SDLL151 - Clean modes of an embedded viscoelastic beam - free

Summary:

The objective of this test is to validate the calculation of complex modes of a beam comprising at the same time a standard elastic material, and a viscoelastic material whose properties depend on the frequency.

1 Problem of reference

1.1 Geometry

Beam of rectangular section, made up of two layers of different materials:

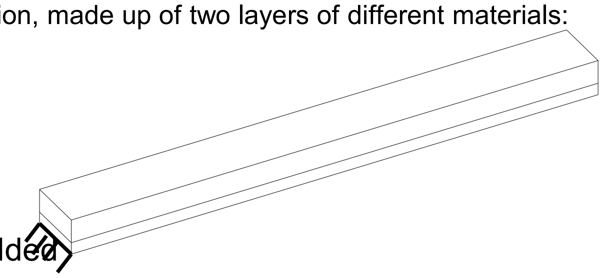

Image 1.1-1: Geometry of the viscoelastic beam.

Width: 0.01 m
Length: 0.15 m
Thickness: Elastic material (n°1 layer of the lower part): 0.001 m
Viscoelastic material (n°2 layer of the top): 0.002 m

1.2 Properties of materials

The material of the n°1 layer of the lower part is elastic isotropic (steel); its properties are constant:

- Young modulus $E=210\ 000\ MPa$
- Poisson's ratio $\nu=0{,}3$
- density $\rho=7800\ kg/m^3$
- damping hysteretic $\eta=0{,}001$

The material of the n°2 layer of the top is viscoelastic (elastomer); some of its properties are dependent on the frequency:

Frequency (Hz)	Real part of the Young modulus E (MPa)	Factor of loss η
1	23.2	1.1
10	58	0.85
50	145	0.7
100	203	0.6
500	348	0.4
1000	435	0.35
1500	464	0.34

Table 1.2-1 : Properties dependent on the frequency of viscoelastic material.

The others are constant:

- Poisson's ratio $\nu=0{,}45$
- density $\rho=1200\,kg/m^3$

1.3 Boundary conditions and loadings

Embedding on a steel edge.

1.4 Initial conditions

Without object (calculation of clean modes).

2 Reference solution

2.1 Method of calculating

The solution is calculated in an analytical and digital mixed way.

The dynamic rigidity (complex) of the viscoelastic beam is calculated according to [1] by the equation:

$$EI^{*}=E_1 I_1\left(1+e_2 h_2^3+\frac{3(1+h_2)^2(e_2\times h_2)}{1+e_2\times h_2}\right) \qquad (1)$$

The significance of the terms of the formula is given in the list below:

E^{*} : Young modulus of the viscoelastic beam
I : second moment of inertia of the cross section of the viscoelastic beam
I_1 : second moment of inertia of the cross section of material of the n°1 layer
E_j^{*} : Young modulus of material of the n°j layer
e_2 : report of the modules E_2^{*}/E_1 (the damping of the material n°1 is regarded as negligible)
H_j : thickness of material of the n° layer$J$
h_2 : report thicknesses H_2/H_1

Depreciation η complex modes are then calculated like the report of the imaginary part of dynamic rigidity on its real part:

$$\eta=\frac{\Im(EI)}{\Re(EI)} \qquad (2)$$

Lastly, Eigen frequencies f_i are calculated according to [2] by the following equation:

$$f_i=\frac{\lambda_i^2}{2\pi L^2}\sqrt{\frac{\Re(EI)}{m}} \qquad (3)$$

With λ_i : modal coefficient (given in [2]) associated with the Eigen frequency f_i
L : length of the viscoelastic beam
m : linear density of the viscoelastic beam

To take into account the frequency response of the mechanical properties of viscoelastic material, depreciation and Eigen frequencies are calculated by an iterative method.

2.2 Sizes and results of reference

One tests the values of the Eigen frequencies and depreciation of some complex modes of the viscoelastic beam.

2.3 Uncertainties on the solution

Digital solution.

2.4 Bibliographical references

[1] A. D. Nashif, D.I.G. Jones, J.P. Henderson, *Damping vibration*. John Wiley and sounds, 1985.

[2] Robert D. BLEVINS PhD, *Formulated for natural frequency and shape mode*, §8.1.2 “Individual-span beams”. Krieger publishing company, Hefty fellow, 2001.

3 Modeling A

3.1 Characteristics of modeling

Viscoelastic material: modeling `3D_SI`.
Material steel: modeling `DKT` (the surface elements are the lower skin of the voluminal elements of the viscoelastic layer).

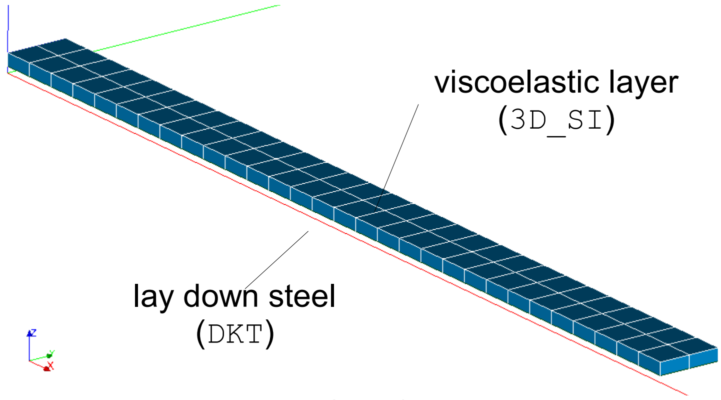

Image 3.1-1 : Grid of the structure.

3.2 Characteristics of the grid

Many nodes 186
Many meshs 376

of which: elements	SEG2	132
	QUAD4	184
	HEXA8	60

3.3 Sizes tested and results

The calculation of the clean modes with the option `TYPE_MODE=' COMPLEXE'` allows to test at the same time the Eigen frequencies and modal depreciation. One tests only the modes of inflection in the thickness of the beam (the n°3 mode is an inflection in the width).

Mode	Eigen frequency (Hz)	Tolerance	Damping	Tolerance
1	33,093	1.0%	0.011782	12.0%
2	211,356	1.0%	0.018138	10.0%
4	601,643	2.0%	0.018834	10.0%

Table 3.3-1 : Values of reference tested.

4 Summary of the results

The got results reveal maximum errors of 1.2% on the Eigen frequencies and 10.4% on depreciation compared to the reference solution.