SDLL10 - Rectangular beam of section variable (embed-embedded)

Summary:

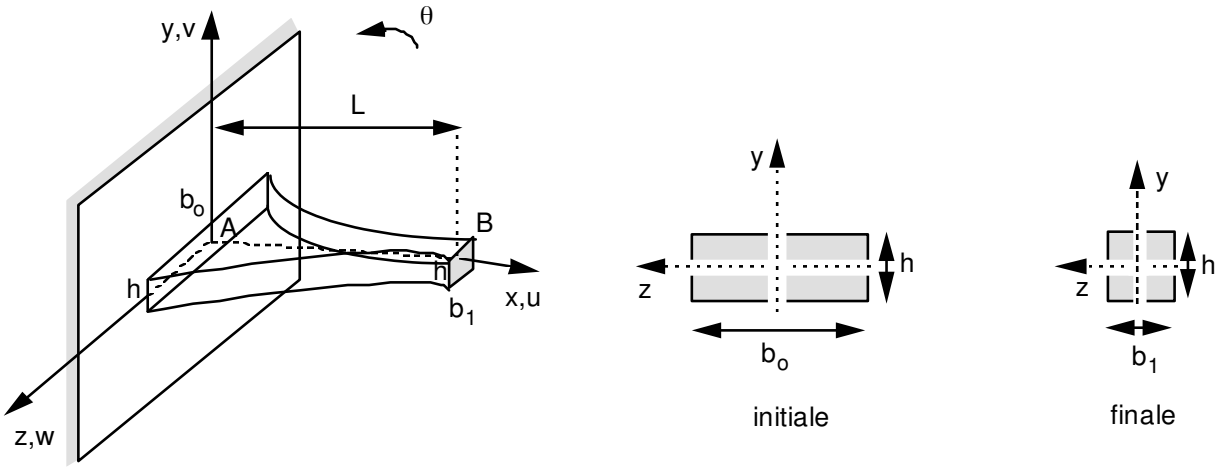
This problem plan consists in seeking the frequencies and the modes of vibration of a mechanical structure made up of an embed-embedded beam whose surface of the cross-section varies exponentially. This test of Mechanics of the Structures corresponds to a dynamic analysis of a linear model having a linear behavior. It understands only one modeling.

Via this problem, one tests the element of beam in inflection of variable Timoshenko of section as well as the calculation of the frequencies and the modes of vibration by the method of Lanczos. One tests also the functionality “normalizes to 1. “at the point of maximum amplitude in translation” of the modes of vibration.

By using a fine space discretization, the got results are in concord with the analytical results given in guide VPCS.

1 Problem of reference

1.1 Geometry



Length of the beam: $L=0.6\,m$
Constant thickness: $h=0.01\,m$

Rectangular section:

Initial cross-section:
width: $b_0=0.03\,m$
surface: $A_0=3.10^{-4}m^2$
moment of inertia: $Iz_0=0.25\,10^{-8}m^4$

Variation of the section:
$b=b_0e^{-2\alpha x}$ with $\alpha=1.$
$A=A_0e^{-2\alpha x}$
$Iz=Iz_0e^{-2\alpha x}$

Coordinates of the points (m):

	A	B
x	0.	0.6
y	0.	0.

1.2 Material properties

$E=2.10^{11}\,Pa$
$\nu=0.3$
$\rho=7\,800.\,kg/m^3$

1.3 Boundary conditions and loadings

Points A and B : embedded, $u=v=0.$, $\theta=0$.

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is that given in card SDLL10/89 of the guide VPCS which presents the method of calculating in the following way:

The pulsation ω_i is given by the roots of the equation:

$$1 - \cos(rL)\,\mathrm{ch}(sL) + \frac{s^2 - r^2}{2rs}\,\mathrm{sh}(sL)\sin(rL) = 0$$

with:

$$\lambda_i^4 = \frac{\rho\, A_0\, \omega_i^2}{E\, I_{z0}} \quad ; \quad r = \sqrt{\alpha^2 + \lambda_i^2} \quad ; \quad s = \sqrt{\lambda_i^2 - \alpha^2} \text{ si } \left(\lambda_i^2 - \alpha^2\right) > 0$$

Components of translation ν mode $F_i(x)$ are then:

$$\Phi_i(x) = e^{\alpha x}\left[\cos(rx) - ch(rx) + \frac{\cos(rL) - ch(sL)}{r\,sh(sL) - s\sin(rL)}\left(s\sin(rx) - r\,sh(sx)\right)\right]$$

2.2 Results of reference

the first 4 Eigen frequencies and own standards modes to 1 for the largest component in translation.

2.3 Uncertainty on the solution

Analytical solution.

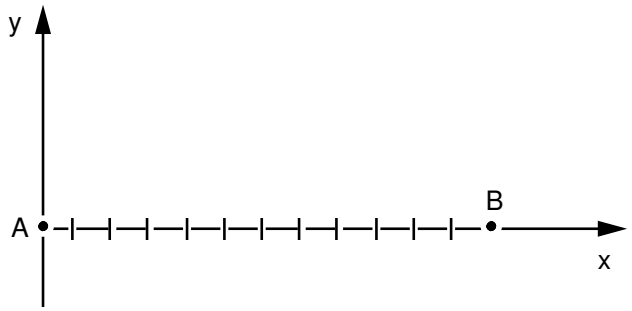
2.4 Bibliographical references

- Working group Analyzes Dynamic. Committee of Validation of the Software packages of Structural analysis. French company of the Mechanics (1988).

3 Modeling A

3.1 Characteristics of modeling

Element of beam POU_D_T (Right Beam of Timoshenko)



Cutting: beam AB : 120 meshs SEG2 of variable section.

Limiting conditions:

in all the nodes	DDL_IMPO	(ALL: 'YES', DZ: 0. , DRX: 0. , DRY MARTINI: 0.)
with the nodes ends		(NODE: (AB) DX: 0. , DY: 0. , DRZ: 0.)

Names of the nodes:

Not A	$x=0.$	$N1$
	$x=0.1$	$N21$
	$x=0.2$	$N41$
	$x=0.3$	$N61$
	$x=0.4$	$N81$
	$x=0.5$	$N101$
Not B	$x=0.6$	$N121$

3.2 Characteristics of the grid

Many nodes:	121
Many meshs and types:	120 SEG2

3.3 Sizes tested and results

Order of the clean mode	Frequency Reference	% tolerance
1	143,303	1.6
2	396,821	0.45
3	779,425	0.3
4	1289.577	0.9

Clean modes of *Aster* were normalized to 1. at the point of maximum amplitude in translation as in the reference.

Clean mode $F_i(x)$ normalized to 1 at the point of maximum amplitude

	i	$x=0.1$	$x=0.2$	$x=0.3$	$x=0.4$	$x=0.5$
Reference		0.2349	0.6962	0.98960	0.8505	0.3507
Aster	1	0.2363	0.6970	0.9895	0.8516	0.3529
% difference		0,583	0,119	0.	0,132	0,631
% tolerance		0.6	0.15	0.1	0.15	0.7
Reference		– 0.4653	– 0.7558	0.	0.9232	0.6941
Aster	2	– 0.4670	– 0.7555	$-2{,}910^{-4}$	0.9226	0.6971
% difference		0.37	– 0,041	$-2{,}910^{-4}$	– 0,063	0,435
% tolerance		0.4	0.1	1.10^{-3}	0.1	0.45
Reference		0.6278	0.1969	– 0.7783	0.2406	0.9366
Aster	3	0.6290	0.1952	– 0.7782	0.2377	0.9387
% difference		0,192	– 0.89	– 0,014	– 1,226	0,228
% tolerance		0.2	0.9	0.1	1.23	0.25
Reference		– 0,666	0.4832	0.	– 0.5901	0.9937
Aster	4	– 0.6656	0.4840	$4{,}610^{-4}$	– 0.5919	0.9928
% difference		– 0,081	0.18	$4{,}610^{-4}$	0.31	– 0,089
% tolerance		0.1	0.2	1.10^{-3}	0.35	0.1

3.4 Remarks

Calculations carried out by:

```
CALC_MODES
              OPTION = 'PLUS_PETITE'
              CALC_FREQ=_F (NMAX_FREQ = 4)
              SOLVEUR_MODAL=_F (METHOD = 'TRI_DIAG')
```

Contents of the file results:

the first 4 Eigen frequencies, clean vectors.

4 Summary of the results

Modeling suitable (frequencies and modes suitable for less 2%) with a fine grid.

A calculation carried out on a coarse grid (12 meshs) shows more important variations with the reference solution. This is especially due to the way in which the modes are normalized.