SDLL127 - Line of trees with rotor with variable circular section

Summary:

This test makes it possible to validate the calculation of the modes in rotation of a system of trees at rest or turning with a rotor with variable section.

In this test, there is a model of rotor with variable circular section resting on two stages discs whose matrices of stiffness and damping are symmetrical. This example is drawn from the file of validation of code CADYRO 2.0 [bib1], software finite elements intended to envisage the dynamic behavior of rotors.

1 Problem of reference

1.1 Geometry

The structure is made up of a rotor length L with variable circular section with two infinitely rigid stages on the level of the supports. The diameters of the section of the rotor vary in a linear way according to the length.

1.2 Properties of material

The geometrical characteristics and material are listed in the following table.

Material	$E=2\,10^{11}\,N/m^2$	$\rho=7800\,kg/m^3$	$\nu=0.3$
Length rotor		$L=1\,m$	
Diameter section	initial external diameter: $De1=0{,}2\,m$		
	final external diameter: $De2=0{,}1\,m$		
Thickness	in any point of the rotor: $De-Di=0{,}04\,m$		

Table 1.2-1

The coefficients in translation of the stages are:

$$Kxx=Kyy=1.0\mathrm{E}+12\,kg.s^{-2}$$
$$Kxy=Kyx=0.0\,kg.s-2$$
$$Cxx=Cyy=Cxy=Cyx=0.0\,kg.s^{-1}$$

The coefficients in torsion of the stages are:

$$Krz=Kry=1.0\mathrm{E}+12\,kg.s^{-2}$$
$$Kxy=Kyx=0.0\,kg.s-2$$
$$Crx=Cry=Cxy=Cyx=0.0\,kg.s^{-1}$$

1.3 Boundary conditions

The rotor is pressed on two rigid stages at the two ends.

2 Reference solution

2.1 Method of calculating

The reference is given by code CADYRO [bib1] for elements of beam to variable section.

In addition, Lbe values resulting from the modeling using of the elements of beam with variable section (Modeling A) are framed by those obtained by those obtained from:

- modeling B, approaching the exact solution by lower values. That amounts assigning to each element **constant circular section** of diameter equal to the diameter of the studied rotor **with the right of the initial node** element.
- modeling C, approaching the exact solution by higher values. This one consists in assigning to each element **constant circular section** of diameter equal to the diameter of the studied rotor **with the right of the final node** element.

Modelings B and C are then equivalent (in the direction of the limit) to modeling A when the number of elements used by those tends towards the infinite one. In this case, 2000 elements of beam with constant circular section are used.

2.2 Sizes and results of reference

The reference variable is the frequency of the modes to the stop and for a number of revolutions of 4000 tr/mn . The number of calculated modes is 12.

Number of frequency	**Number of revolutions (tr/mn)**	**Cadyro calculation with tree with variable section F (Hz)**
1	0	332.14
	4000	331.51
2	0	332.14
	4000	334.10
3	0	1193.81
	4000	1196.30
4	0	1193.81
	4000	1202.05
5	0	1607.19
	4000	1607.19
6	0	2291.51
	4000	2304.73
7	0	2291.51
	4000	2312.08
8	0	2512.22
	4000	2512.22
9	0	3160.39
	4000	3160.39
10	0	3520.23
	4000	3516.43
11	0	3520.23
	4000	3524.02
12	0	4724.14
	4000	4724.14

2.3 Bibliographical references

[1] Dwicker of validation of code CADYRO 2.0, software finite elements intended to envisage the dynamic behavior of rotors, H-P60-1994-04877-FR.

3 Modeling A

3.1 Characteristics of modeling

The tree is modelled with elements beam POU_D_T with variable section. Moreover, discrete elements of type DIS_TR are used for the modeling of the stages

3.2 Characteristics of the grid

Number and type of elements:
100 SEG2
2 POI1

Many nodes: 101

3.3 Sizes tested and results

Values of the first 12 frequencies to the stop and for the number of revolutions $4000\,tr/min$, are presented in the table below

Number of frequency	Number of revolutions (tr/mn)	Cadyro calculation with tree with variable section F (Hz)	Tolerance (%)
1	0	332.14	1.0
	4000	331.51	1.0
2	0	332.14	1.0
	4000	334.10	1.0
3	0	1193.81	1.0
	4000	1196.30	1.0
4	0	1193.81	1.0
	4000	1202.05	1.0
5	0	1607.19	1.0
	4000	1607.19	1.0
6	0	2291.51	1.0
	4000	2304.73	2.0
7	0	2291.51	1.0
	4000	2312.08	2.0
8	0	2512.22	1.0
	4000	2512.22	1.0
9	0	3160.39	1.0
	4000	3160.39	1.0
10	0	3520.23	2.0
	4000	3516.43	2.0
11	0	3520.23	2.0
	4000	3524.02	2.0
12	0	4724.14	2.0
	4000	4724.14	2.0

Moreover, one also checks the calculation of the mass of the elements with homothetic variation of the section.
The calculation of the mass is exact only when the report $R_1/R_2=Ep_1/Ep_2$. In the contrary case, one needs a sufficiently fine grid to attenuate the made mistake. See [R3.08.01].

The error is most important on the last element (M100). The mass of this element is of 0.99549 kg.
The total mass of the rotor is of 63.7115 kg.

4 Modeling B

4.1 Characteristics of modeling

The tree is modelled with elements beam POU_D_T regularly distributed with **constant circular section** of diameter equal to the diameter of the studied rotor **with the right of the initial node** element. Moreover, discrete elements of type DIS_TR are used for the modeling of the stages

4.2 Characteristics of the grid

Number and type of elements:
2000 SEG2
2 POI1

Many nodes: 2001

4.3 Sizes tested and results

Values of the first 12 frequencies to the stop and for the number of revolutions $4000\,tr/min$ are presented in the table below:

Number of frequency	**Number of revolutions (tr/mn)**	**Cadyro calculation with tree with variable section F (Hz)**	**Tolerance (%)**
1	0	332.14	1.0
	4000	331.51	1.0
2	0	332.14	1.0
	4000	334.10	1.0
3	0	1193.81	1.0
	4000	1196.30	1.0
4	0	1193.81	1.0
	4000	1202.05	1.0
5	0	1607.19	1.0
	4000	1607.19	1.0
6	0	2291.51	1.0
	4000	2304.73	2.0
7	0	2291.51	1.0
	4000	2312.08	2.0
8	0	2512.22	1.0
	4000	2512.22	1.0
9	0	3160.39	1.0
	4000	3160.39	1.0
10	0	3520.23	2.0
	4000	3516.43	2.0
11	0	3520.23	2.0
	4000	3524.02	2.0
12	0	4724.14	2.0
	4000	4724.14	2.0

Table 4.3-1 : Eigen frequencies obtained by QZ

5 Modeling C

5.1 Characteristics of modeling

The tree is modelled with elements beam POU_D_T regularly distributed with **constant circular section** of diameter equal to the diameter of the studied rotor **with the right of the final node** element. Moreover, discrete elements of type DIS_TR are used for the modeling of the stages

5.2 Characteristics of the grid

Number and type of elements:
2000 SEG2
2 POI1

Many nodes: 2001

5.3 Sizes tested and results

Values of the first 12 frequencies to the stop and for the number of revolutions $4000\,tr/min$ are presented in the table below

Number of frequency	**Number of revolutions** (tr/mn)	**Cadyro calculation with tree with variable section** F (Hz)	**Tolerance (%)**
1	0	332.14	1.0
	4000	331.51	1.0
2	0	332.14	1.0
	4000	334.10	1.0
3	0	1193.81	1.0
	4000	1196.30	1.0
4	0	1193.81	1.0
	4000	1202.05	1.0
5	0	1607.19	1.0
	4000	1607.19	1.0
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	4000	2304.73	2.0
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	4000	2312.08	2.0
8	0	2512.22	1.0
	4000	2512.22	1.0
9	0	3160.39	1.0
	4000	3160.39	1.0
10	0	3520.23	2.0
	4000	3516.43	2.0
11	0	3520.23	2.0
	4000	3524.02	2.0
12	0	4724.14	2.0
	4000	4724.14	2.0

Table 5.3-1 : Eigen frequencies obtained by QZ

6 Summary of the results

This CAS-test makes it possible to numerically validate the taking into account of the lines of trees to variable circular section. The got results are in concord with the values of reference, resulting DE CADYRO and of two very fine modelings approaching the exact solution by lower and higher values.

The calculation of the mass of the elements whose section varies in a homothetic way is checked with less $10^{-2}\%$.