SDLD107 - Minimization of an energy functional calculus of standard error in relation of behavior in dynamics of the structures

Summary:

The scope of application of this test relates to the dynamics of the structures. It makes it possible to validate the operator of calculation `CALC_ERC_DYN` [u4.53.41] which makes it possible to obtain the fields solution of a problem of minimization of an energy functional calculus of standard error in relation in behavior (ERC) under a modal formulation.

It is a question of solving the problem of ERC for a system made up of 3 masses and 4 springs, embedded at its ends in free vibration. The springs and the masses are modelled by elements of the type `'DIS_T'`.

The results of checked for the studied case were got semi-analytically. Its resolution, which serves as reference, was carried out using the Matlab software.

1 Problem of reference

1.1 Geometry

The studied system is composed of 3 masses (m) and 4 springs (k). The unit is embedded at its ends.

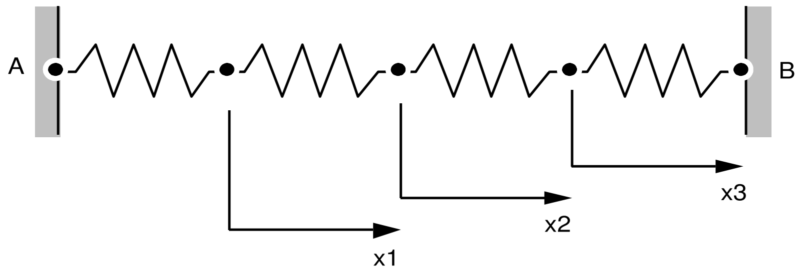

Image 1.1-a: Geometry of the studied system

1.2 Material properties

Stiffness of the springs: $k=1\ N/m$.

Specific masses: $m=1\ kg$.

1.3 Boundary conditions

Points A and B embedded.

1.4 Initial conditions

Structure initially at rest.

1.5 Observation

It is considered that the two translations x1 and x2 are observed perfectly.

It is considered that the first two clean modes are observed:

- For the first clean mode, one supposes the two translations x1 and x2 like his pulsation clean perfectly observed ($\hat{\omega}_1=\omega_1$).
- For the second clean mode, one considers two translations x1 and x2 perfectly observed while information on the own pulsation is supposed to be sullied with an error of 25 % ($\hat{\omega}_2=1,25*\omega_2$) .

1.6 Functional calculus of error

One chooses to minimize the error of the energy functional calculus of standard error in following relation of behavior:

$$e_\omega^2(u,v,w)=\frac{\gamma}{2}(u-v)^T[K](u-v)+\frac{1-\gamma}{2}(u-w)^T\omega^2[M](u-w)+\frac{1-\alpha}{\alpha}(Hu-\hat{u})^T[Gr](Hu-\hat{u})$$

2 Reference solution

2.1 Method of calculating used for the reference solution

2.1.1 Model of reference

For the model of reference, one supposes the vector of following displacement:

$$x=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Thus, the matrices structural of mass (M) and stiffness (K) are:

$$M=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad K=\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

The own pulsations of the system mass-arises are worth:

$$\omega_1^2=(2-\sqrt{2}) \quad \omega_2^2=2 \quad \omega_3^2=(2+\sqrt{2})$$

respective modal deformations (normalized to 1 according to the greatest amplitude):

$$\varphi_1=\frac{1}{2}\begin{pmatrix} \sqrt{2} \\ 2 \\ \sqrt{2} \end{pmatrix} \quad \varphi_2=\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \quad \varphi_3=\frac{1}{2}\begin{pmatrix} -\sqrt{2} \\ 2 \\ -\sqrt{2} \end{pmatrix}$$

The matrix allowing to observe perfectly x1 and x2 are:

$$H=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

In addition the matrix normalizes (gr.) is chosen by combination of the reduced matrices of Guyan of mass and stiffness. For their construction, static modes associated with x1 and x2 are:

$$\psi_1=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \psi_2=\begin{pmatrix} 0 \\ 1 \\ 0.5 \end{pmatrix}$$

forming the base of static modes $\psi=[\psi_1\,\psi_2]$. That led to:

$$G_r=\psi^T[K+M]\psi \;;$$

$$Gr=\begin{pmatrix} 3 & -1 \\ -1 & 2.75 \end{pmatrix}$$

Parameters of weighting of $e_\omega^2(u,v,w)$ are selected $\alpha=0.5$ and $\gamma=0.5$.

2.1.2 Construction of the linear system associated with the problem of the functional calculus of error in relation of behavior.

The problem making it possible to find the fields associated with the functional calculus of type error in relation with behavior brings to the resolution of the linear matric system according to:

$$A\,l=b$$

with, for each own pulsation ω_i :

$$A_i=\begin{pmatrix}\gamma(K+\gamma/(1-\gamma)\omega_i^2 M) & -\gamma(K-\omega_i^2 M)\\ -\gamma(K-\omega_i^2 M) & (-2\alpha/(1-\alpha))H^T G_r H\end{pmatrix} \text{ and } b_i=\begin{pmatrix}0_3\\ (-2\alpha/(1-\alpha))H^T G_r \hat{u}_i\end{pmatrix}$$

where $\hat{u}_i$ represent the observation of the mode φ_i associated with the own pulsation ω_i estimated. α and γ are, as for them, the parameters of weighting associated with the functional calculus with error.
Lastly, the vector solution l is the concatenation of two associated fields has the functional calculus of error so that:

$$l=\begin{pmatrix}u-v\\ u\end{pmatrix}$$

2.2 Results of reference

Two cases of reference are tested:

1) First clean mode, perfect observations:

$$\hat{u}_1=\begin{pmatrix}\frac{\sqrt{2}}{2}\\ 1\end{pmatrix}\ ;\ \hat{\omega}_1=(2-\sqrt{2})$$

In this case, the result is commonplace because it must bring to a result of functional calculus of error perfectly no one associated with:

$$(u-v)=\begin{pmatrix}0\\0\\0\end{pmatrix}\ ;\ u=\frac{1}{2}\begin{pmatrix}\sqrt{2}\\ 2\\ \sqrt{2}\end{pmatrix}$$

2) Second clean mode, perfect displacements, and own pulsation sullied with error:

$$\hat{u}_2=\begin{pmatrix}-1\\ 0\end{pmatrix}\ ;\ \hat{\omega}_2=1.25*2$$

In this case, the construction of the matric problem $A\,l=b$ bring to:

$$A=\begin{pmatrix} 1+1.25^2 & -0.5 & 0 & -1+1.25^2 & 0.5 & 0 \\ -0.5 & 1+1.25^2 & -0.5 & 0.5 & -1+1.25^2 & 0.5 \\ 0 & -0.5 & 1+1.25^2 & 0 & 0.5 & -1+1.25^2 \\ -1+1.25^2 & 0.5 & 0 & -6 & 2 & 0 \\ 0.5 & -1+1.25^2 & 0.5 & 2 & -5.5 & 0 \\ 0 & 0.5 & -1+1.25^2 & 0 & 0 & 0 \end{pmatrix}$$

$$b=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 6 \\ -2 \\ 0 \end{pmatrix}$$

This system of equations was solved using the Matlab software, leading to the solution:

$$l=\begin{pmatrix} 0.223608826207038 \\ 0.107013222975753 \\ -0.095122864867336 \\ -0.957415448053491 \\ 0.038110367724860 \\ 0.494584477951991 \end{pmatrix}$$

In this case, the value of the functional calculus is worth:

$$e^2_{\omega_2}(u,v,w)=0.089643288114668$$

the part associated with the fields with error is:

$$\frac{\gamma}{2}(u-v)^T[K](u-v)+\frac{1-\gamma}{2}(u-w)^T\omega^2[M](u-w)=0.083454681437031$$

2.3 Uncertainty on the solution

First case: analytical solution.

Second case: semi-analytical solution.

3 Modeling A

3.1 Characteristics of the grid

Many nodes: 5 including 2 embedded

Many meshs and types: 4 SEG2

3.2 Sizes tested and results

Calculation of the fields solution and the value of the functional calculus with the operator CALC_ERC_DYN.

Identification	Reference
Formulation: MODAL	
First case	
Value of the functional calculus $e^2_{\omega_1}(u,v,w)$	0.0
Second case	
Value of the functional calculus $e^2_{\omega_2}(u,v,w)$	0.089643288114668
Value of the functional calculus $e^2_{\omega_2}(u,v,w)$	0.083454681437031
Field of displacement u_1 (m)	-0.957415448053491
Field of displacement u_2 (m)	0.038110367724860
Field of displacement u_3 (m)	0.494584477951991
Field of displacement $(u-v)_1$ (m)	0.223608826207038
Field of displacement $(u-v)_2$ (m)	0.107013222975753
Field of displacement $(u-v)_3$ (m)	-0.095122864867336

Table 3.2-1 : Sizes and results tested

4 Summary of the results

The precision on the optimal fields (U) and (UV) as on the value of the functional calculus is very good (errors about 1E-12 with 1E-14)

This test thus validates the operator CALC_ERC_DYN of calculation of research of the acceptable fields associated with a problem of energy functional calculus of standard error in relation of behavior in dynamics under a modal formulation.