SDLD33 - Spectral seismic answer of a system 2 masses - 4 springs multimedia by 2 décorrélés groups of supports

Summary:

The problem consists in calculating the spectral response of a system 2 masses – 4 springs multimedia, subjected to a multiple seismic excitation, by regarding the 3 supports as 2 groups of décorrélés supports.

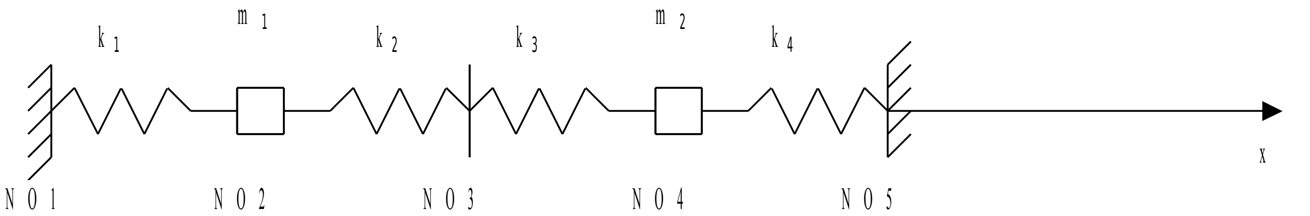
One tests the discrete element in traction, the calculation of the clean modes, the static modes and the spectral response by modal superposition via the operator COMB_SISM_MODAL. This CAS-test makes it possible to validate the order of the combinations to be considered in this case, namely the office plurality intra-group then the office plurality joint committee.

The got results are in very good agreement with the analytical results of reference.

1 Problem of reference

1.1 Geometry

The system is composed of a set of 4 springs, 2 specific masses, is supported by 3 supports.



1.2 Material properties

Stiffness of connection: $k=k_1=k_2=10^3\ N/m$ $k_3=k_4=2.10^3\ N/m$;
specific mass: $m=m_1=m_2=10\ kg$.

1.3 Boundary conditions and loadings

Boundary conditions :
Only authorized displacements are the translations according to the axis x .
Points $NO1$, $NO3$ and $NO5$ are embedded: $dx=dy=dz=drx=dry=drz=0$.
The other points are free in translation according to the direction x : $dy=dz=drx=dry=drz=0$.

Loading :
The structure is subjected to a multiple spectral seismic excitation and differential displacements.
The spectra of answers of oscillator in pseudo-acceleration are simplified. Only the values corresponding to the 2 Eigen frequencies of the system are mentioned. They do not depend on damping:

with the node $NO1$:

$$SRO_{NO1}(f_1)=A_{11}=7\,m/s^2$$
$$SRO_{NO1}(f_2)=A_{21}=5\,m/s^2$$
$$DDS_{NO1}=D_1=-0.04\,m$$

with the node $NO3$:

$$SRO_{NO3}(f_1)=A_{11}=7.7\,m/s^2$$
$$SRO_{NO3}(f_2)=A_{12}=5.5\,m/s^2$$
$$DDS_{NO3}=D_2=-0.044\,m$$

with the node $NO5$:

$$SRO_{NO5}(f_1)=A_{21}=12\,m/s^2$$
$$SRO_{NO5}(f_2)=A_{22}=6\,m/s^2$$
$$DDS_{NO5}=D_3=0.06\,m$$

Excitations with the nodes $NO1$ and $NO3$ are correlated. One sets up 2 groups of décorrélés supports: group 1 is composed of the nodes $NO1$ and $NO3$; group 2 is made up by the only node $NO5$.

1.4 Initial conditions

The system is at rest.

2 Reference solution

2.1 Method of calculating used for the reference solution

One calculates the spectral response by modal superposition of a system 2 masses – 4 springs subjected to three distinct excitations. One determines the displacement of the masses to the nodes $NO2$ and $NO4$ along the axis x.

One calculates analytically:

- Eigen frequencies f_i,
- associated clean vectors ϕ_{N_i} standardized compared to the modal mass,
- static modes of supports Ψ_i system,
- factors of modal participation P_{ij} relating to the supports,
- Rm_{ij} the maximum of answer of each mode starting from the spectra of excitation,
- Re_{ij} the contribution of the movement of training of each support starting from differential displacements,

2.2 Results of reference

2.2.1 characteristic Matrices and vectors

- **matrix of rigidity** K

$$K=\begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & 0 & -k_4 & k_4 \end{bmatrix}$$ matrix relating to the degrees of freedom 1,2,3,4,5

$$K^p=\begin{bmatrix} k_1+k_2 & 0 & -k_1 & -k_2 & 0 \\ 0 & k_3+k_4 & 0 & -k_3 & -k_4 \\ -k_1 & 0 & k_1 & 0 & 0 \\ -k_2 & -k_3 & 0 & k_2+k_3 & 0 \\ 0 & -k_4 & 0 & 0 & k_4 \end{bmatrix}$$

partitionnée matrix degrees of freedom of structure 2.4, degrees of freedom of support 1,3,5

$$\boldsymbol{K}^p=\begin{bmatrix} \boldsymbol{k} & \boldsymbol{k}_{xs} \\ \boldsymbol{k}_{sx} & \boldsymbol{k}_{ss} \end{bmatrix} \quad \boldsymbol{k}=\begin{bmatrix} k_1+k_2 & 0 \\ 0 & k_3+k_4 \end{bmatrix} \quad \boldsymbol{k}_{xs}=\begin{bmatrix} -k_1 & 0 & -k_2 & 0 \\ 0 & -k_3 & 0 & -k_4 \end{bmatrix}$$

- **matrix of mass** M

$$M=\begin{bmatrix}0&0&0&0&0\\0&m_1&0&0&0\\0&0&0&0&0\\0&0&0&m_2&0\\0&0&0&0&0\end{bmatrix}$$

matrix relating to the degrees of freedom 1,2,3,4,5

$$M^p=\begin{bmatrix}m_1&0&0&0&0\\0&m_2&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}$$

partitionnée matrix degrees of freedom of structure 2.4, degrees of freedom of support 1,3,5

- **modal calculation in embedded base**

$$(K-\lambda_i M)\phi_i=0 \quad \lambda_i=\omega_i^2$$

$$dét(K-\lambda_i M)=0 \Leftrightarrow \lambda_i^2-\left(\frac{k_1+k_2}{m_1}+\frac{k_3+k_4}{m_2}\right)\lambda_i+\frac{(k_1+k_2)(k_3+k_4)}{m_1 m_2}=0$$

-

$$\lambda_1=\frac{2k}{m} \qquad \lambda_2=\frac{4k}{m}$$

- Eigen frequencies:

$$\Rightarrow \quad f_1=\frac{\omega_1}{2\pi} \qquad f_2=\frac{\omega_2}{2\pi}$$

- not normalized clean modes:

$$\phi_1=\begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix} \qquad \phi_2=\begin{bmatrix}0\\0\\0\\1\\0\end{bmatrix}$$

- generalized modal masses $\mu_i=\phi_i^T M\,\phi_i$:
  - $\mu_1=\mu_2=m$
- own standards modes with the unit generalized modal mass ϕ_{Nk} :
  - $\Rightarrow \quad \phi_{N1}=\frac{\phi_1}{\sqrt{\mu_1}} \quad \phi_{N2}=\frac{\phi_2}{\sqrt{\mu_2}}$

- **static modes of supports** ψ_{Sj}

Matrix of the static modes reduced to the ddls of structure $\varphi_S=-\boldsymbol{k}^{-1}\boldsymbol{k}_{xs}$

$$\varphi_S=\frac{-1}{4k}\begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}-k & -k & 0\\0 & -2k & -2k\end{bmatrix}=\begin{bmatrix}1/2 & 1/2 & 0\\0 & 1/2 & 1/2\end{bmatrix}$$

- static solution with a unit displacement of the node $NO1$:

displacements: $\psi_{S1}=\begin{bmatrix}1\\1/2\\0\\0\\0\end{bmatrix}$

- static solution with a unit displacement of the node $NO3$:

displacements: $\psi_{S2}=\begin{bmatrix}0\\1/2\\1\\1/2\\0\end{bmatrix}$

- static solution with a unit displacement of the node $NO5$:

displacements: $\psi_{S3}=\begin{bmatrix}0\\0\\0\\1/2\\1\end{bmatrix}$

- **rigid mode of body** ψ_{R1}

Matrix of the rigid modes reduced to the ddls of structure: $\varphi_R=\varphi_S\,\boldsymbol{S_R}$

Mode of rigid body $\psi_{R1}=\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}$

It is checked well that: $\psi_{R1}=\psi_{S1}+\psi_{S2}+\psi_{S3}$

2.2.2 Loading 1 multi-supports

- factors of modal participation $P_{kj}=\phi_{Nk}^T M\,\psi_j$:
- contribution of the dynamic mode 1 to the movement imposed on the node $NO1$:
  $\Rightarrow \quad P_{11}=\phi_{N1}^T M\,\psi_1=\frac{\sqrt{m}}{2}$
- contribution of the dynamic mode 1 to the movement imposed on the node $NO3$:
  $\Rightarrow \quad P_{12}=\phi_{N1}^T M\,\psi_2=\frac{\sqrt{m}}{2}$
- contribution of the dynamic mode 1 to the movement imposed on the node $NO5$:
  $\Rightarrow \quad P_{13}=\phi_{N1}^T M\,\psi_3=0$
- contribution of the dynamic mode 2 to the movement imposed on the node $NO1$:
  $\Rightarrow \quad P_{21}=\phi_{N2}^T M\,\psi_1=\frac{\sqrt{m}}{2}$
- contribution of the dynamic mode 2 to the movement imposed on the node $NO3$:
  $\Rightarrow \quad P_{22}=\phi_{N2}^T M\,\psi_2=\frac{\sqrt{m}}{2}$
- contribution of the dynamic mode 2 to the movement imposed on the node $NO5$:
  $\Rightarrow \quad P_{23}=\phi_{N2}^T M\,\psi_3=\frac{\sqrt{m}}{2}$

- **answer of the mode i with the movement of the support j**

$$Rm_{kj}=\phi_{Nk} P_{kj}\frac{A_{kj}}{\omega_i^2}$$

Combined answers of the modal oscillators

Response of mode 1 to the movement of support 1: $Rm_{11}=\phi_{N1} P_{11}\frac{A_{11}}{\omega_1^2}=\frac{A_{11}}{2\,\omega_1^2}\begin{pmatrix}1\\0\end{pmatrix}$

Response of mode 1 to the movement of support 2: $Rm_{12}=\phi_{N1} P_{12}\frac{A_{12}}{\omega_1^2}=\frac{A_{12}}{2\,\omega_1^2}\begin{pmatrix}1\\0\end{pmatrix}$

Response of mode 1 to the movement of support 3: $Rm_{13}=\phi_{N1} P_{13}\frac{A_{13}}{\omega_1^2}=\begin{pmatrix}0\\0\end{pmatrix}$

Response of mode 2 to the movement of support 1: $Rm_{21}=\phi_{N2} P_{21}\frac{A_{21}}{\omega_2^2}=\begin{pmatrix}0\\0\end{pmatrix}$

Response of mode 2 to the movement of support 2: $Rm_{22}=\phi_{N2} P_{22}\frac{A_{22}}{\omega_2^2}=\frac{A_{22}}{2\,\omega_2^2}\begin{pmatrix}0\\1\end{pmatrix}$

Response of mode 2 to the movement of support 3: $Rm_{23}=\phi_{N2} P_{23}\frac{A_{23}}{\omega_2^2}=\frac{A_{23}}{2\,\omega_2^2}\begin{pmatrix}0\\1\end{pmatrix}$

- **office plurality intra-group (algebraic sum)** mode 1: $$Rm_{1\text{groupe1}}=Rm_{11}+Rm_{12}=\frac{A_{11}+A_{12}}{2\,\omega_1^2}\begin{pmatrix}1\\0\end{pmatrix}$$

mode 2: $$Rm_{2\text{groupe1}}=Rm_{21}+Rm_{22}=\frac{A_{22}}{2\,\omega_2^2}\begin{pmatrix}0\\1\end{pmatrix}$$

- **contribution of the support j with the movement of training**

$$\text{Re}_j=\psi_j D_j$$

contributions of groups 1 and 2 to the movement of training

$$\text{Re}_{groupe1}=\text{Re}_1+\text{Re}_2=\frac{1}{2}\begin{pmatrix}D_1+D_2\\D_2\end{pmatrix} \qquad \text{Re}_{groupe2}=\text{Re}_3=\frac{1}{2}\begin{pmatrix}0\\D_3\end{pmatrix}$$

- **office plurality on the modes (quadratic)**

$$Rm_{groupe1}=\sqrt{(Rm_{11}+Rm_{12})^2+(Rm_{21}+Rm_{22})^2}=\begin{pmatrix}\dfrac{A_{11}+A_{12}}{2\,\omega_1^2}\\ \dfrac{A_{22}}{2\,\omega_2^2}\end{pmatrix}$$

$$Rm_{groupe2}=\sqrt{Rm_{13}^2+Rm_{23}^2}=\frac{A_{23}}{2\,\omega_2^2}\begin{pmatrix}0\\1\end{pmatrix}$$

- **answer of the groups of supports 1 and 2**

$$R_1=\sqrt{Rm_{groupe1}^2+\text{Re}_{groupe1}^2} \qquad R_1^2=\frac{1}{4}\begin{bmatrix}\dfrac{(A_{11}+A_{12})^2}{\omega_1^2}+(D_1+D_2)^2\\ \dfrac{A_{22}^2}{\omega_2^2}+D_2^2\end{bmatrix}$$

$$R_2=\sqrt{Rm_{groupe2}^2+\text{Re}_{groupe2}^2} \qquad R_2^2=\frac{1}{4}\begin{bmatrix}0\\ \dfrac{A_{23}^2}{\omega_2^2}+D_3^2\end{bmatrix}$$

- **office plurality joint committee (quadratic)**

$$R=\sqrt{R_1^2+R_2^2} \qquad R^2=\frac{1}{4}\begin{bmatrix}\dfrac{(A_{11}+A_{12})^2}{\omega_1^2}+(D_1+D_2)^2\\ \dfrac{A_{22}^2+A_{23}^2}{\omega_2^2}+D_2^2+D_3^2\end{bmatrix}$$

2.3 Uncertainty on the solution

No (analytical solution)

3 Modeling A

3.1 Characteristics of modeling

The system is modelled by:

- 4 discrete elements K_T_D_L,
- 2 discrete elements M_T_D_N.

3.2 Characteristics of the grid

The grid consists of 4 meshs SEG2.

3.3 Parameters of modeling

Answer on the first 2 modes without static correction (combination of modal answers SRSS)

4 Results of modeling A

4.1 Eigen frequencies

MODE	Reference	*Code_Aster*	Relative error (%)
1	2.2507900E+00	2.25079079E+00	3.5E-05
2	3.1830000E+00	3.18309886E+00	0,003

4.2 Total answer on complete modal basis

Modes 1 and 2 are taken into account. The components inertial (primary education) and statics (secondary) of the answer are directly cumulated on the level of the supports.

COMB_MODE=' SRSS'

- answer of the support $j=1$ (node $NO1$): $R_1=\sqrt{Rm_1^2+\mathrm{Re}_1^2}$ with $Rm_1=\sqrt{Rm_{11}^2+Rm_{21}^2}$
- answer of the support $j=2$ (node $NO4$): $R_2=\sqrt{Rm_2^2+\mathrm{Re}_2^2}$ with $Rm_2=\sqrt{Rm_{12}^2+Rm_{22}^2}$
- total answer: $R=\sqrt{R_1^2+R_2^2}$

- absolute displacements: DEPL

NODE	Reference	*Code_Aster*	Relative error (%)
NO1	4.00000E-02	4.00000E-02	0,000
NO2	5.58000E-02	5.58083E-02	0,015
NO3	4.40000E-02	4.40000E-02	0,000
NO4	3.85600E-02	3.85683E-02	0,022
NO5	6.00000E-02	6.00000E-02	0,000

- nodal reactions: REAC_NODA

NODE	Reference	*Code_Aster*	Relative error (%)
NO1	3.68000E+01	3.68044E+01	0,012
NO2	0.00000E+00	2.23756E-14	2.2E-14
NO3	8.64900E+01	8.64906E+01	7.0E-04
NO4	0.00000E+00	3.071843E-14	3.1E-14
NO5	7.71400E+01	7.71367E+01	-0,004

5 Summary of the results

Perfect agreement of the results *Aster* with the analytical values of reference.