SDLL141 - Eigen frequencies of a beam alone, subjected to the gyroscopic effect.

Summary:

This problem consists in seeking the frequencies of vibration of a beam pressed on each one of its ends, on infinitely rigid supports. The beam is full, of section circular and subjected at a number of constant revolutions. It comprise any disc.

Cinq modelingS are studied:

- Modeling a: the beam is along the axis x ,
- Modeling b: the beam is along the axis t such as t directing vector of the bisectrix (x, y) .
- Modeling C: the beam is along the axis t such as t directing vector of the bisectrix, and (x, y) mass is distributed by discrete elements installed on each node.
- Modeling D: the beam is along the axis x . The section is circular and variable with the two rays $R1$ and $R2$ identical.
- Modeling E: modeling C with declaration of the characteristics of the discrete elements in the local reference mark begins again.

This problem thus makes it possible to test the effect of the gyroscopic matrix which was developed for a right beam.

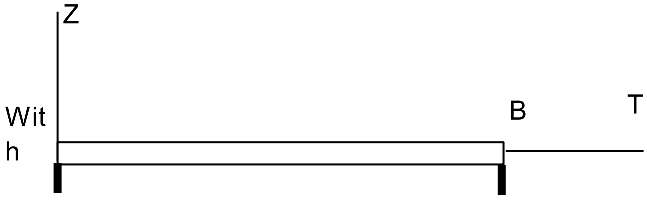
The gyroscopic effect led to the unfolding of the modes. The evolution of the Eigen frequencies according to the number of revolutions makes it possible to build the diagram of Campbell.

The references are based on the theory of Euler-Bernouilli.

The got results are in concord with those given in reference.

1 Problem of reference

1.1 Geometry



Modelings A and D:

$t = x$

Modelings B, C and E:

$\frac{\pi}{4} = (\hat{x}, t)$ and $t.z = 0$

Length of the beam:

$L = AB = 0.9\,m$

Circular section:

Diameter: $D = 0.05\,m$

Coordinates of the points (m):

		Modelings With and D	Modelings B and C
	A	B	B
X	0.	0.9	$0.9\cos(\pi/4)$
Y	0.	0.	$0.9\sin(\pi/4)$
Z	0.	0.	0.

Table 1.1-1 : Coordinates of the points A and B

1.2 Material properties

$E = 2.10^{11}\,Pa$
$\rho = 7\,800\,kg/m^3$ except for Lbe modelingS C and E.

For Lbe modelingS C and E, the density of material is taken equal to zero. The mass is installed via discrete elements installed on each node.

1.3 Boundary conditions and loadings

Not A : supported $u = v = w = \theta_x = 0$
Not B : supported $u = v = w = \theta_x = 0$

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is that presented in the work of Rene-Jean GIBERT.
By adopting the following notations:

- beam according to x
- y and z movements of inflection in the plan xOz and xOy
- S : section of the beam
- I : moment of inertia of inflection compared to the axes y and z
- I_x : moment of inertia per unit of length compared to the axis Ox
- ρ , E characteristics of material
- Ω number of revolutions of the beam

The solution singular are controls by the system of equations according to:

$$EI\frac{\partial^4 Y}{\partial x^4}-\rho S\omega^2 Y+i\omega\Omega I_x\frac{\partial^2 Z}{\partial z^2}=0$$
$$EI\frac{\partial^4 Z}{\partial x^4}-\rho S\omega^2 Z-i\omega\Omega I_x\frac{\partial^2 Y}{\partial z^2}=0$$

by observing the boundary conditions following:

$$\begin{cases} Y=Z=0 \\ \dfrac{\partial^2 Y}{\partial z^2}=\dfrac{\partial^2 Z}{\partial z^2}=0 \end{cases} \text{ in } \begin{cases} x=0 \\ x=L \end{cases}$$

One obtains two families of clean modes:

- Mode retrogresses:

$$Y_1=-i.Z_1=\sin\frac{n\pi x}{L} \text{ with } \left(\frac{\omega_1}{\omega_0}\right)=\sqrt{\lambda^2+1}-\lambda$$

- Direct mode:

$$Y_2=-i.Z_2=\sin\frac{n\pi x}{L} \text{ with } \left(\frac{\omega_2}{\omega_0}\right)=\sqrt{\lambda^2+1}+\lambda$$

while posing:

own pulsation without rotation: $\omega_0=\left(\frac{n\pi}{L}\right)^2\sqrt{\frac{EI}{\rho S}}$

$$\lambda=\frac{1}{2}.\frac{\Omega I_x}{\sqrt{EI\rho S}} \text{ with } I_x=\frac{\rho S D^2}{8} \text{ and } I=\frac{\pi D^4}{64}$$

2.2 Results of reference

the first 4 clean modes of inflection.

2.3 Uncertainty on the solution

Analytical solution with the assumption of beam of Euler.

2.4 Bibliographical references

Rene-Jean GIBERT, Vibrations of the structures, n°69 of the collection R & D of EDF at EYROLLES, p. 235-237 (1988).

3 Modeling A

3.1 Characteristics of modeling

Modeling : 18 Elements équirépartis of beam `POU_D_E`

3.2 Characteristics of the grid

The axis of the beam is directed according to the vector x .

Grid: Many nodes: 19
Many meshs and types: 18 `SEG2`

Names of the nodes: Not A = $N1$
Not B = $N19$

3.3 Sizes tested and results

Rotor with the stop ($\Omega=0$) (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 1	'ANALYTICAL'	122.7475	0.1 %
Mode 2	'ANALYTICAL'	490.9899	0.1 %
Mode 3	'ANALYTICAL'	1104.7273	0.1 %
Mode 4	'ANALYTICAL'	1963.9596	0.1 %

Calculation of the Eigen frequencies using the algorithm of Sorensen

Rotor in rotation ($\Omega=10^4 rd.s^{-1}$) , direct modes (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 2	'ANALYTICAL'	125.8150	0.1 %
Mode 4	'ANALYTICAL'	503.2598	0.1 %
Mode 6	'ANALYTICAL'	1132.3346	0.1 %
Mode 9	'ANALYTICAL'	2013.0393	0.1 %

Rotor in rotation ($\Omega=10^4 rd.s^{-1}$) , retrograde modes (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 1	'ANALYTICAL'	119.7548	0.1 %
Mode 3	'ANALYTICAL'	479.0191	0.1 %
Mode 5	'ANALYTICAL'	1077.7931	0.1 %
Mode 7	'ANALYTICAL'	1916.0765	0.1 %

4 Modeling B

4.1 Characteristics of modeling

Modeling : 18 Elements équirépartis of beam POU_D_E

4.2 Characteristics of the grid

The axis of the beam is directed according to the vector $(\cos(\pi/4), \sin(\pi/4), 0)$.

Grid: Many nodes: 19
Many meshs and types: 18 SEG2

Names of the nodes: Not A = $N1$
Not B = $N19$

3.3 Sizes tested and results

Rotor with the stop ($\Omega=0$) (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 1	'ANALYTICAL'	122.7475	0.1 %
Mode 2	'ANALYTICAL'	490.9899	0.1 %
Mode 3	'ANALYTICAL'	1104.7273	0.1 %
Mode 4	'ANALYTICAL'	1963.9596	0.1 %

Calculation of the Eigen frequencies using the algorithm of Sorensen
Rotor in rotation ($\Omega=10^4 rd.s^{-1}$) , direct modes (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 2	'ANALYTICAL'	125.8150	0.1 %
Mode 4	'ANALYTICAL'	503.2598	0.1 %
Mode 6	'ANALYTICAL'	1132.3346	0.1 %
Mode 9	'ANALYTICAL'	2013.0393	0.1 %

Rotor in rotation ($\Omega=10^4 rd.s^{-1}$) , retrograde modes (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 1	'ANALYTICAL'	119.7548	0.1 %
Mode 3	'ANALYTICAL'	479.0191	0.1 %
Mode 5	'ANALYTICAL'	1077.7931	0.1 %
Mode 7	'ANALYTICAL'	1916.0765	0.1 %

5 Modeling C

5.1 Characteristics of modeling

Modeling :
- 18 Elements équirépartis of beam POU_D_E
- 19 discrete Elements DIS_TR

5.2 Characteristics of the grid

The axis of the beam is directed according to the vector $(\cos(\pi/4), \sin(\pi/4), 0)$.

Grid: Many nodes: 19
Many meshs and types: 18 SEG2, 19 POI1

Names of the nodes: Not A = $N1$
Not B = $N19$

5.3 Mass of the discrete elements

In this modeling, the mass of the beam is not taken into account on the elements of beam but on the discrete elements. One calculates characteristics of mass to be assigned to each element in order to be equivalent if the mass is affected on the elements of beam.

Length of an element: $e=\frac{L}{18}=0.05\,m$

Characteristics of the discrete elements in the base (t, v, z)

Nodes $N2$ with $N18$	Nodes $N1$ and $N19$
$m=\rho\, e\pi\frac{D^2}{4}=0.7657\,kg$	$m'=\rho\frac{e}{2}\pi\frac{D^2}{4}=0.3829\,kg$
$I_{tt}=m.\frac{D^2}{8}=2,393.10^{-4}\ kg.m^2$	$I'_{tt}=m'.\frac{D^2}{8}=1,196.10^{-4}\ kg.m^2$
$I_{vv}=I_{zz}=\frac{I_{tt}}{2}+m.\frac{e^2}{12}=2,791.10^{-4}\ kg.m^2$	$I'_{vv}=I'_{zz}=\frac{I'_{tt}}{2}+m'.\frac{e^2}{3}=1,395.10^{-4}\ kg.m^2$

Table 5.3-1 : Calculation of the characteristics of the discrete elements

Two solutions are possible to define the characteristics in the base:
- that is to say to carry out a basic change of the local reference mark of the beam (t, v, z) with the total reference mark (x, y, z) . For that, it is necessary to carry out a basic change by a rotation of axis z and of value $-45°$. One obtains:

$$\overline{\overline{I}}=\begin{bmatrix} I_{xx} & I_{xy} & 0 \\ I_{xy} & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$ with:

$$I_{xx}=\cos^2(\pi/4)I_{tt}+\sin^2(\pi/4)I_{vv}$$
$$I_{yy}=\sin^2(\pi/4)I_{tt}+\cos^2(\pi/4)I_{vv}$$
$$I_{xy}=\cos(\pi/4)\sin(\pi/4)(I_{tt}-I_{vv})$$

Characteristics of the discrete elements in the base (x, y, z)

Nodes $N2$ with $N18$	Nodes $N1$ and $N19$
$m=\rho.e.\pi.\frac{D^2}{4}=0,7657\ kg$	$m'=\rho.\frac{e}{2}.\pi.\frac{D^2}{4}=0,3829\ kg$
$I_{xx}=I_{yy}=\frac{1}{2}\left(m.\frac{D^2}{8}+\frac{1}{2}m.\frac{D^2}{8}+m.\frac{e^2}{12}\right)$ $=2,592.10^{-4}\ kg.m^2$	$I'_{xx}=I'_{yy}=\frac{1}{2}\left(m'.\frac{D^2}{8}+\frac{1}{2}m'.\frac{D^2}{8}+m'.\frac{e^2}{12}\right)$ $=1,296.10^{-4}\ kg.m^2$
$I_{zz}=\frac{I_{tt}}{2}+m.\frac{e^2}{12}=2,792.10^{-4}\ kg.m^2$	$I'_{zz}=\frac{I'_{tt}}{2}+m'.\frac{e^2}{12}=1,396.10^{-4}\ kg.m^2$
$I_{xy}=I_{yx}=\frac{1}{2}\left\{m.\frac{D^2}{8}-\left(\frac{1}{2}m.\frac{D^2}{8}+m.\frac{e^2}{12}\right)\right\}$ $=-1,994.10^{-5}\ kg.m^2$	$I'_{xy}=I'_{yx}=\frac{1}{2}\left\{m'.\frac{D^2}{8}-\left(\frac{1}{2}m'.\frac{D^2}{8}+m'.\frac{e^2}{12}\right)\right\}$ $=-9,971.10^{-6}\ kg.m^2$

Table 5.3-2 : Calculation of the characteristics of the discrete elements

– that is to say to declare the characteristics in the local reference mark of the beam and to use the nautical angles to lay down the direction of the local reference mark. This method is usedE for modeling E.

3.3 Sizes tested and results

Rotor with the stop ($\Omega=0$) (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 1	'ANALYTICAL'	122.7475	0.1 %
Mode 2	'ANALYTICAL'	490.9899	0.1 %
Mode 3	'ANALYTICAL'	1104.7273	0.1 %
Mode 4	'ANALYTICAL'	1963.9596	0.1 %

Calculation of the Eigen frequencies using the algorithm of Sorensen
Rotor in rotation ($\Omega=10^4 rd.s^{-1}$) , direct modes (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 2	'ANALYTICAL'	125.8150	0.1 %
Mode 4	'ANALYTICAL'	503.2598	0.1 %
Mode 6	'ANALYTICAL'	1132.3346	0.1 %
Mode 9	'ANALYTICAL'	2013.0393	0.1 %

Rotor in rotation ($\Omega=10^4 rd.s^{-1}$) , retrograde modes (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 1	'ANALYTICAL'	119.7548	0.1 %
Mode 3	'ANALYTICAL'	479.0191	0.1 %

Mode 5	`'ANALYTICAL'`	1077.7931	0.1 %
Mode 7	`'ANALYTICAL'`	1916.0765	0.1 %

6 Modeling D

6.1 Characteristics of modeling

Modeling : 18 Elements équirépartis of beam POU_D_E

The elements are of variable circular section with the two rays $R1$ and $R2$ identical.

6.2 Characteristics of the grid

The axis of the beam is directed according to the vector x .

Grid: Many nodes: 19
Many meshs and types: 18 SEG2

Names of the nodes: Not A = $N1$
Not B = $N19$

3.3 Sizes tested and results

Rotor with the stop ($\Omega=0$) (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 1	'ANALYTICAL'	122.7475	0.1 %
Mode 2	'ANALYTICAL'	490.9899	0.1 %
Mode 3	'ANALYTICAL'	1104.7273	0.1 %
Mode 4	'ANALYTICAL'	1963.9596	0.1 %

Calculation of the Eigen frequencies using the algorithm of Sorensen

Rotor in rotation ($\Omega=10^4 rd.s^{-1}$) , direct modes (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 2	'ANALYTICAL'	125.8150	0.1 %
Mode 4	'ANALYTICAL'	503.2598	0.1 %
Mode 6	'ANALYTICAL'	1132.3346	0.1 %
Mode 9	'ANALYTICAL'	2013.0393	0.1 %

Rotor in rotation ($\Omega=10^4 rd.s^{-1}$) , retrograde modes (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 1	'ANALYTICAL'	119.7548	0.1 %
Mode 3	'ANALYTICAL'	479.0191	0.1 %
Mode 5	'ANALYTICAL'	1077.7931	0.1 %
Mode 7	'ANALYTICAL'	1916.0765	0.1 %

7 Modeling E

7.1 Characteristics of modeling

Modeling :

- 18 Elements équirépartis of beam `POU_D_E`
- 19 discrete Elements `DIS_TR`

7.2 Characteristics of the grid

The axis of the beam is directed according to the vector $(\cos(\pi/4),\sin(\pi/4),0)$.

Grid: Many nodes: 19
Many meshs and types: 18 `SEG2`, 19 `POI1`

Names of the nodes: Not A = $N1$
Not B = $N19$

7.3 Mass of the discrete elements

In this modeling, the mass of the beam is not taken into account on the elements of beam but on the discrete elements. One calculates characteristics of mass to be assigned to each element in order to be equivalent if the mass is affected on the elements of beam.

Length of an element: $e=\frac{L}{18}=0.05\,m$

Characteristics of the discrete elements in the base (t, v, z)

Nodes $N2$ with $N18$	Nodes $N1$ and $N19$
$m=\rho\, e\pi\frac{D^2}{4}=0.7657\,kg$	$m'=\rho\frac{e}{2}\pi\frac{D^2}{4}=0.3829\,kg$
$I_{tt}=m.\frac{D^2}{8}=2{,}393.10^{-4}\ kg.m^2$	$I'_{tt}=m'.\frac{D^2}{8}=1{,}196.10^{-4}\ kg.m^2$
$I_{vv}=I_{zz}=\frac{I_{tt}}{2}+m.\frac{e^2}{12}=2{,}791.10^{-4}\ kg.m^2$	$I'_{vv}=I'_{zz}=\frac{I'_{tt}}{2}+m'.\frac{e^2}{3}=1{,}395.10^{-4}\ kg.m^2$

Table 7.3-1 : Calculation of the characteristics of the discrete elements

In this modeling contrary to modeling C, one declares the characteristics in the local reference mark of the beam and one use the nautical angles to lay down the direction of the local reference mark.

3.3 Sizes tested and results

Rotor with the stop ($\Omega=0$) (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 1	'ANALYTICAL'	122.7475	0.1 %
Mode 2	'ANALYTICAL'	490.9899	0.1 %

Mode 3	`'ANALYTICAL'`	1104.7273	0.1 %
Mode 4	`'ANALYTICAL'`	1963.9596	0.1 %

Calculation of the Eigen frequencies using the algorithm of Sorensen

Rotor in rotation ($\Omega=10^4 rd.s^{-1}$) , direct modes (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 2	`'ANALYTICAL'`	125.8150	0.1 %
Mode 4	`'ANALYTICAL'`	503.2598	0.1 %
Mode 6	`'ANALYTICAL'`	1132.3346	0.1 %
Mode 9	`'ANALYTICAL'`	2013.0393	0.1 %

Rotor in rotation ($\Omega=10^4 rd.s^{-1}$) , retrograde modes (frequencies in Hz)

Identification	Type of reference	Value of reference	Tolerance
Mode 1	`'ANALYTICAL'`	119.7548	0.1 %
Mode 3	`'ANALYTICAL'`	479.0191	0.1 %
Mode 5	`'ANALYTICAL'`	1077.7931	0.1 %
Mode 7	`'ANALYTICAL'`	1916.0765	0.1 %

8 Summary of the results

Good establishment of the gyroscopic effect for the element of beam. Change of axis of the beam x (direction according to which the elements were established) (modeling A) with a direction $x+y$ (modeling B) do not generate variations of results.

In analytical absence of reference for the validation of the discrete elements subjected to the gyroscopic effect, Lbe modelingS C and E allowstent all the same to check the gyroscopic matrix installation of a presumedly indeformable disc. The movement of each disc is fixed by that of the nodes and thus follows the deformation of neutral fibre only in a discrete way. This explains the variations noted on modeling C, all the more for the high modes characterized by a concavity of the more important modal deformation.