

SDLD325 - Transitory dynamic response of a system mass-arises deadened with 2 degrees of freedom

Summary:

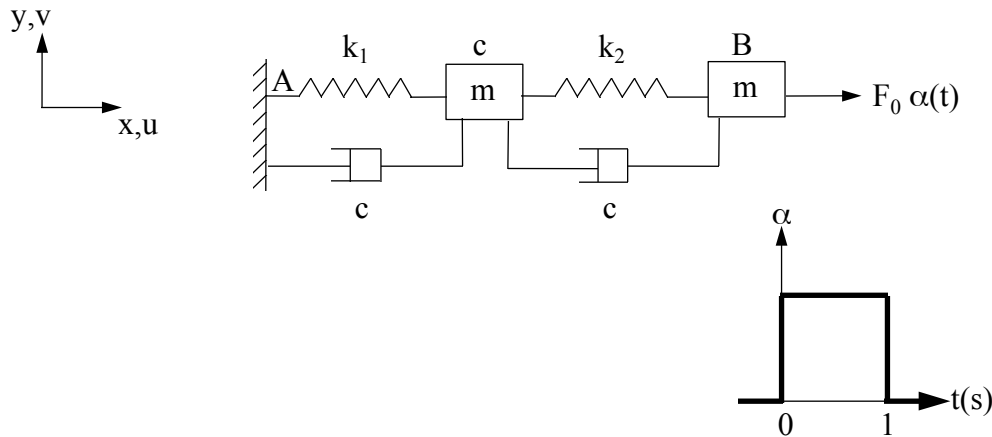
This problem consists in analyzing the dynamic response of a system made up of a set of masses - spring-shock absorbers to 2 degrees of freedom from which the stiffnesses of the springs are very different under excitation of type crenel into 1 degree of freedom.

Via this problem, one tests the sensitivity of diagrams of integration on physical space or modal space screw - to - screw of the report of rigidities.

The results in displacement and speed are compared with an average of results coming from industrial codes and a method of digital integration of improved Newmark type.

1 Problem of reference

1.1 Geometry



1.2 Material properties

Stiffnesses of connection: $k = 28.10^3 \text{ N.m}^{-1}$

2 cases:

- $k_1 = k/10$, $k_2 = 10k$
- $k_1 = 10k$, $k_2 = k/10$

Specific mass: $m = 10 \text{ kg}$

One-way viscous damping: $c = 50 \text{ kg.s}^{-1}$

1.3 Boundary conditions and loadings

End A embedded.

Force applied at the end B : $F(t) = F_0 \alpha(t)$ with
$$\begin{cases} \alpha(t) = 1 \text{ si } 0 \leq t \leq 1 \text{ s} \\ \alpha(t) = 0 \text{ sinon} \end{cases}$$

and $F_0 = 5 \text{ N}$.

1.4 Initial conditions

The system is at rest with $t=0$: $u(0) = 0$ and $\frac{du}{dt}(0) = 0$

2 Reference solution

2.1 Method of calculating used for the reference solution

The research of the transitory response of this problem to damping nonproportional can be undertaken by digital integration in real space:

$$[M]\{\ddot{u}_n\} + [C]\{\dot{u}_n\} + [K]\{u_n\} = \{F\}$$

For that, the answer was calculated with two industrial codes:

- PERMAS: Diagram of integration of Newmark ($\alpha=0,25$ and $\delta=0,5$) $\Delta t=10^{-4s}$;
- ABAQUS: Diagram of integration of Hilbert-Hugues-Taylor [bib1] ($\alpha=-0,05$) $\Delta t=10^{-4s}$;

and method of integration of β - Newmark improved [bib2]:

$$\left[\frac{[M]}{\Delta t^2} + \frac{[C]}{2\Delta t} + \frac{[K]}{3} \right] \{u_{n+2}\} = \left[\frac{\{F_{n+2}\} + \{F_{n+1}\} + \{F_n\}}{3} \right] + \left[\frac{2[M]}{\Delta t^2} - \frac{[K]}{3} \right] \{u_{n+1}\} + \left[\frac{[M]}{\Delta t^2} + \frac{[C]}{2\Delta t} - \frac{[K]}{3} \right] \{u_n\}$$

where n , $n+1$, $n+2$ the calculations carried out at times indicate respectively t_n , $t_{n+1}=t_n+\Delta t$ and $t_{n+2}=t_n+2\Delta t$ where Δt is the increment of appointed time.

To start, one takes:

- u_0 and $u_{-1}=u_0-\Delta t \dot{u}_0$
- $F_{-1}=2F_0-F_1$

The step of adopted time is $\Delta t=10^{-5s}$.

2.2 Results of reference

Displacement and speed of the point end B .

2.3 Uncertainty on the solution

Average of digital solutions.

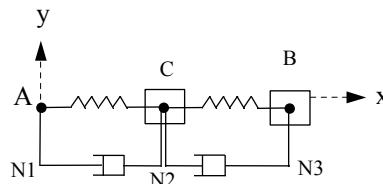
2.4 Bibliographical references

- 1) H.M. HILBERT, T.J.R HUGUES and R.L. TAYLOR "Improved numerical dissipation for time integration algorithms in structural dynamics" Earthquake Engineering and Structural Dynamics, Vol.5, 1977, pp. 283-292
- 2) N.M. Structural NEWMARK "with method of computation for dynamics" Proceeding ASCE J.Eng.Mech. DIV E-3, July 1959, pp. 67-94

3 Modeling A

3.1 Characteristics of modeling

Discrete elements of rigidity, damping and mass.



Characteristics of the elements:

DISCRETE:	nodal mass	M_T_D_N
	linear rigidity	K_T_D_L ($k_{N1N2} = k/10$, $k_{N2N3} = 10k$)
	straight-line	A_T_D_L
	depreciation	

Boundary conditions: with the node *NI* **DDL_IMPO** $DX = DY = DZ = 0$.

Names of the nodes: $A = N1$, $C = N2$, $B = N3$.

Methods of calculating:

- Integration on physical space with Newmark ($\alpha = 0,25$, $\delta = 0,5$)
Pas de time $\Delta t = 10^{-3} s$
- Integration on the modal basis supplements with Euler
Pas de time $\Delta t = 10^{-3} s$ then modal recombination
- Integration on the modal basis supplements with Δt adaptive of order 2
Pas de initial time $\Delta t = 10^{-3} s$ then modal recombination
- Integration on the modal basis supplements with Δt adaptive by the method of the Runge-Kutta type of order (32). The tolerance of relative error is of 10^{-5} .
- Integration on the modal basis supplements with Δt adaptive by the method of the Runge-Kutta type of order (54). The tolerance of relative error is of 10^{-6} .

Duration of observation: 3 S.

3.2 Characteristics of the grid

Many nodes: 3

Number of meshes and type: 2 meshes SEG2

3.3 Sizes tested and results

- Displacement (m) point *B*

Time (s)	Reference
0.27	3.0927 E-3
0.53	8.7953 E-4
0.80	2.4669 E-3
1.25	-1.0980 E-3
1.51	7.8754 E-4
1.78	-5.6508 E-4
2.05	4.0502 E-4
2.31	-2.9012 E-4
2.58	2.0831 E-4
2.85	-1.4943 E-4

- Speed ($m.s^{-1}$) point *B*

Time (s)	Reference
0.11	1.8347 E-2
0.39	-1.3140 E-2
0.66	9.3509 E-3
0.93	-6.7080 E-3
1.11	-1.5863 E-2
1.37	1.1157 E-2
1.64	-7.9838 E-3
1.90	5.7108 E-3
2.17	-4.0998 E-3
2.44	2.9405 E-3
2.71	-2.1073 E-3
2.97	1.5105 E-3

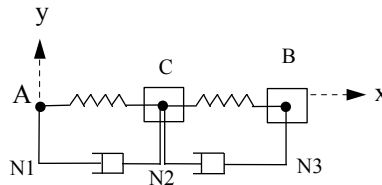
3.4 Remarks

The results are tested on the level of the respective peaks of displacement and speed where the values are most significant.

4 Modeling B

4.1 Characteristics of modeling

Discrete elements of rigidity, damping and mass.



Characteristics of the elements:

DISCRETE	nodal mass	M_T_D_N
:		
	linear rigidity	K_T_D_L ($k_{N1N2}=10k$, $k_{N2N3}=k/10$)
	straight-line	A_T_D_L
	depreciation	

Boundary conditions: with the node $N1$ DDL_IMPO $DX = DY = DZ = 0$.

Names of the nodes: $A = N1$, $C = N2$, $B = N3$.

Methods of calculating:

- Integration on physical space with Newmark ($\alpha=0,25$, $\delta=0,5$)
Pas de time $\Delta t = 10^{-3} s$
- Integration on the modal basis supplements with Euler
Pas de time $\Delta t = 10^{-3} s$ then modal recombination
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- Integration on the modal basis supplements with Δt adaptive by the method of the Runge-Kutta type of order (54). The tolerance of relative error is of 10^{-6} .

Duration of observation: $2,5 s$.

4.2 Characteristics of the grid

Many nodes: 3

Number of meshes and type: 2 meshes SEG2

4.3 Sizes tested and results

- Displacement (m) point B

Time (s)	Reference
0.19	2.9334 E-3
0.38	1.0959 E-3
0.57	2.2468 E-3
0.76	1.5260 E-3
0.95	1.9773 E-3
1.19	-1.2107 E-3
1.38	7.5880 E-4
1.57	-4.7553 E-4
1.76	2.9796 E-4
1.95	-1.8668 E-4
2.14	1.1694 E-4
2.33	-7.3246 E-5

- Speed ($m.s^{-1}$) point B

Time (s)	Reference
0.09	2.4261 E-2
0.28	-1.5210 E-2
0.47	9.5332 E-3
0.66	-5.9745 E-3
0.85	3.7438 E-3
1.08	-2.6037 E-2
1.27	1.6302 E-2
1.46	-1.0204 E-2
1.66	6.3887 E-3
1.85	-4.0059 E-3
2.04	2.5114 E-3
2.23	-1.5743 E-3
2.42	9.8676 E-4

4.4 Remarks

The results are tested on the level of the respective peaks of displacement and speed where the values are most significant.

5 Summary of the results

For two modelings, the results are precise with an error lower than 1%.

Integration on modal basis with a diagram with adaptive step of order 2 gives the best results for a restricted computing time.