SDLD321 - Transitory dynamic response of a harmonic oscillator with variable damping

Summary:

The system considered is a harmonic oscillator with 1 degree of freedom under harmonic excitation to resonance. Various depreciation will be considered:

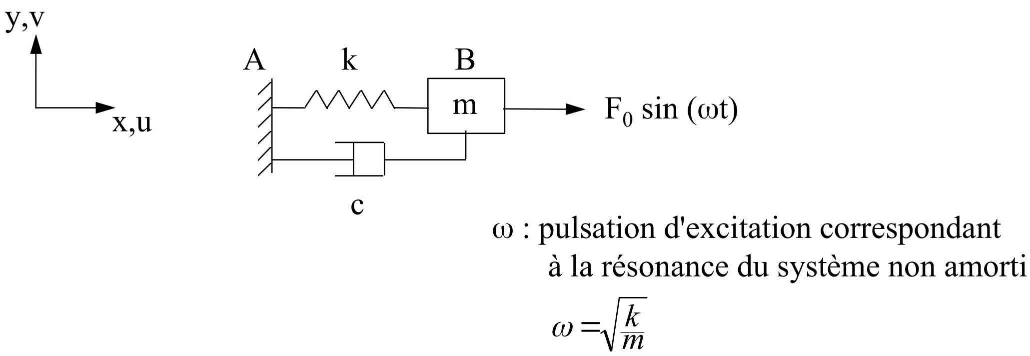
- critical damping,
- average damping,
- very weak damping.

Via this problem, the various algorithms of the order are tested `DYNA_TRAN_MODAL` [U4.54.03] and their capacities to deal with problems with extreme damping. The results are compared with the exact analytical solutions.

1 Problem of reference

1.1 Geometry

The system is composed of a mass, a spring and a shock absorber. He admits a single degree of freedom in translation.



1.2 Material properties

Stiffness of connection: $k=25.10^3\ N.m^{-1}$

Specific mass: $m=10\,kg$

Viscous damping:

$c=c_{critique}$; $c=0{,}01\,c_{critique}$; $c=10^{-5}c_{critique}$
with $c_{critique}=1\,000\,kg.s^{-1}$

1.3 Boundary conditions and loadings

End A embedded.

Harmonic force according to X at the frequency of resonance at the point B :

$F(t)=F_0\sin(\omega t)$ for $t\geq 0$ with $F_0=5\,N$ and $\omega=\sqrt{\frac{k}{m}}=50\,rad.s^{-1}$.

1.4 Initial conditions

The system is at rest with $t=0$: $u(0)=0$ and $\frac{du}{dt}(0)=0$.

2 Reference solution

2.1 Method of calculating used for the reference solution

The simple oscillator checks the following equation:

$$m\ddot{u}+c\dot{u}+ku=F_0\sin(\omega t)$$

with $u(0)=0$ and $\dot{u}(0)=0$

ω : own pulsation of the oscillator $\omega=\sqrt{\frac{k}{m}}$

Critical damping is $c_{critique}=2\mathrm{m}\omega$.

The solution for $c=c_{critique}$ is:

$$u(t)=\frac{F_0}{2\mathrm{k}}\left[e^{-\omega t}(1+\omega t)-\cos(\omega t)\right]$$

The solution for a subcritical damping such as $\frac{c}{c_{critique}}=\xi$ is:

$$u(t)=e^{-\xi\omega t}\left(\frac{F_0}{2\xi k}\cos(\omega_D t)+\frac{F_0\omega}{2k\omega_D}\sin(\omega_D t)\right)-\frac{F_0}{2\xi k}\cos(\omega t)$$

with $\omega_D=\omega\sqrt{1-\xi^2}$

2.2 Results of reference

Displacement and speed of the point B .

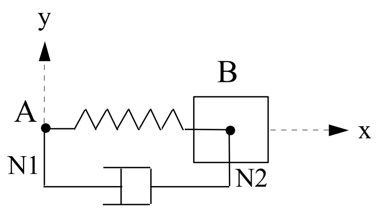
2.3 Uncertainty on the solution

Exact analytical solution.

3 Modeling A

3.1 Characteristics of modeling

Discrete elements of rigidity, damping and mass.



Characteristics of the elements:

DISCRETE :	nodal mass	M_T_D_N
	linear rigidity	K_T_D_L
	straight-line depreciation	A_T_D_L ($c=c_{critique}$)

Boundary conditions: with the node $N1$ DDL_IMPO DX = DY = DZ = 0.

Names of the nodes: $P_1=N1$, $P_2=N2$.

Methods of calculating:

- Integration on the modal basis with Newmark ($\alpha=0,25$, $\delta=0,5$)
  Pas de time $\Delta t=10^{-3} s$
- Integration on the modal basis with Euler
  Pas de time $\Delta t=10^{-3} s$

Duration of observation: $0,5 s$.

3.2 Characteristics of the grid

Many nodes: 2

Number of meshs and type: 1 mesh SEG2

3.3 Sizes tested and results

- Displacement of the point B

Time (s)	Displacement Reference (m)	Displacement NEWMARK **Aster** (m)	Tolerance (%)	Displacement EULER **Aster** (m)	Tolerance (%)
0.06	1.18914 E-4	1.18886 E-4	0.5%	1.18886 E-4	0.5%

0.12	– 9.42819 E-5	– 9.42574 E-5	0.5%	– 9.47822 E-5	0.6%
0.19	9.97958 E-5	9.97765 E-5	0.5%	9.96206 E-5	0.5%
0.25	– 9.97748 E-5	– 9.97526 E-5	0.5%	– 9.99152 E-5	0.5%
0.31	9.78457 E-5	9.78210 E-5	0.5%	9.83436 E-5	0.6%
0.38	– 9.88705 E-5	– 9.88530 E-5	0.5%	– 9.84730 E-5	0.5%
0.44	9.99961 E-5	9.99754 E-5	0.5%	9.99525 E-5	0.5%

- Speed of the point B

Time (s)	**Speed Reference** ($m.s^{-1}$)	**Speed NEWMARK** ***Aster*** ($m.s^{-1}$)	**Tolerance** (%)	**Speed EULER** ***Aster*** ($m.s^{-1}$)	**Tolerance** (%)
0.03	3.31400 E-3	3.31363 E-3	0.5%	3.32568 E-3	0.5%
0.09	– 5.13760 E-3	– 5.13729 E-3	0.5%	– 5.13627 E-3	0.65%
0.16	4.93337 E-3	4.93354 E-3	0.5%	4.93088 E-3	0.5%
0.22	– 5.00087 E-3	– 5.00087 E-3	0.5%	– 5.00133 E-3	0.5%
0.28	4.95298 E-3	4.95284 E-3	0.5%	4.95297 E-3	0.5%
0.35	– 4.87813 E-3	– 4.87836 E-3	0.5%	– 4.87801 E-3	0.5%
0.41	4.98415 E-3	4.98423 E-3	0.5%	4.98409 E-3	0.5%
0.47	– 4.99041 E-3	– 4.99035 E-3	0.5%	– 4.99043 E-3	0.5%

3.4 Remarks

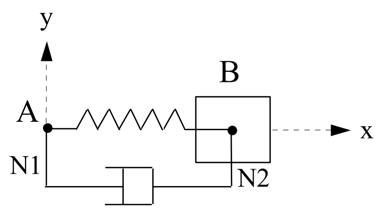
The results are tested on the level as of peaks for the grain of observation selected (10^{-2}s) where the values are most significant.

The mode becomes quasi-permanent after the first period, it is what one must observe by carrying out a transitory analysis.

4 Modeling B

4.1 Characteristics of modeling

Discrete elements of rigidity, damping and mass.



Characteristics of the elements:

DISCRETE :	nodal mass	M_T_D_N
	linear rigidity	K_T_D_L
	straight-line depreciation	A_T_D_L ($c=0{,}01\,c_{critique}$)

Boundary conditions: with the node $N1$ DDL_IMPO DX = DY = DZ = 0.

Names of the nodes: $P_1=N1$, $P_2=N2$.

Methods of calculating:

- Integration on the modal basis with Fu-Devogelaere
  Pas de time $\Delta t=10^{-3}s$
- Integration on the modal basis with Δt adaptive of order 2
  Pas de initial time $\Delta t=10^{-5}s$
  Not maximum $\Delta t=10^{-3}s$

Duration of observation: $5\,s$.

4.2 Characteristics of the grid

Many nodes: 2

Number of meshs and type: 1 mesh SEG2

4.3 Sizes tested and results

- Displacement of the point B

Time (s)	Displacement Reference (m)	Displacement DEVOG Aster (m)	Tolerance (%)	Displacement ADAPT_ORDRE2 Aster (m)	Tolerance (%)
0.06	3.06503 E-4	3.06503 E-4	0.5%	3.06521 E-4	0.5%
0.13	– 5.93807 E-4	– 5.93807 E-4	0.5%	– 5.93729 E-4	0.5%
0.25	– 1.17872 E-3	– 1.17872 E-3	0.5%	– 1.17890 E-3	0.5%
0.69	2.91788 E-3	2.91788 E-3	0.5%	2.91744 E-3	0.5%
1.01	– 3.83901 E-3	– 3.83901 E-3	0.5%	– 3.83567 E-3	0.5%
2.32	6.68206 E-3	6.68206 E-3	0.5%	6.68656 E-3	0.5%
3.64	– 8.19821 E-3	– 8.19821 E-3	0.5%	– 8.204 E-3	0.5%
4.96	9.00847 E-3	9.00847 E-3	0.5%	9.0143 E-3	0.5%

Time (s)	Displacement Reference (m)	Displacement RUNGE_KUTTA_54 Aster (m)	Tolerance (%)	Displacement RUNGE_KUTTA_32 Aster (m)	Tolerance (%)
0.06	3.06503 E-4	3.06420E-04	0.5%	3.06443E-04	0.5%
0.13	– 5.93807 E-4	-5.93619E-04	0.5%	-5.93713E-04	0.5%
0.25	– 1.17872 E-3	-1.178373E-3	0.5%	-1.17845E-3	0.5%
0.69	2.91788 E-3	2.91701E-3	0.5%	2.91706E-3	0.5%
1.01	– 3.83901 E-3	-3.83786E-3	0.5%	-3.83772E-3	0.5%
2.32	6.68206 E-3	6.68009E-3	0.5%	6.67939E-3	0.5%
3.64	– 8.19821 E-3	-8.19578E-3	0.5%	-8.19318E-3	0.5%
4.96	9.00847 E-3	9.00579E-3	0.5%	9.00479E-3	0.5%

- Speed of the point B

Time (s)	Speed Reference ($m.s^{-1}$)	Speed DEVOG Aster ($m.s^{-1}$)	Tolerance (%)	Speed *ADAPT_ORDRE2* Aster ($m.s^{-1}$)	Tolerance (%)
0.04	8.95997 E-3	8.95997 E-3	0.5%	8.9722 E-3	0.5%
0.10	– 2.33271 E-2	– 2.33271 E-2	0.5%	– 2.33499 E-2	0.5%
0.22	– 5.20590 E-2	– 5.20590 E-2	0.5%	– 5.2113 E-2	0.5%
0.66	1.40500 E-1	1.40500 E-1	0.5%	1.40591 E-1	0.5%
1.04	1.99889 E-1	1.99889 E-1	0.5%	1.99933 E-1	0.5%
2.36	– 3.39933 E-1	– 3.39933 E-1	0.5%	– 3.39725 E-1	0.5%
3.68	4.10585 E-1	4.10585 E-1	0.5%	4.10008 E-1	0.5%
5.00	– 4.4531 E-1	– 4.45308 E-1	0.5%	– 4.44429 E-1	0.5%

Time (s)	**Speed Reference** ($m.s^{-1}$)	**Speed** `RUNGE_KUTTA_54` ***Aster*** ($m.s^{-1}$)	**Tolerance** (%)	**Speed** ***`RUNGE_KUTTA_32`*** ***Aster*** ($m.s^{-1}$)	**Tolerance** (%)
0.04	8.95997 E-3	8.89561E-3	0.5%	8,95719E-3	0.5%
0.10	– 2.33271 E-2	-2,33194E-2	0.5%	-2,33211E-2	0.5%
0.22	– 5.20590 E-2	-5,20435E-2	0.5%	-5,20573E-2	0.5%
0.66	1.40500 E-1	1,40458E-1	0.5%	1,40475E-1	0.5%
1.04	1.99889 E-1	1,99829E-1	0.5%	1,99809E-1	0.5%
2.36	– 3.39933 E-1	-3,39832E-1	0.5%	-3,39767E-1	0.5%
3.68	4.10585 E-1	4,10463E-1	0.5%	4,10403E-1	0.5%
5.00	– 4.4531 E-1	-4,45308E-1	0.5%	-4.45145E-1	0.5%

4.4 Remarks

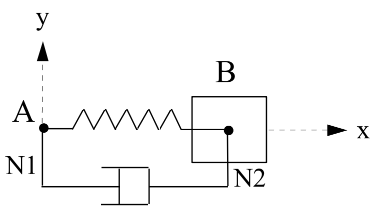
The results are tested on the level as of peaks where the values are most significant.

The duration of selected observation makes it possible to see the effect of damping. However, in this interval, the answer of the point B remain always transitory but one is close to the permanent mode whose scale of displacement is $10^{-2} m$.

5 Modeling C

5.1 Characteristics of modeling

Discrete elements of rigidity, damping and mass.



Characteristics of the elements:

DISCRETE :	nodal mass	M_T_D_N
	linear rigidity	K_T_D_L
	straight-line depreciation	A_T_D_L ($c=10^{-5} c_{critique}$)

Boundary conditions: with the node $N1$ DDL_IMPO DX = DY = DZ = 0.

Names of the nodes: $P_1=N1$, $P_2=N2$.

Methods of calculating:

- Integration on the modal basis with Newmark ($\alpha=0,25$, $\delta=0,5$)
  Pas de time $\Delta t=10^{-3s}$
- Integration on the modal basis with Euler
  Pas de time $\Delta t=10^{-3s}$

Duration of observation: $5 s$.

5.2 Characteristics of the grid

Many nodes: 2

Number of meshs and type: 1 mesh SEG2

5.3 Sizes tested and results

- Displacement of the point B

Time (s)	Displacement Reference (m)	Displacement NEWMARK **Aster** (m)	Tolerance (%)	Displacement EULER **Aster** (m)	Tolerance (%)
0.06	3.11105 E-4	3.10936 E-4	0.5%	3.11181 E-4	0.5%
0.13	– 6.13250 E-4	– 6.13016 E-4	0.5%	– 6.13380 E-4	0.5%

0.25	– 1.25380 E-3	– 1.25304 E-3	0.5%	– 1.25418 E-3	0.5%
0.69	3.44945 E-3	3.44691 E-3	0.5%	3.45069 E-3	0.5%
1.01	– 4.88729 E-3	– 4.89081 E-3	0.5%	– 4.88547 E-3	0.5%
2.32	1.12876 E-2	1.12475 E-2	0.5%	1.13069 E-2	0.5%
3.64	– 1.77960 E-2	– 1.77100 E-2	0.5%	– 1.78360 E-2	0.5%
4.96	2.43613 E-2	2.42198 E-2	0.5%	2.44242 E-2	0.5%

- Speed of the point B

Time (s)	**Speed Reference** ($m.s^{-1}$)	**Speed NEWMARK** ***Aster*** ($m.s^{-1}$)	**Tolerance** (%)	**Speed EULER** ***Aster*** ($m.s^{-1}$)	**Tolerance** (%)
0.04	9.09284 E-3	9.08897 E-3	0.5%	9.08230 E-3	0.5%
0.10	– 2.39724 E-2	– 2.39637 E-2	0.5%	– 2.40269 E-2	0.5%
0.22	– 5.49964 E-2	– 5.49680 E-2	0.5%	– 5.48752 E-2	0.5%
0.66	1.64958 E-1	1.64879 E-1	0.5%	1.64882 E-1	0.5%
1.04	2.56456 E-1	2.56547 E-1	0.5%	2.57280 E-1	0.5%
2.36	– 5.79010 E-1	– 5.80019 E-1	0.5%	– 5.81033 E-1	0.5%
3.68	8.97631 E-1	9.00729 E-1	0.5%	9.00668 E-1	0.5%
5.00	– 1.21164	– 1.21829	0.5%	– 1.21531	0.5%

5.4 Remarks

The results are tested on the level as of peaks where the values are most significant.

In the interval of observation, one remains very below permanent mode in resonance whose scale of displacement is $10\,m$.

6 Summary of the results

For modeling A, the results got as well in displacement as of speed have an absolute error largely lower than 1% compared to the analytical solution.

The diagram of integration of Newmark is shown more precise than the diagram of Euler.

With 1% critical damping (modeling B), the diagram of Fu-Devogelaere integration is of a frightening precision (not error compared to the reference solution).

The diagram with step of adaptive time of order 2 also gives results to very small percentage of error.

For very weak depreciation (modeling C), one will note a better precision for the diagram of integration of the type Euler than for a diagram of the Newmark type. For this last, the error increases according to time but remains lower all the same than 1%.