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## SDLL11 - Free-free thin circular ring

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### Summary:

This CAS-test makes it possible to test the frequencies and the modes of vibration of a circular ring into free-free.

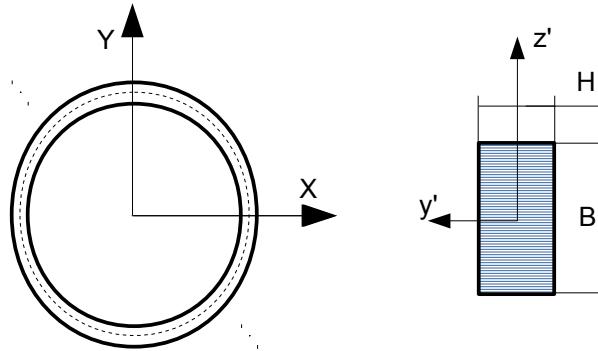
Eight modelings are carried out, for each one among it one specifies the modeling and the type of mesh tested.

Method of research of the Eigen frequencies used is the method of SORENSEN for modelings  $A, B, C, D, E, F, G$ , and method of LANCZOS for modeling  $I$ .

- Modelings 3D
  - Modeling  $A$  : mesh HEXA20
  - Modeling  $B$  : mesh HEXA8
  - Modeling  $C$  : mesh PENTA15
  - Modeling  $D$  : mesh TETRA10
- Modeling POU\_D\_T
  - Modeling  $E$  : mesh SEG2
- Modelings 2D
  - Modeling  $F$  : mesh QUAD8
  - Modeling  $G$  : mesh QUAD8
  - Modeling  $I$  : mesh QUAD8

## 1 Problem of reference

### 1.1 Geometry



Dimension in (m)

- Average radius of curvature  $R=0.1\text{m}$
- Thickness:  $h=0.005\text{m}$
- Width:  $b=0.010\text{m}$  (perpendicular plan)
- Surface:  $A=5\times 10^{-5}\text{m}^2$
- Moment of inertia:  $I_z=1.042\times 10^{-10}\text{m}^4$   $I_y=4.167\times 10^{-10}\text{m}^4$
- Torque:  $J=2.859\times 10^{-10}\text{m}^4$

### 1.2 Properties of material

- Rubber band
  - $E=7.2\times 10^{10}\text{Pa}$  Young modulus
  - $\nu=0.3$  Poisson's ratio
  - $\rho=2700\text{kg/m}^3$  Density

### 1.3 Boundary conditions and loadings

- Displacements: all the points of the ring are free
- Loading: none

## 2 Reference solution

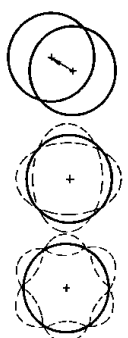
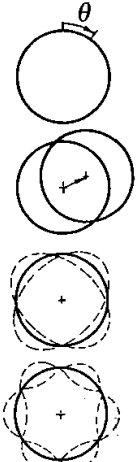
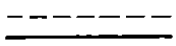
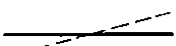
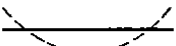
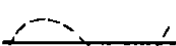
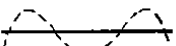
### 2.1 Method of calculating used for the reference solution

- Modes of vibration in the plan  
For these modes of vibration, the equation of inflection of the curved beams of V. Boussinesq (1883), without extension of neutral fibre leads to:

$$f_i = \frac{1}{2\pi} \sqrt{\left( \frac{i^2(i^2-1)^2}{i^2+1} \times \frac{EI_z}{\rho AR^4} \right)} \quad i=0,1,2,\dots$$

The reference solution is established for thin arcs such as  $\alpha R \geq 100 \sqrt{\left( \frac{I_z}{A} \right)}$  with  $\alpha$ , angle in the center in radians.

- Modes of vibration except plan  
For the transverse modes of vibrations with rectangular section, the solution was established starting from the results of two computer codes, using different formulations.

| Clean modes in the plan<br>(polar coordinates $(i, \theta)$ )   |  | Clean modes except plan   |
|---|--|---|
| <p>Symmetrical</p> $u'_i = i \cos(i\theta)$ $v'_i = \sin(i\theta)$ $\theta'_i = -\frac{1-i^2}{R} \sin(i\theta)$ | <p>Antisymmetric</p> $u'_i = i \sin(i\theta)$ $v'_i = -\cos(i\theta)$ $\theta'_i = -\frac{1-i^2}{R} \cos(i\theta)$ |   |
|                              |                                 | <p><math>i=0</math></p>  <p><math>i=1</math></p>  <p><math>i=2</math></p>  <p><math>i=3</math></p>  <p><math>i=4</math></p>  |

### 2.2 Reference variable

- FREQ* : frequency

## 2.3 Size and result of reference

|                   | Component   | Nature of the clean mode |       | Reference<br>(Hz) |
|-------------------|-------------|--------------------------|-------|-------------------|
|                   |             | <i>i</i>                 | order |                   |
| Modes in the plan | <i>FREQ</i> | 2                        | 4,5   | 318.36            |
|                   |             | 3                        | 6,7   | 900.46            |
|                   |             | 4                        | 8,9   | 1726.55           |
|                   |             | 5                        | 10,11 | 2792.21           |
| Transverse modes  | <i>FREQ</i> | 2                        | 4,5   | 511.              |
|                   |             | 3                        | 6,7   | 1590.             |
|                   |             | 4                        | 8,9   | 3184.             |

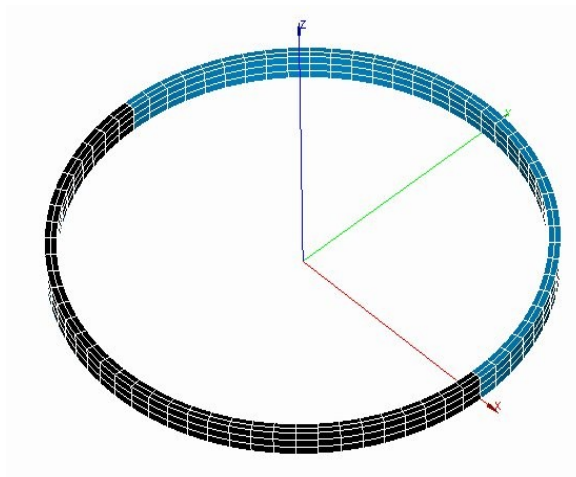
## 2.4 Bibliographical references

- [1] Guide of Validation of the Software packages of Calculations of the Structures: SFM, technical AFNOR, ISBN: 2-12-486611-7

## 3 Modeling A

### 3.1 Characteristics of modeling A

- Modeling 3D



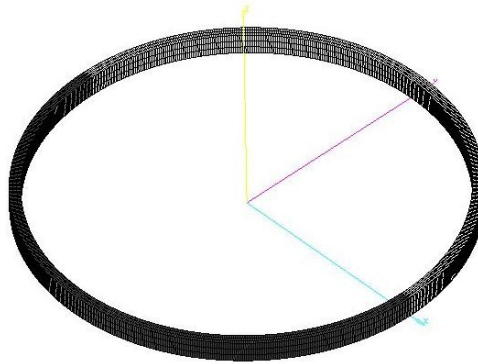
Many nodes                    2952  
Many meshes                    432                    HEXA20

### 3.2 Sizes tested and results

|                   | Component   | Nature of the clean mode |       | Reference (Hz) | Tolerance (%) |
|-------------------|-------------|--------------------------|-------|----------------|---------------|
|                   |             | <i>i</i>                 | order |                |               |
| Modes in the plan | <i>FREQ</i> | 2                        | 4,5   | 318.36         | 0.1           |
|                   |             | 3                        | 6,7   | 900.46         | 0.3           |
|                   |             | 4                        | 8,9   | 1726.55        | 0.5           |
|                   |             | 5                        | 10,11 | 2792.21        | 0.8           |
| Transverse modes  | <i>FREQ</i> | 2                        | 4,5   | 511.           | 0.7           |
|                   |             | 3                        | 6,7   | 1590.          | 1.4           |
|                   |             | 4                        | 8,9   | 3184.          | 2.3           |

## 4 Modeling B

### 4.1 Characteristics of modeling B



Modeling 3D:

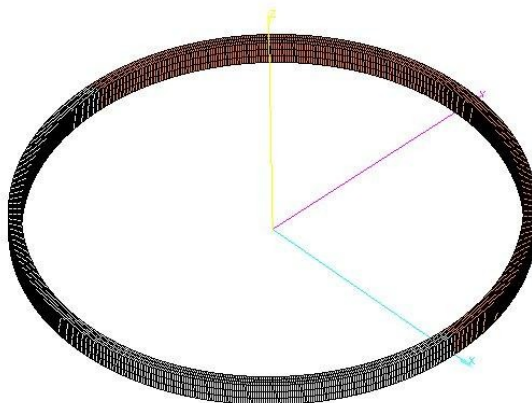
Many nodes 12800  
Many meshes 7200 HEXA8

### 4.2 Sizes tested and results

|                   | Component   | Nature of the clean mode |       | Reference (Hz) | Tolerance (%) |
|-------------------|-------------|--------------------------|-------|----------------|---------------|
|                   |             | <i>i</i>                 | order |                |               |
| Modes in the plan | <i>FREQ</i> | 2                        | 4,5   | 318.36         | 1.8           |
|                   |             | 3                        | 6,7   | 900.46         | 1.6           |
|                   |             | 4                        | 8,9   | 1726.55        | 1.35          |
|                   |             | 5                        | 10,11 | 2792.21        | 1.            |
| Transverse modes  | <i>FREQ</i> | 2                        | 4,5   | 511.           | 1.7           |
|                   |             | 3                        | 6,7   | 1590.          | 0.4           |
|                   |             | 4                        | 8,9   | 3184.          | 0.8           |

## 5 Modeling C

### 5.1 Characteristics of modeling C



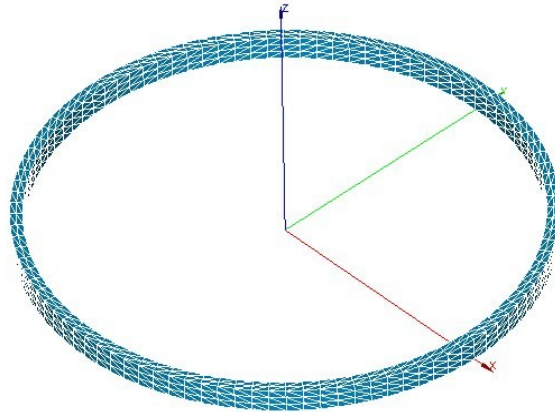
Modeling 3D:  
 Many nodes                    3528  
 Many meshes                    864                    PENTA15

### 5.2 Sizes tested and results

|                   | Component   | Nature of the clean mode |       | Reference (Hz) | Tolerance (%) |
|-------------------|-------------|--------------------------|-------|----------------|---------------|
|                   |             | <i>i</i>                 | order |                |               |
| Modes in the plan | <i>FREQ</i> | 2                        | 4,5   | 318.36         | 0.1           |
|                   |             | 3                        | 6,7   | 900.46         | 0.2           |
|                   |             | 4                        | 8,9   | 1726.55        | 0.35          |
|                   |             | 5                        | 10,11 | 2792.21        | 0.6           |
| Transverse modes  | <i>FREQ</i> | 2                        | 4,5   | 511.           | 0.7           |
|                   |             | 3                        | 6,7   | 1590.          | 1.4           |
|                   |             | 4                        | 8,9   | 3184.          | 2.3           |

## 6 Modeling D

### 6.1 Characteristics of modeling D



Modeling 3D:

Many nodes                    5824  
Many meshes                    2728      TETRA10

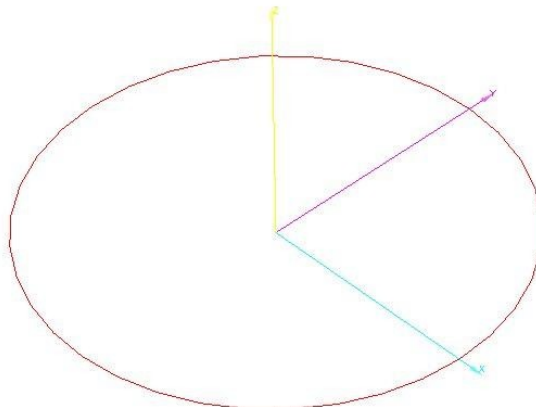
### 6.2 Sizes tested and results

|                   | Component   | Nature of the clean mode |       | Reference (Hz) | Tolerance (%) |
|-------------------|-------------|--------------------------|-------|----------------|---------------|
|                   |             | l                        | order |                |               |
| Modes in the plan | <i>FREQ</i> | 2                        | 4,5   | 318.36         | 0.1           |
|                   |             | 3                        | 6,7   | 900.46         | 0.2           |
|                   |             | 4                        | 8,9   | 1726.55        | 0.4           |
|                   |             | 5                        | 10,11 | 2792.21        | 0.7           |
| Transverse modes  | <i>FREQ</i> | 2                        | 4,5   | 511.           | 0.25          |
|                   |             | 3                        | 6,7   | 1590.          | 1.1           |
|                   |             | 4                        | 8,9   | 3184.          | 2.0           |



## 7 Modeling E

### 7.1 Characteristics of modeling



Modeling POU\_D\_T:

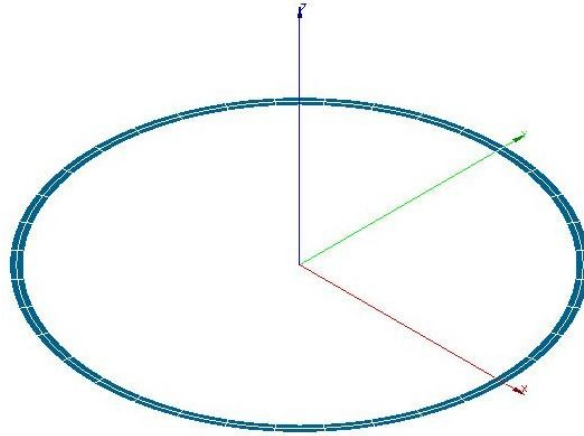
Many nodes 72  
Many meshes 72 SEG2

### 7.2 Sizes tested and results

|                   | Component   | Nature of the clean mode |       | Reference (Hz) | Tolerance (%) |
|-------------------|-------------|--------------------------|-------|----------------|---------------|
|                   |             | <i>i</i>                 | order |                |               |
| Modes in the plan | <i>FREQ</i> | 2                        | 4,5   | 318.36         | 0.25          |
|                   |             | 3                        | 6,7   | 900.46         | 0.6           |
|                   |             | 4                        | 8,9   | 1726.55        | 1.1           |
|                   |             | 5                        | 10,11 | 2792.21        | 1.7           |
| Transverse modes  | <i>FREQ</i> | 2                        | 4,5   | 511.           | 0.85          |
|                   |             | 3                        | 6,7   | 1590.          | 1.5           |
|                   |             | 4                        | 8,9   | 3184.          | 2.4           |

## 8 Modeling F

### 8.1 Characteristics of modeling F



Modeling D\_PLAN:

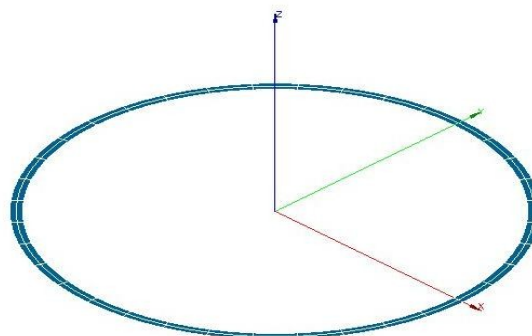
Many nodes 288  
Many meshes 72 QUAD8

### 8.2 Sizes tested and results

|                   | Component   | Nature of the clean mode |       | Reference (Hz) | Tolerance (%) |
|-------------------|-------------|--------------------------|-------|----------------|---------------|
|                   |             | <i>i</i>                 | order |                |               |
| Modes in the plan | <i>FREQ</i> | 2                        | 4,5   | 318.36         | 1.4           |
|                   |             | 3                        | 6,7   | 900.46         | 1.6           |
|                   |             | 4                        | 8,9   | 1726.55        | 1.7           |
|                   |             | 5                        | 10,11 | 2792.21        | 2.0           |

## 9 Modeling G

### 9.1 Characteristics of modeling G



Modeling C\_PLAN :

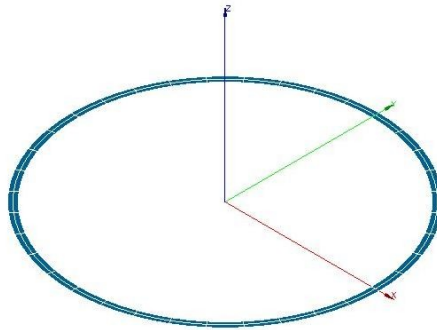
Many nodes 293  
Many meshes 72 QUAD8

### 9.2 Sizes tested and results

|                   | Component   | Nature of the clean mode |       | Reference (Hz) | Tolerance (%) |
|-------------------|-------------|--------------------------|-------|----------------|---------------|
|                   |             | <i>i</i>                 | order |                |               |
| Modes in the plan | <i>FREQ</i> | 2                        | 4,5   | 318.36         | 1.4           |
|                   |             | 3                        | 6,7   | 900.46         | 1.6           |
|                   |             | 4                        | 8,9   | 1726.55        | 1.7           |
|                   |             | 5                        | 10,11 | 2792.21        | 2.0           |

## 10 Modeling I

### 10.1 Characteristics of modeling I



Modeling C\_PLAN:

Many nodes 293  
Many meshes 72 QUAD8

### 10.2 Sizes tested and results

|                   | Component   | Nature of the clean mode |       | Reference (Hz) | Tolerance (%) |
|-------------------|-------------|--------------------------|-------|----------------|---------------|
|                   |             | <i>i</i>                 | order |                |               |
| Modes in the plan | <i>FREQ</i> | 2                        | 4,5   | 318.36         | 1.4           |
|                   |             | 3                        | 6,7   | 900.46         | 1.6           |
|                   |             | 4                        | 8,9   | 1726.55        | 1.7           |
|                   |             | 5                        | 10,11 | 2792.21        | 2.0           |

## 11 Summary of the results

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The got results are satisfactory.