SDLL15 - Beam hurled, embed-free, with mass or offset inertia

Summary:

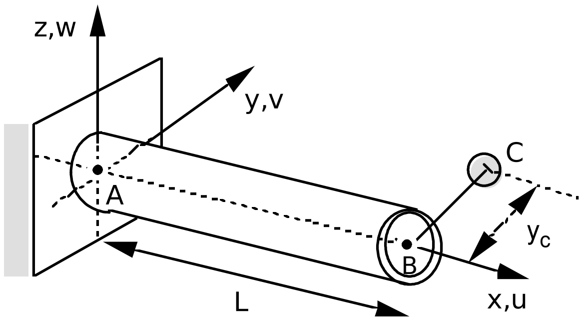
This three-dimensional problem consists in calculating the frequencies and the modes of vibration of a mechanical structure made up of a right beam slim, embed-free, with tubular section and of an unbalance attached at the loose lead of the beam. This test of Mechanics of the Structures corresponds to a dynamic analysis of a linear model having a linear behavior. It comprises only one modeling.

This problem makes it possible to test the element of beam of Euler Bernouilli, the model of specific mass and modal calculation by the method of Lanczos.

The got results are in concord with those of guide VPCS. Two calculations carried out (eccentricity of the specific mass worthless or different from zero) make it possible to highlight the coupling of the various modes when the specific mass is offset.

1 Problem of reference

1.1 Geometry



Coordinates of the points (in m) :

	A	B	C
x	0.	10.	10.
y	0.	0.	yc
z	0.	0.	0.

length of the beam: $AB=L=10\,m$

specific mass in C : $m_c=1000\,kg$

Tubular section:

external diameter	$de=0.350\,m$
internal diameter	$di=0.320\,m$
surface	$A=1.57865\,10^{-2}\,m^2$
inertia	$Iy=Iz=2.21899\,10^{-4}\,m^4$
polar inertia	$Ip=4.43798\,10^{-4}\,m^4$

2 studied cases:
1) $yc=0.$
2) $yc=1.\,m$

1.2 Material properties

$E=2.1\,10^{11}\,Pa$

$\rho=7800\,kg/m^3$

1.3 Boundary conditions and loadings

Not A embedded: ($u=v=w=0$, $\theta_x=\theta_y=\theta_z=0$).

1.4 Initial conditions

Without object for the modal analysis.

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is that given in card SDLL15/89 of the guide VPCS which presents the method of calculating in the following way:

The problem with not offset mass leads to uncoupled modes:

- traction and compression (effect of the mass alone),
- torsion (effect of inertia around neutral fibre),
- inflection in the plans x, y and x, z (effect of the mass).

The various Eigen frequencies are given with a model by finite elements of beam of Euler (slim beam).

For the first mode with an unbalance, a method of Rayleigh gives the approximate formula:

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{3 E I_z}{L^3 (m_c + 0.24 M)}}$$

with M = total mass of the beam.

When the mass is offset, modes of inflection (x, z) and of torsion are coupled, as well as the modes of inflection (x, y) and of traction and compression.

For the clean mode, the components at the point B allow to calculate the components in the centre of gravity of the mass (not C) by:

$$\begin{bmatrix} u_c \\ v_c \\ w_c \end{bmatrix} = \begin{bmatrix} u_B \\ v_B \\ w_B \end{bmatrix} + \begin{bmatrix} 0 & z_c & -y_c \\ -z_c & 0 & +x_c \\ +y_c & -x_c & 0 \end{bmatrix} \begin{bmatrix} \theta_{xB} \\ \theta_{yB} \\ \theta_{zB} \end{bmatrix}$$

For this test:

$$u_c = u_B = -\theta_{zB}$$

$$v_c = v_B$$

$$w_c = w_B + \theta_{xB}$$

2.2 Results of reference

Case 1: the first 10 clean modes.

Case 2: the first 8 clean modes.

2.3 Uncertainty on the solution

Problem 1:	$f1$ analytical solution
	other frequencies $\pm 1\%$
Problem 2:	$\pm 1\%$

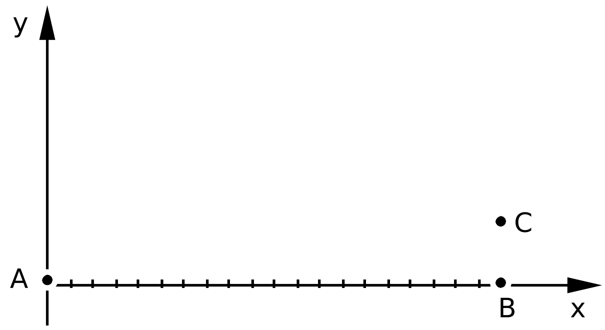
2.4 Bibliographical references

1) Working group Analyzes Dynamic. Commission of Validation of the Software packages of Structural analysis. French company of Mécaniens. (1988)

3 Modeling A

3.1 Characteristics of modeling

Element of beam POU_D_E and discrete element DIS_TR



Cutting: beam AB : 20 meshs SEG2.

Limiting conditions:

with the node end A
DDL_IMPO: (NODE: With DX: 0. , DY: 0. , DZ: 0. , DRX: 0. , DRY MARTINI: 0. , DRZ: 0.)

Nodal mass in B with an eccentricity	$ey=0.$	Case 1
	$ey=1.$	Case 2

Names of the nodes:	Points	$A=N100$	$B=N200$

3.2 Characteristics of the grid

Many nodes:	21
Many meshs and types:	20 SEG2

3.3 Sizes tested and results

Case	Nature of the clean mode	Frequency Hz Reference	Aster	% difference
CA 1	inflection 1.2	1.65	1.6554	0.33
	inflection 3.4	16.07	16.0712	0.
	inflection 5.6	50.02	50.0240	0.
	traction 1	76.47	76.4727	0.
$yc=0.$	torsion 1	80.47	80.4688	0.
	inflection 7.8	103.20	103.20444	0.
CA 2	f_z+t_o 1	1,636	1.6363	0.
	f_y+t_r 2	1,642	1.6416	0.
	f_y+t_r 3	13.46	13.4551	0.
	f_z+t_o 4	13.59	13.5919	0.
$yc=1.$	f_z+t_o 5	28.90	28.8972	0.
	f_y+t_r 6	31.96	31.9594	0.
	f_z+t_o 7	61.61	61.6091	0.
	f_y+t_r 8	63.93	63.9289	0.
Mode	θ_{xB}	0.03	$3,039\ 10^{-2}$	1,321
1	w_C/w_B	1,030	1,030	0.
2	u_C/v_B	– 0,148	– 0,148	0.
3	u_C/v_B	– 2,882	– 2,880	0.07
4	w_C/w_B	– 0,922	– 0,923	0,108
5	θ_{xB}	– 1,922	– 1.92268	0,036

with: $f_z+t_o=flexion\ x,z+torsion$ $f_y+t_r=flexion\ x,y+traction$

3.4 Remarks

Calculations carried out by:

```
CALC_MODES
            OPTION = 'PLUS_PETITE'
            CALC_FREQ=_F (NMAX_FREQ = N)
            SOLVEUR_MODAL=_F (METHOD = 'TRI_DIAG')
```

Case 1: n=10, Case 2: n=8

In the test, one cannot check the values of the reports $\frac{u_C}{v_B}$ for modes 2 and 3 (except manually). With regard to the values of $\frac{w_C}{w_B}$, the technique is the following one: if one imposes $w_B=1$ (order `NORM_MODE`), one has then $\frac{w_C}{w_B}=1+\theta_{xB}$ and one can make checks on the values of θ_{xB}.

Contents of the file results:

Case 1: the first 11 Eigen frequencies, clean vectors and modal parameters.
Case 2: the first 9 Eigen frequencies, clean vectors and modal parameters.

4 Summary of the results

The modeling of unbalance gives exact results for the 8 frequencies of reference.

The precision of the clean modes is about 0.1% until mode 4.