SDLD320 - Transitory answer of a free system of 3 masses and 2 springs under harmonic excitation

Summary:

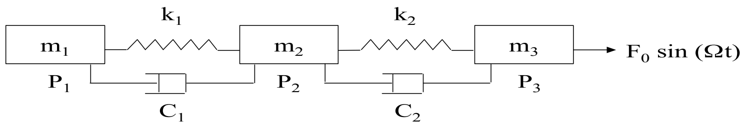
One considers the transitory analysis of a discrete system masses/arises linear with three degrees of completely free freedom. This system has a non-proportional damping. A sinewave excitation is applied at an end of the system.

In this problem, one tests, through a discrete model, the calculation of the transitory answer of a system whose rigid modes are not fixed. One is interested only in the transitory mode. For that, one will seek the solution by an integration on the complete modal basis.

The got results (displacement, speed and acceleration) are compared with an average of results coming from industrial codes and a method of digital integration of type β - Newmark improved.

1 Problem of reference

1.1 Geometry



1.2 Properties of materials

Stiffnesses of connection: $k_1 = 4.10^9 N.m^{-1}$, $k_2 = 5.33\,10^8 N.m^{-1}$

Specific masses: $m_1 = 10^6 kg$, $m_2 = m_3 = 12.10^6 kg$

One-way viscous damping: $C_1 = 1.2566\,10^6 kg.s^{-1}$, $C_2 = 9.0478\,10^6 kg.s^{-1}$

1.3 Boundary conditions and loadings

Completely free system.

Loading at the point P_3 along the axis x : $F(t)0 = F_0 \sin(\Omega t)$ for $t \geq 0$ with $F_0 = 5.10^4 N$ and $\Omega = 19\pi\, rad.s^{-1}$.

1.4 Initial conditions

The system is at rest with $t=0$: $u(0)=0$ and $\frac{du}{dt}(0)=0$.

2 Reference solution

2.1 Method of calculating used for the reference solution

The research of the transitory response of this problem to damping nonproportional, and where the rigid modes are not fixed, can be undertaken by digital integration in real space:

$$[M]\{\ddot{u}_n\}+[C]\{\dot{u}_n\}+[K]\{u_n\}=\{F\} \quad .$$

For that, the answer was calculated with two industrial codes:

- PERMAS: Diagram of integration of Newmark ($\alpha=0{,}25$, $\delta=0{,}5$), $\Delta t=10^{-4s}$,
  Diagram of integration with cubic interpolation of Hermit [bib1], $\Delta t=10^{-4s}$,
- ABAQUS: Diagram of integration of Hilber-Hughes-Taylor [bib2] ($\alpha=-0{,}05$), $\Delta t=10^{-4s}$,

and method of integration of β - Newmark improved [bib3]:

$$\left(\frac{[M]}{\Delta t^2}+\frac{[C]}{2\Delta t}+\frac{[K]}{3}\right)\{u_{n+2}\} = \left(\frac{\{F_{n+2}\}+\{F_{n+1}\}+\{F_n\}}{3}\right)+\left(\frac{2[M]}{\Delta t^2}-\frac{[K]}{3}\right)\{u_{n+1}\}$$

$$+\left(-\frac{[M]}{\Delta t^2}+\frac{[C]}{2\Delta t}-\frac{[K]}{3}\right)\{u_n\}$$

where n , $n+1$, $n+2$ the calculations carried out at times indicate respectively t_n , $t_{n+1}=t_n+\Delta t$ and $t_{n+2}=t_n+2\Delta t$ where Δt is the increment of appointed time.

To start, one takes:

- u_0 et $u_{-1}=u_0-\Delta t\ \dot{u}_0$
- $F_{-1}=2F_0-F_1$

The step of adopted time is $\Delta t=10^{-5s}$.

2.2 Results of reference

Displacement, speed and acceleration of the point P_3 .

Differential of displacement enters the points P_3 and P_1 .

2.3 Uncertainty on the solution

Average of digital solutions.

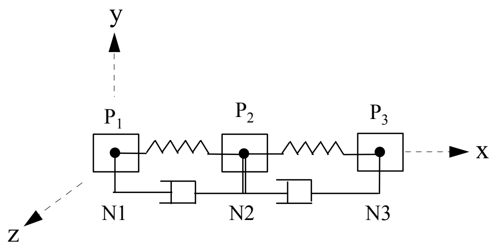
2.4 Bibliographical references

1) J.H. ARGYRIS, PC DUNE and T. ANGELOPOULOS "Non-linear oscillations using the finite technical element" comp. Meth. Appl. Mech. Engng., Vol.2, 1972, pp. 203-254
2) H.M. HILBER, T.J.R. HUGHES and R.L. TAYLOR "Improved numerical dissipation for time integration algorithms in structural dynamics" Earthquake Engineering and Structural Dynamics, Vol.5, 1977, pp. 283-292
3) N.M. Structural NEWMARK "with method of computation for dynamics" Proceeding ASCE J.Eng.Mech. Div E-3, July 1959, pp. 67-94

3 Modeling A

3.1 Characteristics of modeling

Discrete elements of rigidity, damping and mass.



Characteristics of the elements:

DISCRETE :	nodal masses	M_TR_D_N
	linear rigidities	K_TR_D_L
	straight-line depreciations	A_TR_D_L

Pas de boundary conditions, in all the nodes: DX , DY , DZ , DRX , DRY , DRZ free.

Names of the nodes: $P_1 = N1$, $P_2 = N2$, $P_3 = N3$.

Method of calculating:

Integration on the modal basis supplements with Newmark ($\alpha = 0{,}25$, $\delta = 0{,}5$),
Pas de time: $\Delta t = 10^{-4s}$ then modal recombination.

Duration of observation: $5s$.

3.2 Characteristics of the grid

Many nodes: 3

Number of meshs and type: 2 meshs **SEG2**

3.3 Sizes tested and results

- Displacement of the point P_3

Time (s)	Displacement Reference (m)	Displacement Aster (m)	Difference (%)
0.09	6.7395 E-6	6.73326 E-6	-0.093
0.32	1.1019 E-5	1.10002 E-6	-0.171
1.18	3.6683 E-5	3.66122 E-5	-0.193
4.92	1.6615 E-4	1.65849 E-4	-0.181

- Speed of the point P_3

Time (s)	Speed Reference ($m.s^{-1}$)	Speed Aster ($m.s^{-1}$)	Difference (%)
0.05	1.3425 E-4	1.34131 E-4	-0.088
0.32	-6.4111 E-5	-6.41097 E-4	-0.002
1.18	1.6104 E-5	1.60598 E-5	-0.274
3.55	4.4262 E-5	4.41720 E-5	-0.203

- Acceleration of the point P_3

Time (s)	Acceleration Reference ($m.s^{-2}$)	Acceleration Aster ($m.s^{-2}$)	Difference (%)
0.09	-3.5694 E-3	-3.56634 E-3	-0.086
0.18	-4.3924 E-3	-4.38933 E-3	-0.070
0.55	4.3766 E-3	4.37283 E-3	-0.086
1.18	4.2459 E-3	4.24264 E-3	-0.077
4.92	-4.2233 E-3	-4.21962 E-3	-0.087

- Relative displacement of the point P_3 compared to the point P_1

Time (s)	u_3-u_1 Reference (m)	u_3-u_1 Aster (m)	Difference (%)
0.18	8.0987 E-6	8.04800 E-6	-0.626
0.55	-6.2246 E-6	-6.21194 E-6	-0.203
0.82	5.3064 E-6	5.34121 E-6	0.656
1.18	-4.5552 E-6	-4.52071 E-6	-0.757
1.92	-3.0416 E-6	-3.04417 E-6	0.085
3.55	1.8448 E-6	1.82742 E-6	-0.942
4.92	1.4832 E-6	1.47526 E-6	-0.535

3.4 Remarks

Besides the comparison for the values tested, one checks that variable kinematics other than those related to the translation according to x remain worthless.

4 Summary of the results

- To obtain a good precision of the results, it is initially necessary to obtain a precise and perfectly orthogonal modal base (`CALC_MODES`) :
  - by avoiding the multiple modes (different rigidity on the nonexcited degrees of freedom),
  - by calculating the rigid modes of body correctly (to prefer the option `'CENTER'` in `CALC_MODES` with the other options),
  - by specifying the method `'JACOBI'` (in the keyword factor `SOLVEUR_MODAL`) for a complete modal extraction.
- The precision of the results is good as well for displacements as for speeds and accelerations.

For the elastic answer of the system (relative displacements $u_3 - u_1$), the digital precision is a little less good because of the digital office plurality of the errors on the absolute values.