SDLL401 - Tilted right beam with 20°, subjected with sinusoidal efforts

Summary:

This test is resulting from the validation independent of version 4 of the models of beams.

It makes it possible to check the internal efforts on an inclined beam, for sinusoidal loadings according to time (a modeling with elements POU_D_T, right beam of Timoshenko).

1 Problem of reference

1.1 Geometry

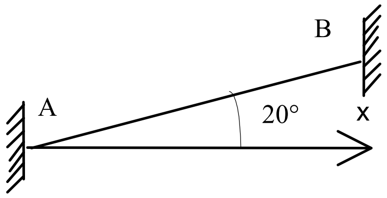

Figure -1.1-a

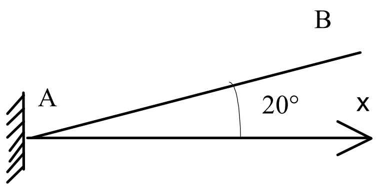

Figure -1.1-b

Right beam length $1\,m$.
slope $20°$ compared to x (trigonometrical direction).

Characteristics of the section:

$S=\pi\times 0.01^2 m^2$

1.2 Properties of materials

Young modulus	$E=2.10^{11}\,Pa$
Poisson's ratio	$\nu=0,3$
Density	$\rho=7800\,kg/m^3$

1.3 Boundary conditions and loading

Boundary condition:

- For the loading distributed [Figure -1.1-a]
  Nodes A and B embedded: DX, DY, DZ, DRX, DRY, DRZ blocked
- For the specific loading [Figure -1.1-b]
  Node A embedded: DX, DY, DZ, DRX, DRY, DRZ blocked

Loadings:

- $f(t)=1000\times\cos(t)$ according to the direction AB
  either distributed or applied at the end B
- $M_T(t)=1000\times\cos(t)$ applied at the end B

2 Reference solutions

2.1 Method of calculating used for the reference solutions

2.1.1 Loading distributed of traction and compression

A right beam length L working only in traction and compression is subjected to a loading distributed constant according to x but varying in a sinusoidal way according to time. It is embedded at its two ends.

$$\begin{cases} \rho S \dfrac{\partial^2 u}{\partial t^2} - ES \dfrac{\partial^2 u}{\partial x^2} = f(t) \\ u(0)=0,\ u(L)=0 \end{cases}$$

To solve, one applies to the equation the transform of Fourier in time:

$$\frac{\partial^2 \hat{u}}{\partial x^2} = -\frac{\rho}{E} 4\pi^2 \omega^2 \hat{u} + \frac{1}{ES} \hat{f}(\omega)$$

$\hat{u}$: transform of Fourier of u,
$\hat{f}$: transform of Fourier of f.

Thus, we have for $f(t)=F\cos(2\pi\omega_0 t)$:

$$u(x,t) = \frac{a^2 F}{ES\, 4\pi^2 \omega_o^2} \left[\left(\cos\left(\frac{2\pi\omega_o}{a} L \right) - 1 \right) \frac{\sin\left(\frac{2\pi\omega_o}{a} x \right)}{\sin\left(\frac{2\pi\omega_o}{a} L \right)} - \left(\cos\left(\frac{2\pi\omega_o}{a} x \right) - 1 \right) \right] \cos(2\pi\omega_o t).$$

with: $a^2 = \dfrac{E}{\rho}$.

The use of the law of behavior gives us the tractive effort compression:

$$N(x,t) = \frac{aF}{2\pi\omega_0} \left\{ \left[1 - \cos\left(\frac{2\pi\omega_0}{a} L \right) \right] \frac{\cos\left(\frac{2\pi\omega_0}{a} x \right)}{\sin\left(\frac{2\pi\omega_0}{a} L \right)} - \sin\left(\frac{2\pi\omega_0}{a} x \right) \right\} \cos(2\pi\omega_0 t)$$

2.1.2 Specific loadings

A beam comforts length L working only in traction compression (or torsion) is subjected to a sinusoidal force in time, (or a moment) applied at its loose lead.

2.1.2.1 Traction

$$\begin{cases} \rho S \dfrac{\partial^2 u}{\partial t^2} - \mathrm{ES} \dfrac{\partial^2 u}{\partial x^2} = 0 \\ u(0) = 0, \quad \dfrac{\partial u}{\partial x}(\mathrm{L}) = \dfrac{1}{\mathrm{ES}} \mathrm{f(t)}. \end{cases}$$

The technique of resolution is equivalent to that of the paragraph [§ 2.1.1.1].

For $f(t) = F\cos(2\pi\,\omega_0 t)$, we have:

$$u(\mathrm{x,\ t}) = \frac{\mathrm{a\,F}}{\mathrm{ES}\,2\pi\,\omega_o} \frac{\sin\left(\dfrac{2\pi\,\omega_o}{\mathrm{a}}\,\mathrm{x}\right)}{\cos\left(\dfrac{2\pi\,\omega_o}{\mathrm{a}}\,\mathrm{L}\right)} \cos(2\pi\,\omega_o\,\mathrm{t})$$

avec $a^2 = \dfrac{E}{\rho}$

and $N(\mathrm{x,\ t}) = F \dfrac{\cos\left(\dfrac{2\pi\,\omega_o}{\mathrm{a}}\,\mathrm{x}\right)}{\cos\left(\dfrac{2\pi\,\omega_o}{\mathrm{a}}\,\mathrm{L}\right)} \cos(2\pi\,\omega_o\,\mathrm{t})$

2.1.2.2 Torsion

$$\begin{cases} G\,I_p \dfrac{\partial^2 \theta x}{\partial x^2} - I_{\theta_x} \dfrac{\partial^2 \theta_x}{\partial t^2} = f(t) \\ u(0) = 0, \quad u(L) = 0 \end{cases}$$

$$G = \frac{E}{2(1+\nu)}$$

$$I_p = \frac{\pi\ 0{,}01^4}{2} m^4, \qquad \theta_x(x,t) = \frac{b\,F}{G\,I_p\,2\pi\omega_0} \frac{\sin\dfrac{2\pi\omega_0}{b}x}{\cos\dfrac{2\pi\omega_0}{b}L} \cos(2\pi\omega_0 t)$$

$$I_{\theta_x} = \rho\,I_p \qquad M_T(x,t) = F \frac{\cos\left(\dfrac{2\pi\omega_0}{b}x\right)}{\cos\left(\dfrac{2\pi\omega_0}{b}L\right)} \cos(2\pi\omega_0 t)$$

avec $b = \dfrac{G}{\rho}$

2.2 Results of reference

Interior efforts (N and MT)

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

1) Report n° 2314/A of the Institute Aerotechnics “Proposal and realization for new cases tests missing with the validation of the beams ASTER”

3 Modeling A

3.1 Characteristics of modeling

The model is composed of 2 elements right beam of Timoshenko.

3.2 Characteristics of the grid

2 elements POU_D_T

3.3 Sizes tested and results

3.3.1 Distributed load in traction

		Analytical results	Tolerance
Normal effort for $x=0$	$t=1/3\,s$	472.478 NR	1E-3 %
	$t=2/3\,s$	392.944 NR	1E-3 %
Normal effort for $x=L/2$	$t=1/3\,s$	0 NR	1E-6 NR (*)
	$t=2/3\,s$	0 NR	1E-6 NR (*)

(*) absolute Deviation

3.3.2 Concentrated loading

3.3.2.1 Loading in traction

Normal effort for $x=0$

	Analytical results	Tolerance
$t=1/3\,s$	944.957 NR	1E-3 %
$t=2/3\,s$	785.887 NR	1E-3 %

3.3.2.2 Loading in torsion

Torque for $x=0$

	Analytical results	*Tolerance*
$t=1/3\,s$	944.957 N.m	1E-3 %
$t=2/3\,s$	785.887 N.m	1E-3 %

4 Modeling B

It is noted that the beam is very stiff on its modes of traction and compression:

$$f_{0traction/compression}=\frac{1}{2\pi}\sqrt{(a)}=\frac{1}{2\pi}\sqrt{(\frac{E}{\rho})}=806\,Hz$$

In comparison, the frequency of the efforts of traction and compression with $\frac{1}{2\pi}Hz$ can be regarded as quasi-static. It is what is made in modeling b: one compares the results of a calculation with the linear operator of statics of Code_Aster with the results of modeling A on the case of the force distributed.

Modeling finite elements being identical to that of modeling A, the results expected are the same ones:

Distributed load in traction

		Analytical results	Tolerance
Normal effort for $x=0$	$t=1/3\,s$	472.478 NR	1E-3 %
	$t=2/3\,s$	392.944 NR	1E-3 %
Normal effort for $x=L/2$	$t=1/3\,s$	0 NR	1E-6 NR (*)
	$t=2/3\,s$	0 NR	1E-6 NR (*)

(*) absolute Deviation

5 Summary of the results

This test makes it possible to check that the internal efforts of the elements of beam in dynamics are correct. The results show very a good agreement with the analytical solution, for a grid only made up of two elements POU_D_T.

It is also shown that for a very stiff system compared to the frequency of the request, a quasi-static calculation gives as good performances as dynamic calculation.