SDLL09 - Vibration of a slim beam of variable rectangular section (embed-free)

Summary:

This problem plan consists in seeking the frequencies of vibration of a free fixed beam with rectangular variable section. This test comprises only one modeling.

The variation of section of the beam is either homothetic, or nonhomothetic. The characteristics of the section of the beam are given according to the meshs in two different ways:

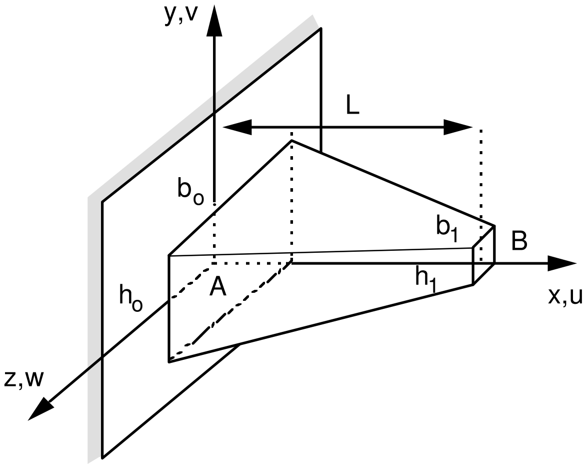
- section and inertias,
- height and width.

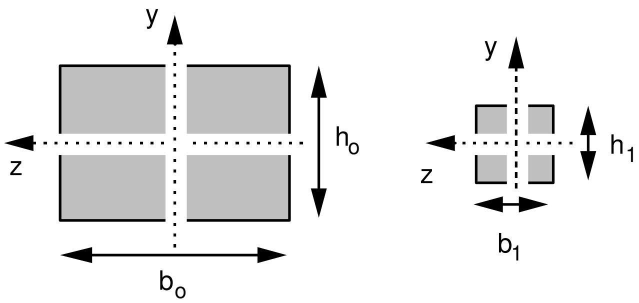
This problem thus makes it possible to test the element of beam with variable section for a prismatic structure as well as calculation of the frequencies of vibration by iterations opposite. In addition, in the operator AFFE_CARA_ELEM, the remanence of certain keywords is tested.

The got results are in concord with those given in guide VPCS.

1 Problem of reference

1.1 Geometry





Sections rectangulaires

Length of the beam:

$$L=1\,m$$

Rectangular section:

	Initial cross-section Case 1	Case 2	Final cross-section
height:	$h_o=0.04\,m$	$=0.04\,m$	$h_1=0.01\,m$
width:	$b_o=0.04\,m$	$=0.05\,m$	$b_1=0.01\,m$
surface:	$A_o=1.6\,10^{-3}\,m^2$	$=2.10^{-3}\,m^2$	$A_1=1.10^{-4}\,m^2$
inertia:	$lz_o=2.1333\,10^{-7}\,m^4$	$=2.6667\,10^{-7}\,m^4$	$Iz_1=8.3333\,10^{-10}\,m^4$

Coordinates of the points (m):

	A	B
x	0.	1.
y	0.	0.
z	0.	0.

1.2 Material properties

$$E=2.10^{11}\,Pa$$

$$\rho=7\,800\,kg/m^3$$

1.3 Boundary conditions and loadings

Not A : embedded $u=v=0$ $\theta=0.$

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is that given in card SDLL09/89 of the guide VPCS which presents the method of calculating in the following way:

Exact calculation by digital integration of the differential equation of the inflection of the beams (Theory of Euler-Bernouilli).

$$\frac{\partial^2\left(EI_z\frac{\partial^2 v}{\partial x^2}\right)}{\partial x^2}=-\rho A\frac{\partial^2 v}{\partial t^2}$$

where I_z and A vary with the X-coordinate.

One obtains:

$$f_i=\frac{1}{2\pi}\lambda_i(\alpha,\beta)\frac{h_1}{L^2}\sqrt{\frac{E}{12\rho}}$$

with:

$$\alpha=\frac{h_0}{h_1}=4$$
$$\beta=\frac{b_0}{b_1}=4\ ou\ 5$$

	λ_1	λ_2	λ_3	λ_4	λ_5
$\beta=4$	23,289	73.9	165.23	299.7	478.1
$\beta=5$	24,308	75.56	167.21	301.9	480.4

2.2 Results of reference

the first 5 clean modes of inflection.

2.3 Uncertainty on the solution

Semi-analytical solution.

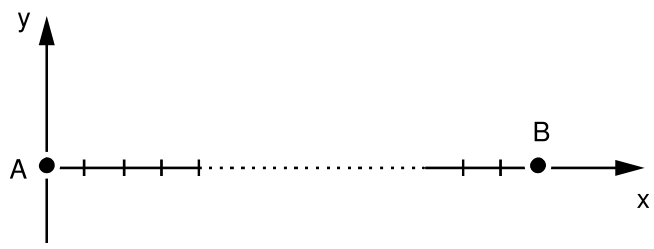
2.4 Bibliographical references

H.H. MABIE, C.B. ROGERS, Transverse vibrations of double-tapered cantilever beams - Newspaper of the Acoustical Society of America, n° 51, p. 1771-1774 (1972).

3 Modeling A

3.1 Characteristics of modeling

Modeling : Elements of beam `POU_D_E`



Cutting: beam AB : 30 meshs SEG2 of section variable
15 meshs in "General section"
15 meshs in "Rectangular section"

Limiting conditions:
in all the nodes
at the end A

`DDL_IMPO: (ALL: 'YES' DZ: 0. , DRX: 0. , DRY MARTINI: 0. )`
`(Node: With DX: 0. , DY: 0. , DRZ: 0. )`

Names of the nodes: Not $A=N100$
Not $B=N200$

3.2 Characteristics of the grid

Grid: Many nodes: 31
Many meshs and types: 30 `SEG2`

3.3 Sizes tested and results

Identification	**Reference**
	Frequency in HZ
Case 1 $h_0/h_1=4$ $b_0/b_1=4$	
homothetic	
inflection 1	54.18
inflection 2	171.94
inflection 3	384.40
inflection 4	697.24
inflection 5	1112.28
Case 2 $h_0/h_1=4$ $b_0/b_1=5$	
nonhomothetic	
inflection 1	56.55
inflection 2	175.19
inflection 3	389.01
inflection 4	702.36
inflection 5	1117.63

4 Summary of the results

Good establishment of the element of non-prismatic beam with a fine grid.

A coarser modeling would be sufficient.