_e=r_2\times DDSe_{NO4}$

4 combinations are calculated:

- **combination n°1**

linear office plurality of the cases has and b: `TYPE_COMBI=' LINE' NUME_ORDRE=200`

secondary answer: $RII_1=Ra+Rb$

absolute displacements: `DEPL`

NODE	Reference
NO1	-4,00000E-02
NO2	7,61905E-03
NO3	5,52381E-02
NO4	6,00000E-02

nodal reactions: `REAC_NODA`

NODE	Reference
NO1	-4,76190E+01
NO4	4,76190E+01

- **combination n°2**

absolute office plurality of the cases has and C: `TYPE_COMBI=' ABS' NUME_ORDRE=201`

secondary answer: $RII_2 = |Ra| + |Rc|$

absolute displacements: `DEPL`

NODE	Reference
NO1	4,00000E-02
NO2	3,52381E-02
NO3	3,04762E-02
NO4	3,00000E-02

nodal reactions: `REAC_NODA`

NODE	Reference
NO1	3,33333E+01
NO4	3,33333E+01

- **combination n°3**

quadratic office plurality of the cases D and E: `TYPE_COMBI=' QUAD' NUME_ORDRE=202`

secondary answer: $RII_3 = \sqrt{Rd^2 + Re^2}$

absolute displacements: `DEPL`

NODE	Reference
NO1	7,00000E-02
NO2	4,37189E-02
NO3	4,77356E-02
NO4	5,00000E-02

nodal reactions: `REAC_NODA`

NODE	Reference
NO1	4,09635E+01
NO4	4,09635E+01

- **combination n°4**

linear office plurality of the cases has and E: `TYPE_COMBI=' LINE' NUME_ORDRE=203`

secondary answer: $RII_4 = Ra + Re$

absolute displacements: `DEPL`

NODE	Reference
NO1	-4,00000E-02
NO2	2,85714E-03

NO3	4,57143E-02
NO4	5,00000E-02

nodal reactions: `REAC_NODA`

NODE	Reference
NO1	-4,28571E+01
NO4	4,28571E+01

The total secondary answer is established by the quadratic office plurality of the 4 preceding combinations:

$$RII=\sqrt{RII_1^2+RII_2^2+RII_3^2+RII_4^2}$$ `NUME_ORDRE=204`

absolute displacements: `DEPL`

NODE	Reference
NO1	9,84886E-02
NO2	5,67386E-02
NO3	9,13703E-02
NO4	9,74679E-02

nodal reactions: `REAC_NODA`

NODE	Reference
NO1	8,30266E+01
NO4	8,30266E+01

5 Summary of the results

Results got with *Code_Aster* are in conformity with the analytical results of reference.