SDLL101 - Vibration of a beam with prestressed

Summary:

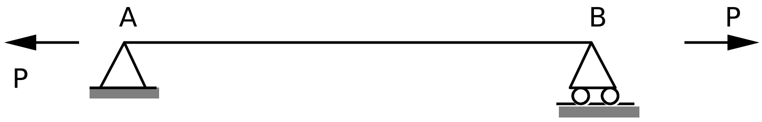
This problem plan consists in seeking the frequencies of vibration of a mechanical structure made up of a hurled beam, of circular section, under tension embed-slide. This test of Mechanics of the Structures corresponds to a dynamic analysis of a linear model having a linear behavior. This test comprises two modelings.

In the first modeling, one tests the element of beam of Timoshenko subjected to a prestressing, the calculation of geometrical rigidity and the calculation of the Eigen frequencies by the method of Lanczos. In the second modeling, one tests the element of beam of Euler - Bernouilli subjected to a prestressing, the calculation of geometrical rigidity and the calculation of the Eigen frequencies by the method of Bathe and Wilson.

The got results are in concord with the results of guide VPCS. One notices a shift to the top of the frequencies of vibration when prestressing in the beam increases.

1 Problem of reference

1.1 Geometry



Full circular section
diameter $d=0.01\,m$

Length of the beam
$L=2\,m$

1.2 Material properties

$E=2\,10^{11}\,N/m^2$
$\nu=0.3$
$\rho=7\,800.\,kg/m^3$

1.3 Boundary conditions and loadings

- Beam pose-posed,
- 4 loadings are studied $P=0.$, $P=10.$, $P=100.$, $P=1\,000.\,N$

2 Reference solution

2.1 Method of calculating used for the reference solution

The equation of vibration of a prestressed beam is:

$$EI_z \frac{\partial^4 y}{\partial x^4} + P \frac{\partial^2 y}{\partial x^2} = -\rho S \frac{\partial^2 y}{\partial x^2}$$

prestressed traction if $P > 0$, of compression if $P < 0$, and led to the Eigen frequencies of inflection (assumption of Euler-Bernoulli)

$$f_i = \frac{i^2 \pi}{2L^2} \left(1 + \frac{PL^2}{EI_z i^2 \pi^2} \right)^{1/2} \left(\frac{EI_z}{\rho S} \right)^{1/2}, \; i = 1,2,3,\ldots$$

2.2 Results of reference

the first 5 Eigen frequencies.

2.3 Uncertainty on the solution

Analytical solution (assumption of the beams of Euler-Bernouilli).

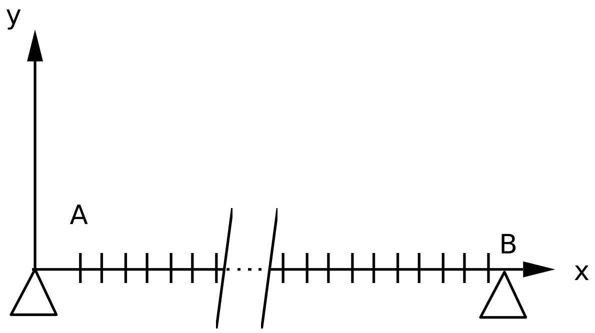
2.4 Bibliographical references

1) Robert D. BLEVINS Formulated for natural frequency and shape mode - 1979 p.144 (rectified formula 8.20).

3 Modeling A

3.1 Characteristics of modeling

Elements of beam POU_D_T (Right Beam of Timoshenko)



Cutting: 10 elements of beam

node A : translations in x and y blocked
node B : translation in y blocked.

Note:

The force P applied in B generate a reaction $-P$ in A .

3.2 Characteristics of the grid

Many nodes:	21
Many meshs and types:	20 SEG2

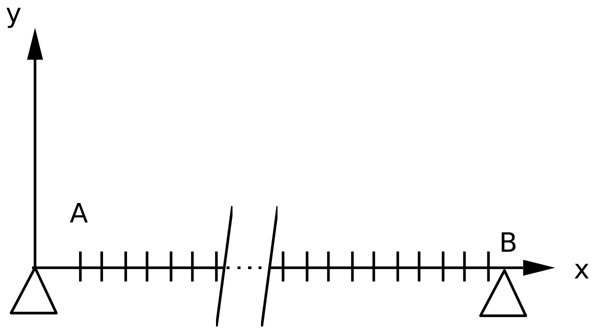
3.3 Sizes tested and results

Pre constraint/order of the clean mode		Reference
$P=0$	1	4.97137
	2	19.8851
	3	44.7414
	4	79.5403
	5	124.2818
$P=10$	1	5.0728
	2	19.9874
	3	44.8439
	4	79.6429
	5	124.3844
$P=100$	1	5.9090
	2	20.8860
	3	45.7561
	4	80.5600
	5	125.3037
$P=1\,000$	1	11.2577
	2	28.3462
	3	54.0370
	4	89.2134
	5	134.1511

4 Modeling B

4.1 Characteristics of modeling

Elements of beam POU_D_E (Beam of Euler-Bernouilli)



Cutting: 19 elements of beam

node A : translations in x and y blocked

node B : translation in y blocked.

Note:

The force P applied in B generate a reaction $-P$ in A .

4.2 Characteristics of the grid

Many nodes:	21
Many meshs and types:	20 SEG2

4.3 Sizes tested and results

Pre constraint/order of the clean mode		Reference
$P=0$	1	4.97137
	2	19.8851
	3	44.7414
	4	79.5403
	5	124.2818
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	4	80.5600
	5	125.3037
$P=1\,000$	1	11.2577
	2	28.3462
	3	54.0370
	4	89.2134
	5	134.1511

5 Summary of the results

The got results are in concord with the results of reference. It is noticed well that the frequencies of vibration increase when prestressing increases.