

## SDLL137 – Structural modification of a beam

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### Summary

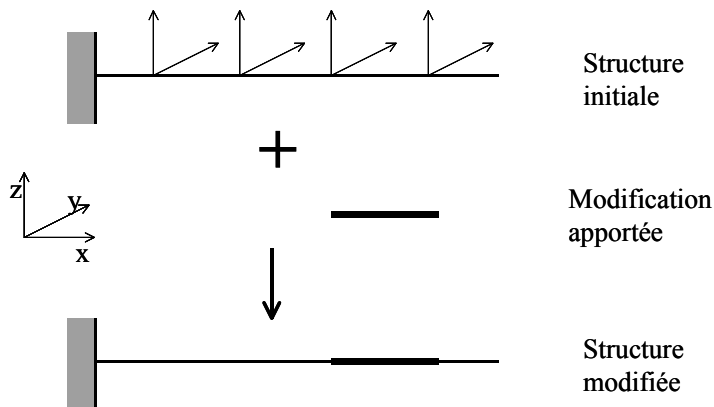
The purpose of this case test is to validate the procedure and the calculation of structural modification starting from measured information.

The method of structural modification used is based on the joint exploitation of measured data and the digital model of the modification made to the initial structure. One makes use then of the technique of under-structuring for the coupling of the two models.

For this case test, measurement was simulated numerically and the results of reference are got by direct calculation on the complete structure.

## 1 Problem of reference

### 1.1 Geometry



On the diagram above, the points of measurement on the initial structure are materialized by arrows, of which the point indicates the significant direction of the sensor. These points of measurement are localised with the X-coordinates  $0.2\text{ m}$ ,  $0.4\text{ m}$ ,  $0.6\text{ m}$  and  $0.8\text{ m}$  according to the directions  $y$  and  $z$ . Measured information is available only in these points of measurement.

The initial structure is a beam with rectangular section ( $9\text{ mm} \times 38\text{ mm}$ ) of length  $0.9\text{ m}$ .

The modification is an additional beam, with rectangular section ( $9\text{ mm} \times 38\text{ mm}$ ), that one applies between the coordinates  $0.6\text{ m}$  and  $0.8\text{ m}$ .

### 1.2 Properties of material

Young modulus:  $E = 2.1 \cdot 10^{11} \text{ N/m}^2$

Poisson's ratio:  $\nu = 0.3$

Density:  $\rho = 7800 \text{ kg/m}^3$

### 1.3 Boundary conditions and loadings

The beam is embedded with the one of its ends (origin of the axes) and free at the other end.

### 1.4 Initial conditions

Without object.

## 2 Reference solution

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### 2.1 Method of calculating

It is a question here of coupling an experimental model representing the real structure and a digital model of the modification which one wishes brought to the initial structure according to the technique suggested by Mr. Corus [1]. This coupling is done via model "a support" making it possible to condense measurement with the interfaces between the real structure and the modification. One assembles then the two models by using the technique of under-structuring. A more detailed description of the procedure of modification used is presented in [U2.07.03]. The extension of this technique on damping structures was studied by B. Groult [2].

The data input of calculation are: identified clean modes of the initial structure, the digital model of the "support" and the digital model of the "modification".

One proposes to calculate the variations of the first two Eigen frequencies of the embed-free beam, following the modification made on a portion of the beam.

Model "the support" chosen for this case test is a digital model with the finite elements of the embed-free beam described in the preceding paragraph. The first two Eigen frequencies of this beam are:  $9.31\text{ Hz}$  and  $39.32\text{ Hz}$ .

The condensation of the information measured with the interfaces is obtained by carrying out an expansion of measurement via a base of beforehand selected projection, defined on this model "support". The quality of the result depends on the choice of this base of projection.

Measurement was simulated numerically starting from a calculation resulting from the model support. The modification was modelled numerically by finite elements.

### 2.2 Sizes and results of reference

One compares the first two Eigen frequencies of the structure modified with the Eigen frequencies obtained by a direct calculation on the complete model. The first two Eigen frequencies of the modified structure are:  $7.78\text{ Hz}$  and  $32.85\text{ Hz}$ .

One also checks the good progress of the procedure of structural modification by comparing the field obtained with the interfaces of the modified structure in two different ways.

The first calculation corresponds to the calculation of the field to the interfaces on the coupled model. The second calculation corresponds to the calculation of the field to the interfaces by static expansion of the field obtained at the points measures modified structure.

The difference between these two fields can be evaluated by the calculation of the sum of the terms of the matrix of MAC (Modal Criterion Insurance) between these two fields. A matrix of MAC close to the matrix identity indicates that the two vectors are almost parallel. Criterion IERI is also evaluated (Indicating Energy of Regularity of Interface). This energy criterion tends towards 0 if the two fields are very close.

### 2.3 Uncertainties

The reference solution on the Eigen frequencies of the modified structure is obtained by direct calculation on the modified structure.

We consider that the selected discretization led to results very close to the analytical solution.

## 3 Modelings has with D

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The first 4 modelings rigorously use the method of structural modification as suggested by Mr. Corus, namely that the basic structure is described in modal form (a base of wide clean modes on a simplified digital model) and the modification is described in the form of a physical model. The elements of calculations implemented here make it possible to couple the two aspects. For more details, one will refer to U2.07.03 documentation (section 3).

## 4 Modeling E

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Modeling E implements same calculation in the form of a classical calculation per under-structuring. The principal structure and the modification are described on modal basis. The assembly of the two substructures is done classically with order `DEFI_MODELE_GENE`. For more precise details, one will refer to U2.07.03 documentation (section 4).

### 4.1 Bibliographical reference

- [1] Mr. Corus, Thesis ECP n° 2003-23, Improvement of the methods of structural modification per use of techniques of expansion and reduction of model.
- [2] B. Groult, Thesis ECP n° 2008-14, Extension of a method of structural modification for the design of dissipative devices integrating of viscoelastic materials.

## 5 Modeling A

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### 5.1 Characteristics of modeling

Model "the support" and the modification were modelled by elements `POU_D_E`. The base of projection chosen for the expansion of measured information, corresponds to the static deformations obtained by application of a loading in each point of measurement according to the significant direction of the sensor. The model "measures" was built starting from the first five identified modes of the initial structure.

### 5.2 Characteristics of the grid

Model "support":

Number and type of mesh: 9 elements of the type `SEG2`.

Model "modification":

Number and type of mesh: 2 elements of the type `SEG2`.

### 5.3 Features tested

The procedure of structural modification proceeds in several stages. The good progress of this procedure and the following features are tested:

`LIRE_RESU`, `PROJ_MESU_MODAL`, `MACR_ELEM_STAT`, `ASSE_MAILLAGE`, `DEPL_INTERNE`.

### 5.4 Sizes tested and results

One checks the values of the first two Eigen frequencies of the modified structure.

One also checks the sum of the terms of the matrix of MAC (Modal Criterion Insurance) obtained between the deformations with the interfaces for the model coupled and the deformations with the interfaces by static expansion of the deformations of the model coupled obtained with the points of measurement. This indicator indicates the relevance of the reconstruction of the field to the interfaces.

Size tested	Reference	Aster	Difference
$f1$	7.7807 Hz	7.7852 Hz	0.058%
$f2$	32.852 Hz	32.845 Hz	0.022%
MAC (Somme)	2	1,992	0,008

## 6 Modeling B

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### 6.1 Characteristics of modeling

It is a modeling identical to modeling A, but this time, one launches the procedure of modification via the macro-order `CALC_ESSAI`. One uses the method ES (Static Expansion) for calculation of the base of expansion.

### 6.2 Characteristics of the grid

Model "support":

Number and type of mesh: 9 elements of the type `SEG2`.

Model "modification":

Number and type of mesh: 2 elements of the type `SEG2`.

### 6.3 Features tested

The procedure of structural modification proceeds in several stages. The good progress of this procedure and the following features are tested:

`LIRE_RESU`, `PROJ_MESU_MODAL`, `MACR_ELEM_STAT`, `ASSE_MAILLAGE`, `DEPL_INTERNE`,  
`MAC_MODES`, `CALC_ESSAI`.

### 6.4 Sizes tested and results

One checks the values of the first two Eigen frequencies of the modified structure.

One also checks the sum of the terms of the matrix of MAC (Modal Criterion Insurance) obtained between the deformations with the interfaces for the model coupled and the deformations with the interfaces by static expansion of the deformations of the model coupled obtained with the points of measurement. This indicator indicates the relevance of the reconstruction of the field to the interfaces.

Size tested	Reference	Aster	Difference
$f_1$	7.7807 Hz	7.7852 Hz	0.058%
$f_2$	32.852 Hz	32.845 Hz	0.022%
MAC (Somme)	2	1.99994	$6 \cdot 10^{-5}$

## 7 Modeling C

### 7.1 Characteristics of modeling

In this modeling, one adds a damping in the model ( $AMOR\_ALPHA = 10^{-4}$ ,  $AMOR\_BETA = 1$ ). One launches the procedure of modification via the macro-order `CALC_ESSAI`. One uses method LMME (Local Model Modeshapes Expansion) for calculation of the base of expansion.

### 7.2 Characteristics of the grid

Model "support":

Number and type of mesh: 9 elements of the type `SEG2`.

Model "modification":

Number and type of mesh: 2 elements of the type `SEG2`.

### 7.3 Features tested

The procedure of structural modification proceeds in several stages. The good progress of this procedure and the following features are tested:

`LIRE_RESU`, `PROJ_MESU_MODAL`, `MACR_ELEM_STAT`, `ASSE_MAILLAGE`, `DEPL_INTERNE`,  
`MAC_MODES`, `CALC_ESSAI`.

### 7.4 Sizes tested and results

One checks the values of the first two Eigen frequencies of the modified structure.

One also checks the sum of the terms of the matrix of *MAC* (Modal Criterion Insurance) obtained between the deformations with the interfaces for the model coupled and the deformations with the interfaces by static expansion of the deformations of the model coupled obtained with the points of measurement. This indicator indicates the relevance of the reconstruction of the field to the interfaces. One calculates also the criterion *IERI* for the first two modes of the structure.

Size tested	Reference	Aster	Difference
$f1$	7.7807 Hz	7.7842 Hz	0.044%
$f2$	32.852 Hz	32,848 Hz	0.009%
<i>MAC</i> (Somme)	2	1.99994	$6 \cdot 10^{-5}$
<i>IERI</i> rigidity (1,1)	0	$4.60 \cdot 10^{-7}$	$4.60 \cdot 10^{-7}$
<i>IERI</i> mass (2,2)	0	$3.17 \cdot 10^{-12}$	$3.17 \cdot 10^{-12}$

## 8 Modeling D

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### 8.1 Characteristics of modeling

It is the same modeling as modeling C. This time, one does not call on the macro-order CALC\_ESSAI.

### 8.2 Characteristics of the grid

Model "support":

Number and type of mesh: 9 elements of the type SEG2.

Model "modification":

Number and type of mesh: 2 elements of the type SEG2.

### 8.3 Features tested

The procedure of structural modification proceeds in several stages. The good progress of this procedure and the following features are tested:

LIRE\_RESU, PROJ\_MESU\_MODAL, MACR\_ELEM\_STAT, ASSE\_MAILLAGE, DEPL\_INTERNE,  
MAC\_MODES, CALC\_ESSAI.

### 8.4 Sizes tested and results

One checks the values of the first two Eigen frequencies of the modified structure.

One also checks the sum of the terms of the matrix of *MAC* (Modal Criterion Insurance) obtained between the deformations with the interfaces for the model coupled and the deformations with the interfaces by static expansion of the deformations of the model coupled obtained with the points of measurement. This indicator indicates the relevance of the reconstruction of the field to the interfaces.

One calculates also the criterion *IERI* for the first two modes of the structure.

Size tested	Reference	Aster	Difference
$f1$	7.7807 Hz	7.7835 Hz	0.036%
$f2$	32.852 Hz	32.848 Hz	0.01%
<i>MAC</i> (Somme)	2	1.99994	$6. 10^{-5}$
<i>IERI</i> rigidity (1,1)	0	$4.92 10^{-7}$	$4.92 10^{-7}$
<i>IERI</i> mass (2,2)	0	$3.17 10^{-12}$	$3.17 10^{-12}$



## 9 Summary of the results

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The got results depend on measured information and the base of projection chosen for the expansion of measurement to the degrees of freedom interfaces. Here, the first five identified clean modes were exploited and bases it selected expansion being static answers. The got results are correct.

Relative uncertainties on the first two Eigen frequencies obtained by the technique of structural modification used, are lower than 1 % solution obtained by direct calculation.

The fields of displacement to the interfaces of the model coupled and the field with the interfaces obtained by static expansion of the fields at the points of measurement obtained on the modified structure are very close. The associated matrix of MAC is very close to the matrix identity. Criterion IERI is also close to 0 (modelings C and D).